PHYSICS

A TEXTBOOK FOR SECONDARY SCHOOLS

131

PART ONE

First Edition: *February* 1968
Magha 1889

Reprinted: *May* 1972

Jyaishtha 1894

P. U 10 T.

Rs. 5·20

Published at the Publication Unit, by Shri S. A. Abidin, Secretary, National Council of Educational Research and Training, Sri Aurobindo Marg, New Delhi 16 and printed by Today and Tomorrow's Printers and Publishers, Faridabad (Haryana)

FOREWORD

The teaching of science in schools almost the world over, in the wake of World War II and the Sputnik, is undergoing a major revolution. The basic aim is to transform—and it is no easy or quick task—what has been routine and dull teaching into something which is genuinely stimulating and inspiring : To lay special emphasis on the experimental approach, on the open-endedness of science and on its universality, and continuing and deepening creativity.

In our country, the NCERT has made a valuable contribution towards improvement of science education in schools. A most important element in the process has been the active cooperation between university and school teachers. The present textbook of Physics represents a most promising and splendid effort. The Editorial Board consisted of Dr. A. R. Verma, Director of the National Physical Laboratory, Delhi, Dr. B. D. Nagchaudhuri, Director of the Saha Institute of Nuclear Physics, Calcutta (now Member of the Planning Commission, New Delhi), and Dr. R. N. Rai, Head of the Department of Science Education, NCERT. Our deep gratitude goes to them and to their many associates who have, I know, spared no pains in the preparation of this book. All the same, it must be regarded as the beginning, the first step, in the long and continuing process to place in the hands of school teachers and students the best textbook that Indian physicists and teachers could provide, making full use of the developments and experience of other educationally advanced countries. The effort to prepare such a book is in the nature of an exciting research project. It involves experimental trials of the book in selected schools and a feedback from teachers and students to those engaged in its preparation.

In the modern world, physics is one of the most exciting and exhilerating of all subjects. This is true both from the point of view of the understanding and insight which physics gives regarding the working of nature, and also from the point of view of its applications to enrich man's material life. Any one who finds himself engaged in the study of physics should regard himself as truly fortunate. It must be our constant endeavour and obligation to ourselves and to our subject to continually improve and deepen our understanding and to contribute to the advancement of knowledge. To learn physics is to do physics—to think and to experiment—and this applies to all of us, teachers and students in universities, colleges and schools. There is no other way of learning physics.

I wish the book success. And a real measure of its success would be that the present book is soon replaced by a still better book; and this would need not only a continued effort on the part of the distinguished authors of the book, but also a response, active cooperation and help from school teachers and school students.

D. S. KOTHARI

PREFACE

For the last few years all over the world there have been attempts to change and reform the teaching of physics. For example, the Physical Sciences Study Committee in USA has brought out a book: similar attempts have been made in UK, Australia and other countries. It is recognized that each country must bring out suitable books taking into account the conditions existing there. Accordingly the work of writing a textbook of physics for secondary schools was started by a panel in 1963 with Dr. D. S. Kothari as Chairman. The panel met several times where we discussed not only the contents but also the approach that should be adopted in the new textbook.

As is well known, knowledge in science doubles itself over a period of 20 years. With the development of physics, the textbooks continue getting fatter and fatter, packing in more and more pieces of information. But the time at the disposal of the student and teacher is limited. Therefore we must first recognize the need for a change both in the courses and the methods of teaching. What should these changes be? A few general principles which have come out as a result of discussion with several colleagues are illustrated below.

First, we must not omit the fundamentals in favour of the details. The fundamentals of physics must be introduced very early. One is surprised to see how quickly a fresh young mind understands the fundamentals of a topic like Bohr's model of the atom. Of course the elaborate mathematical details are not to be given at school level. Once the fundamentals are clear, the details of any specific topic may be read by the student himself later on. For example, it may be interesting to know how a thermometer is manufactured or the details of the dozen different kinds of calorimeter based on just one principle. One can give many such examples. But if these details are going to come in the way of learning fundamentals, *e.g.* that heat is a molecular motion or introducing the laws of thermodynamics, there can be no two opinions that the details should be left out.

A connected point is that one must resist the temptation to make the textbook a complete and self-sufficient book which will not require the help of practical physics book or a numerical examples book, or a teacher's guide, etc. As an example, while giving the principle of a spectrometer the exact details of its adjustments and its applications should be left for a practical physics book.

The second most important principle is not to teach the student anything that is incorrect, or tell him half the truth. Surprisingly this is what has very often been done. We tell the students at high school that light travels in straight lines, and later that it does not. We then teach them that it consists of waves and later still, that it has a particle aspect and light has a dual nature. Take the case of magnetism. We first teach as though magnetic forces are quite separate from electric forces and something fundamentally different. We even invent hypothetical poles that do not exist: only when we come to electromagnetism is the student told that magnetism is after all only an effect of the electric current. The whole point is that we must teach every topic in the light of the correct knowledge as we have it today. Thus the limitation of Newton's Laws of Motion should be stated. The student must be made to understand what is an experimentally verified result, what is the conclusion or an assumption or a theory or conjecture or what is not clearly understood even today. His sense of wonder and curiosity must not be dulled by presenting to him results as if they are gospel truths.

One may ask that there is hardly anything which is taught at high school level that is not fully and clearly understood. This is far from true. Atoms and the forces between them, the behaviour and properties of liquids, the elastic and plastic properties of solids, gravitational forces, and so on are all described to them and they must realise the gaps in our knowledge and its incompleteness. It is no compliment to science teachers that most young persons with fresh and curious minds after attending lectures for a few years begin to lose their creative spirit and the spirit of enquiry. Any book or lecture that arouses this spark has served its purpose. The student will do the rest.

The third principle is that we must teach in a logical sequence rather than a historical sequence. Physics has not developed in the rational manner in which it should be taught. To give one pertinent example, we use several different systems of units which arose for historical and geographical reasons and this causes a lot of unnecessary difficulties. We have, therefore, adopted throughout in this book one rational system of units, *viz.* MKSA system which is accepted internationally.

There are several other points to be borne in mind. Physics should be made to appear real and closely linked with everyday life. Therefore, lots of examples from life in India should be given. Similarly, the laboratory experiments described should be simplified so as to be within the means of a school. These and several other ideas should be the guiding principles in writing this new textbook.

The members of the panel who wrote the different parts of the book are as follows :

1.	Mechanics	Shri B. P. Srivastava
2.	General Properties of Matter	Dr. C. K. M. Murty

3.	Heat	Prof. S. Ramaseshan
4.	Optics	Prof. S. Chandrasekhar
5.	Electricity	Prof. A. R. Verma
6.	Magnetism	Prof. V. G. Bhide

The first draft of some of the parts especially Mechanics, Electricity and Magnetism and Optics was discussed at several sittings of the panel. In place of the panel, the system of an Editorial Board was introduced and I was given the task of Chief Editor. Prof. B. D. Nagchaudhury kindly agreed to help us and agreed to edit the portions on Heat, Optics and Waves. Dr. R. N. Rai and I have edited the Mechanics and General Properties of Matter. In this we have been helped by Dr. H. S. Mani The editing has been time-consuming and general properties of matter has been modified almost to the extent of rewriting. The book is being brought out in parts and it is hoped that the entire book will soon be out.

It would have been ideal if some school teachers had also been associated with this task but that has not been practicable. Therefore, we welcome suggestions from the teachers so that by their experience, improvements may be made. Indeed, they may themselves like to rewrite the book.

A. R. Verma

SECTION I

MECHANICS

Contents

CHAPTER 1

What Physics Deals with

What is Physics

Physics deals with the fundamental properties of matter and energy. A major achievement of physics has been to show that all matter consists of atoms. The study of the constituents of atoms has received great attention in recent years. It has led to the discovery of many new kinds of fundamental particles.

Physics provides us not only with the knowledge of the fundamental laws of nature and an insight into the natural phenomena, but it also has been of the greatest service in the development of engineering sciences and of industry. The tools of physics and the experimental techniques which have been developed in the study of physics now find application in almost every branch of science.

Physics has always been a most exciting subject, but perhaps it has never been more so than in the world of today. In the present century physics has undergone a profound revolution due largely to the pioneering work of Max Planck, Albert Einstein and Niels Bohr. Indian scientists, C. V. Raman, Meghnad Saha and others, have made a notable contribution to modern physics.

Ancient science was largely qualitative and speculative. What distinguishes modern science from ancient science is the close connection between theory and experiment and in no branch of science this connection is more intimate than in physics. The theories in physics make use of mathematics; often the mathematics is simple, but sometimes the most developed concepts and techniques in mathematics are employed.

With the growth of science, instruments and experimental techniques are becoming more elaborate and expensive, but some spectacular work has been done with relatively simple apparatus. This is true even now. For learning physics, as also for research in certain fields, it is possible to do a great deal with simple equipment, provided it is used intelligently and with care.

Physics today can be called the fundamental science of the natural world. The study of physics is basic to a proper understanding of chemical phenomena and it is finding increasing applications in the study of the biological world.

If we take a spring and stretch it slightly and then leave it, the spring regains its original length. Similarly, if we stretch a wire to a small extent and leave it, the wire regains its original length. These are examples of application of force to change the shape of a body. Application of force always changes the shape and size of a body or changes its position or it may do both. In the above two examples, the force may have been applied by our muscles or by hanging a small weight. The second kind of force is an example of what we call the gravitational force or the force arising out of the attraction of the earth. But there are other kinds of forces and physics deals with the nature and application of all these forces.

When we strike a bell, it produces sound which we hear. The sound is transmitted through the air. But it can be transmitted through liquids and solids as well Sound requires a medium for its transmission. But the light that we receive from the sun and the stars, and other sources or light does not require a medium for its transmission. Visible light is one form of a radiation which is called electromagnetic radiation. Sound and electromagnetic radiation are different forms of energy and are propagated in the form of waves. Physics studies the different kinds of waves, their propagation through different media and their interaction with matter.

In the rainy season you may have observed the rainbow with its beautiful colours. The same colours you may have observed when a drop of oil spreads over the surface of water. You may also have observed that the sun is dazzling white during the day but is reddish when it is rising in the morning and setting in the evening. The explanation of all these interesting phenomena is included in the study of physics.

We are now using electrical energy on a very large scale. This energy is mainly generated by burning coal and oil. But you might have read in the papers that India is building two power stations (one at Tarapur in Maharashtra and the other near Kota in Rajasthan) where atomic energy (more properly nuclear energy) will be converted into electrical energy. Physicists have, through their study of the atom and its nucleus, been able to unravel the secrets of the atomic nucleus and utilize its energy for useful purposes

Physics is concerned with the study of all these interesting topics which deal with laws according to which matter reacts with matter or energy. Thus physics may be called the science of matter and energy.

Although Physics is one whole and can not be compartmentalized into different parts, it is divided into different subjects like mechanics, heat, *etc*. for the sake of convenience of reference.

The development of physics and physical instruments has had a tremendous influence on other sciences and technology. The study of the atom and its structure has thrown a new light on the arrangement of atoms in the periodic table and the nature of valency and the chemical bond. The study of radioactivity and radioactive elements now enables the chemist to detect substances in as small quantities as 10^{-10} to 10^{-20} gram. The application of X-ray and neutron diffraction methods and nuclear magnetic resonance methods has been responsible for the elucidation of complex chemical structures. These methods have enabled the biochemists to understand the structure of nucleic acids which may allow us to control the vital processes of life activities.

The use of the optical microscope had already revolutionized the study of biology. The more powerful electron microscope has

made it possible to penetrate inside the tiniest structures of a cell. The irradiation of fungi and bacteria produces new useful strains which can form antibiotics, enzymes and vitamins widely used for practical purposes.

The optical telescope enabled Sir William Herschel to add a planet to the five planets known to the ancients Coupled with the photographic plate, it made possible many fundamental researches in astronomy. Now the application of the radio-telescope has enabled the astronomers to see up to the farthest limits of the universe and detect new sources of radiation like the quasars which may throw new light on cosmology.

Scientific Method

Science is a body of organized knowledge about nature and how it functions. It is, thus, a record of man's accomplishments. It is, however, far more than description It is an approach to problems, a way of thinking. There can be several methods to solve a problem.

To attack a problem it is necessary to define it precisely and to formulate some hypotheses or tentative solutions. To select the most probable hypotheses different ways or methods such as observation, exploration, experimentation and the like are used. On the basis of the data so collected, conclusions are made. It is always necessary to verify conclusions before arriving at generalizations. Science thus demands freedom from bias and the ability to judge facts objectively.

This book is intended to initiate you to science, the methods of science and to the process of unfolding the secrets of nature so that you may realize how the present structure of modern science has been built up little by little by generations of men who combined experimental skill and logical thinking in establishing what are called laws of science.

In ancient times the progress of science was rather slow because scientists working in different countries could not easily communicate with each other. However, with improvements in the means of communication, the tempo of scientific progress in recent times has gone up considerably. Thus scientists from different countries have been able to co-operate so as to piece together their findings Modern science is, therefore, an international activity It cannot be segregated within boundaries of narrow nationalism.

For example, atomic energy could be harnessed in 1945 only as a result of a collective effort of scientists like Einstein and Meitner from Germany, Fermi from Italy, Bohr from Denmark and many others from England, Canada and USA who pooled their experience and knowledge, and worked together for a number of years in America.

In the end it is important to remember that the progress of science is a continuing process. As soon as old problems get solved, new ones crop up. As our knowledge about nature increases, new horizons open up presenting us with fresh problems. It will be the privilege of the present generation to provide an answer to these problems. We hope this book will help you to prepare for this fascinating task.

Questions

1. What problems are studied under Physics ?
2. What is the difference between an experimental Physicist and a theoretical Physicist ? Give a few examples under each class.
3. How is Physics related with other branches of Natural Science ?
4. What is the difference between a Scientist and a Technologist ?
5. How has Physics helped in the advancement of Astronomy ?
6. Distinguish between Physical Sciences and Biological Sciences.
7. How will you demonstrate that measuring instruments are more reliable than our senses ? Hence distinguish between subjective measurement and objective measurement.
8. Physics is an objective method. Comment on this statement.
9. Why is Physics called an exact science? Illustrate your answer with examples.

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CHAPTER II

Units of Physical Quantities

2.1 Units

In physics, as in other sciences, we are often required to make measurements. Suppose we want to measure the length of a room. For this we take a metre scale and lay it along the length of the room successively. If we find that the length of the room is four times that of the metre scale used as the measure, we say that the length of the room is 4 metres. A measurement then means the comparison of an unknown quantity with some chosen quantity of the same kind. The latter is called the *unit*

In the example given above, the metre scale has been used as the unit. The length of the room has been compared with the length of this unit—the metre rod. Thus the measurement involves two things, the numerical measure called the *magnitude*, and the quantity chosen as the *unit* The magnitude tells us how many times the length of the metre is contained in the length which has been measured, while the unit gives us the name of the standard employed for comparison. For example, if we express the length of a table as four without attaching any unit to it, the statement makes no sense.

In the example of the measurement of the length of a room, we could have arbitrarily chosen any suitable length as our unit of length. Indeed this is what had been done in different parts of the world, *e.g.*, the length of a human hand used to be a common unit in India for the measurement of length. Other examples of units of length are the yard in England, and the toise in France. However, if there are different units of length in different parts of the world, it will be inconvenient for exchange of scientific information. Therefore, by international agreement in 1889, the metre was adopted as the unit of length

Now suppose that we wish to determine the area of the floor of the room. For this again we could arbitrarily choose any area, say the area of a sheet of paper, as the unit. As before we lay down this sheet successively so as to cover the whole floor. If the floor can be covered by laying down the sheet 72 times, we say that the area of the floor is 72 units of area. However, having chosen

the unit of length, it was not necessary to arbitrarily choose the unit of area, because area is length times breadth, each one of which is measurable in terms of the metre The unit of area could be chosen an area x metres in length y metres in breadth But it is more convenient to choose 1 metre square as unit of area. Thus the unit of area has been derived from the unit of length. The unit of length is called a fundamental unit and the unit of area is a derived unit.

Fundamental units

If we now wish to measure time, you will see that it is not possible to do so in terms of length described above. We have therefore to choose arbitrarily some interval of time as the unit of time. This is another fundamental unit like the unit of length. Now, the question arises how many different units we have to choose to express conveniently the different physical quantities which we will come across in physics. For quantities encountered in mechanics it is convenient to choose three fundamental units, *viz.* the units of length, mass and time. Other units in mechanics are derived from those units and are called the derived units. For instance, the unit of volume is expressible in terms of $(\text{length})^3$ and the unit of speed as the unit of length divided by the unit of time.

2.2 MKS System

The internationally agreed fundamental unit of length is the **metre**, the fundamental unit of mass is the **kilogram** and for time the unit used is the **second**. Hence the system is called the MKS system. Some books use as the fundamental units the *centimetre*, *gram* and *second*. This system is called the CGS system. But in this book the MKS system will be followed.

(Three additional fundamental units are introduced in some other branches of physics. These are units of temperature, intensity of light and the strength of electric current. These will be considered later.)

2.3 The Metre

The standard metre was defined in 1889 as the distance between *two lines drawn near the ends* of a platinum-iridium bar (at 0°C) preserved in the vaults of the International Bureau of Weights and Measures at Sevres, a suburb of Paris. Originally the metre was intended to be the length

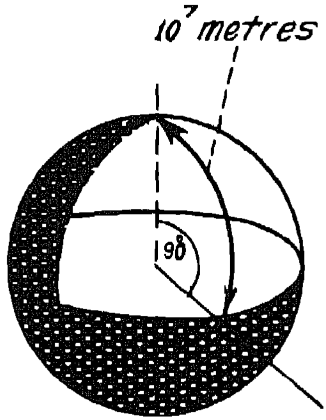
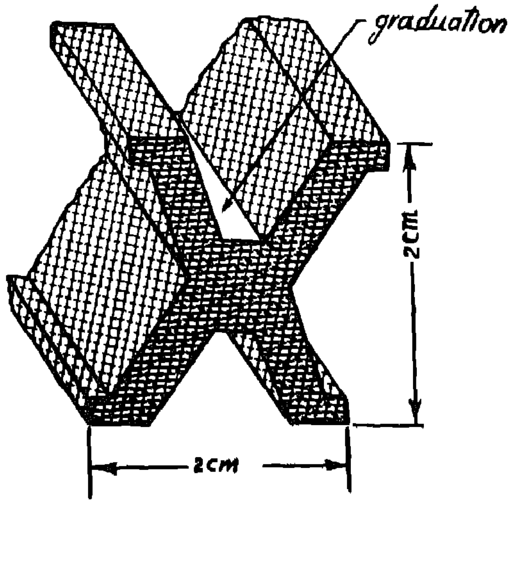


FIG. 2.1. *The distance from the pole to the equator is about* 10^7 *metres.*

FIG 2.2. *A sectional view of the International Prototype Metre in the National Physical Laboratory, Delhi*



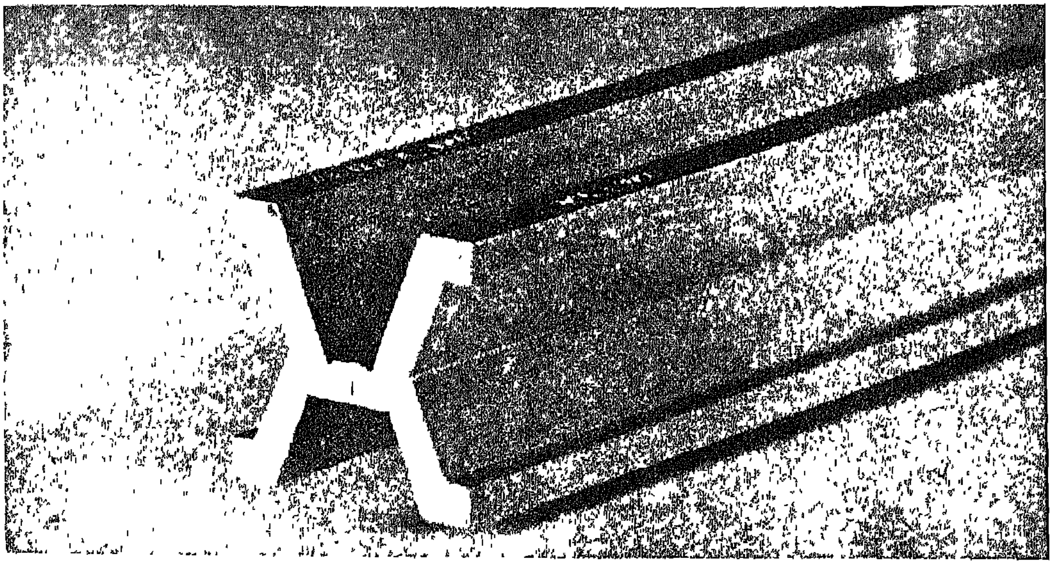


FIG. 2·3. *Photograph of the International Prototype Metre in the National Physical Laboratory, Delhi*

equal to one ten-millionth part of the distance from one of the earth's poles to the equator. But now we know that the standard metre is a bit shorter than one ten-millionth part of the distance mentioned above. Actually, the distance from the pole to the equator is equal to 1,00,00,880 standard metres.

A copy of the above International Prototype Metre is owned by the National Physical Laboratory, Delhi. This National Prototype Metre is made of platinum-iridium alloy. It is the official standard of length in India.

Scientists had long been concerned about the problem of reproducing the metre correctly. It appeared highly desirable to evolve an alternative definition, which could enable one to realize the metre in any scientific laboratory without recourse to the reference bars. So a more precise definition of the metre has been obtained as given below.

We know that light is a wave motion. The wave length of light (the distance from one crest to the adjacent crest) can be taken as the measure of length for defining the metre. Thus in 1960 the International General Conference on Weights and Measures redefined the metre as the length equal to 16,50,763·73 wave lengths of the orange colour light produced by a vapour lamp containing the gas krypton (86)[1].

To express quantities which are the multiples and sub-multiples of the units, it has become customary to use Greek and Latin prefixes. These are given in the Table 2.1.

TABLE 2.1

deca for 10^1	deci for 10^{-1}
hecto for 10^2	centi for 10^{-2}

1. The wave length of krypton has been chosen as the unit because it is sharper than any other light emitted in the ordinary way. It operates at a very low temperature (–210°C) and therefore the broadening due to the Doppler effect, about which you will read later, is negligible. Physicists have now invented a device known as the **laser** the light from which can be used to define the metre more accurately than the light from wave length krypton (86). The wave length of this light is thousands of times sharper than that of krypton (86). It may be that in future the metre may be defined in terms of a laser wave length.

kilo for 10^3	milli for 10^{-3}
mega for 10^6	micro for 10^{-6}
giga for 10^9	nano for 10^{-9}
tera for 10^{12}	pico for 10^{-12}

The millionth part of a metre has been given a special name the micron and is expressed by the symbol μ.

In optics we generally use a still smaller unit the Angström ($\mathring{A}$).

1 Angström $= 10^{-8}$ cm $= 10^{-10}$ m.

MKS unit of volume

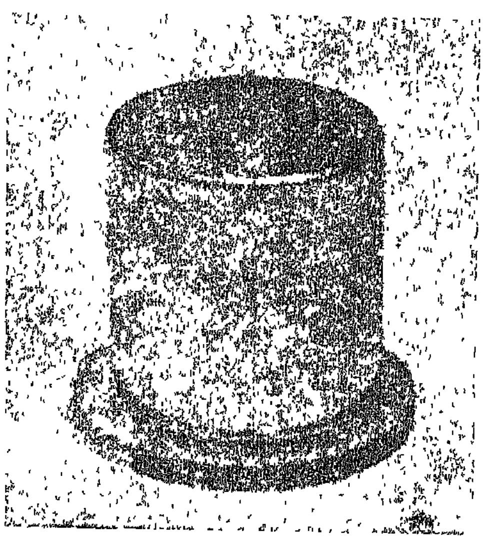
The unit of volume is the cubic metre. This has been derived from the metre. Thus a cubic metre is the volume of a cube whose edges measure 1 metre each.

A convenient unit of volume often used is the litre which is equal to one cubic decimetre $= 10^{-3} m^3$. A thousandth part of a litre is one cubic centimetre (cc), also called one millilitre (ml).

2.4 The Kilogram

The standard kilogram (kg) is the mass of a certain platinum-iridium cylinder which is preserved in the vaults of the International Bureau of Weights and Measures. A kilogram is very nearly equal to the mass of one litre (1,000 cc) of water at 4°C and at one atmosphere pressure.

FIG. 2.4. *Photograph of the standard Kilogram in the National Physical Laboratory, Delhi*



As the name indicates, 1 kilogram is equal to 1,000 grams. So that for all practical purposes 1 g can be taken as the mass of 1 cc of water at 4°C.

2.5 The Second

To measure a given time interval we must have a unit of time. This unit must be based on an event which takes the same time in repeating again and again. Naturally, scientists, in this connection, thought of the rotation of the Earth about its axis. The rotation is reckoned with reference to the Sun. Thus, the solar day is the time interval that elapses while the sun appears to move from its noon position on one day to its noon position the next day at any point on the surface of the earth. This solar day then

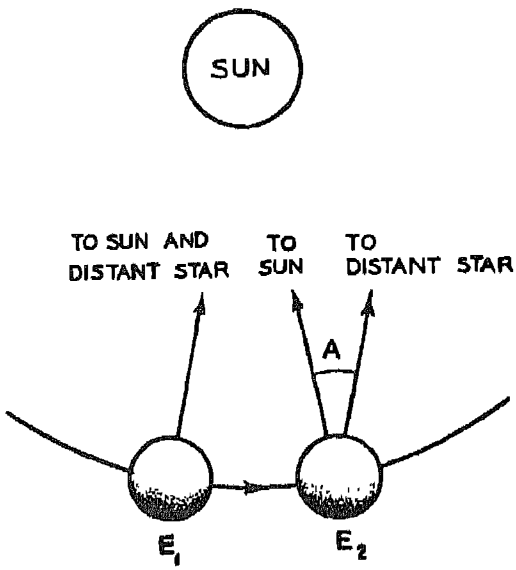


FIG. 2.5. *The solar day is slightly longer than the sidereal day as the earth has to rotate through about 361° around its axis, so that the Sun is exactly overhead at any place on the earth on two consecutive days.*

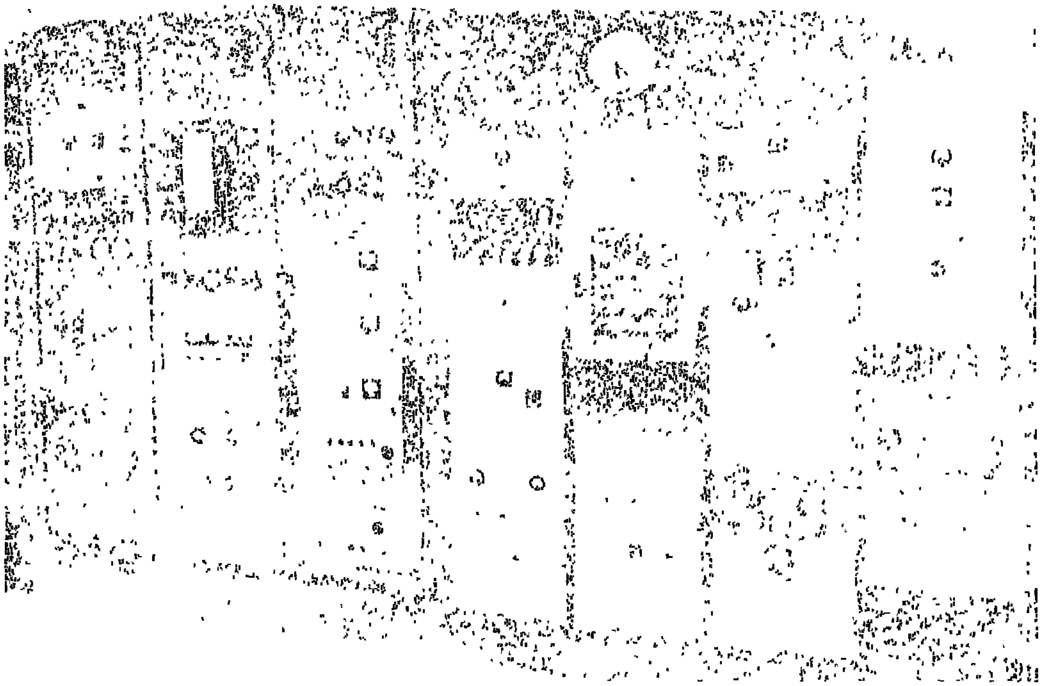


FIG. 2.6. *Quartz clock unit at National Physical Laboratory, Delhi.*

forms the basis for defining the unit of time. As a day is divided into 24 hours, the hour into 60 minutes and each minute into 60 seconds, we can say that 1 solar day contains $24 \times 60 \times 60$ seconds or,

$$1 \text{ sec} = \frac{1}{24 \times 60 \times 60} \text{ solar day},$$
$$= \frac{1}{86400} \text{ solar day}.$$

More accurate measurement of time has become possible using the quartz crystal clocks and atomic and molecular clocks. The atomic and molecular clocks can keep time with an accuracy of 1 second in 300 years.

But the duration of a solar day is not constant throughout the year Thus, the solar day in September is shorter than the solar day in December by about 1 minute[1]. To overcome the defect due to this variation we take the mean solar day, which is the average value of the solar day taken for the whole year[2]. As we have 365·2422 days in a year (tropical year), we have $365 \cdot 2422 \times 24 \times 3600 = 31{,}556{,}926$ seconds in a tropical year. But even the tropical year, varies slightly from year to year. Hence, the General Conference on Weights and Measures, Paris, adopted (on October 14, 1960) the unit of time, the second as 1/31,556,925·9747 part of the tropical year 1900.

1. The day from noon to noon in September is 23 hours 59 minutes 39 seconds of a mean solar day whereas in December it is 24 hours 30 seconds.

2. The tropical or equinoctial year is the time between two successive returns of the Sun to the vernal (spring) equinox which occurs around 21st of March In addition we have the sidereal year and the anomalistic year. The observation of accurate intervals of time is the duty of astronomical observatories. In actual practice, it is intervals of sidereal time that are directly observed and afterwards converted into intervals of mean solar time by division by 1·00273790926. This is because for the successive passages of a star across a line running north and south which passes directly overhead, the earth rotates on its axis through 360° but must rotate through 361° to bring the Sun over head again. The sidereal day is therefore shorter than the mean solar day and is equal to 23 hours 56 minutes 4·09054 seconds of a mean solar day.

Classroom Activities

Construct a second's pendulum.

Questions

1. What is the advantage of expressing numbers in powers of 10 ? Express the results in the following cases in powers of ten :
   (a) $5000 + 700000$; (b) $0 \cdot 00003 \div 0 \cdot 000000002$; (c) $0 \cdot 000012 \times 0 \cdot 0000008$.
2. Calculate the number of seconds in 60 years and express the result in powers of ten.
3. What is meant by sidereal time ? How does the sidereal second differ from the solar second ?
4. Give the result in powers of ten :
   (a) $2 \cdot 4 \times 10^5 \times 6 \cdot 0 \times 10^2$
   (b) $1 \cdot 6 \times 10^{-2} \div 1 \cdot 2 \times 10^{-6}$
   (c) $4 \cdot 0 \times 10^8 - 3 \cdot 0 \times 10^8$
   (d) $5 \cdot 2 \times 10^7 + 2 \cdot 0 \times 10^7$.
5. Express the following in powers of ten :
   (a) The number of milligrams in 10 kilograms
   (b) The number of microseconds in 1 hour.
6. Given 1 pound $= 453 \cdot 6$ g express 10 kilograms in pounds.
7. Why is the temperature specified in defining the metre ?
8. What do you understand by the term Local Time and the Indian Standard Time ?
9. Why is the platinum-iridium alloy used to define the kilogram and the metre ?

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CHAPTER III

Measurement of Length

3.1 Measurement of Small Lengths—Use of Vernier Callipers

We have seen in the previous chapter that one can use a metre scale for measuring the length of a room It is not always convenient to use a metre scale For example, it is very tedious to use a metre scale to measure the distance from Delhi to Agra, or to measure the width of a river. Again, it is impossible to measure the thickness of a piece of paper (which is also a measurement of length) with a metre scale. To suit the varying circumstances, we use different methods and instruments. We will study, in this chapter, some simple methods and a few common instruments used in the measurement of length.

While measuring the length of a given straight line, AB, the metre scale (which you know is graduated into cm and mm) is laid along the line in such a way that the end A of the line touches some fixed mark on the scale. The position of the end B is then observed on the scale. The difference between the readings at the two ends of the line gives the required length. In taking the readings at A or B, we must keep our eye vertically above the points as shown in figure 3.1. If the eye is displaced either to the left or to the right, then an error will be produced in the reading. This error is known as error due to parallax.

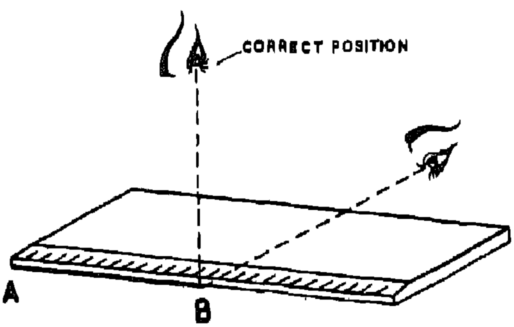


FIG. 3 1 *Error due to parallax.*

You will learn later about atoms, molecules and their parts which are very small in size indeed (10^{-8} cm). For measuring still smaller distances we have to use methods about which you will learn later.

The smallest division on a metre scale is that of a millimetre. Suppose we set out

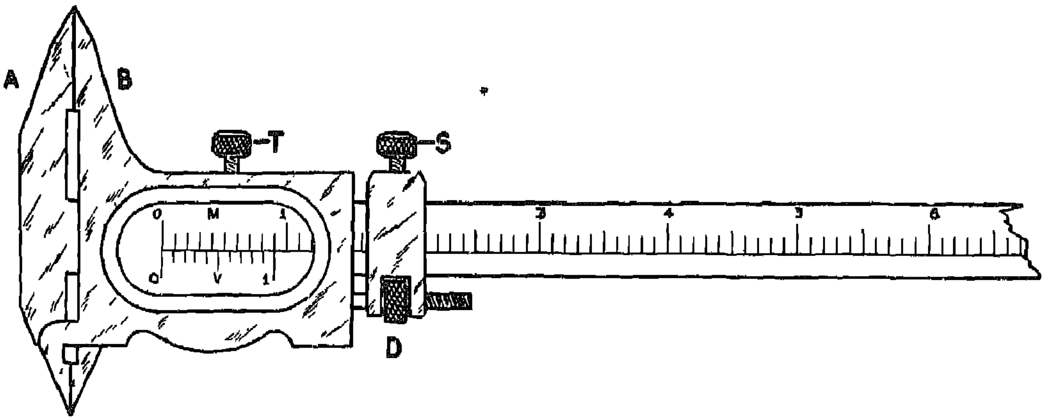


FIG 3 2(a). *Vernier calliper.*

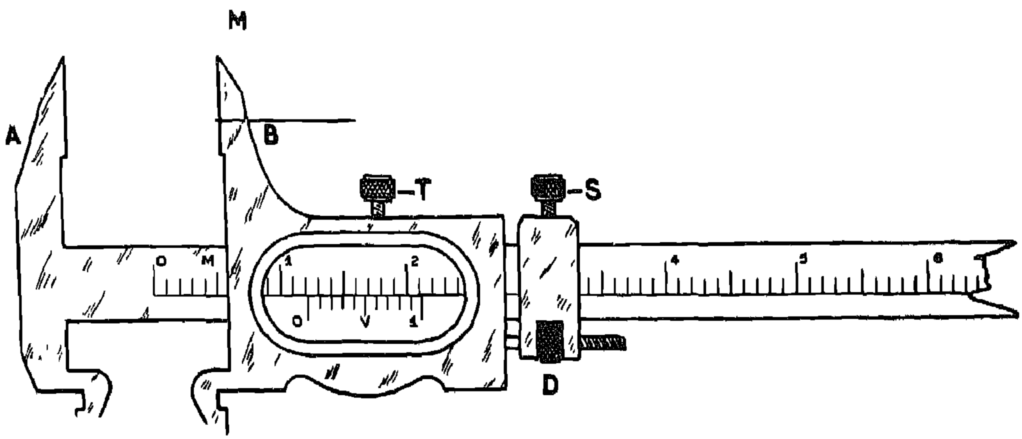
to measure the length of a pencil with the help of a metre scale. Perhaps we find that its length is more than 11·4 cm but less than 11·5 cm. Obviously with the metre scale we cannot find what fraction of a millimetre beyond 11·4 cm is included in its length.

In order to do so we make use of the vernier calliper invented by a French scientist Pierre vernier in the beginning of the 17th century.

Vernier calliper

A vernier calliper is shown in fig 3.2a. In this M is the main scale graduated in millimetres and centimetres. At one end of the main scale is attached the jaw A which is at right angles to the scale. The other jaw B carries the vernier scale V both of which can slide together alongside the main scale (Fig 3.2b). When the two jaws are in contact, the zero mark of the vernier scale coincides with the zero mark of the main scale. T is the clamping screw. On tightening this screw the vernier can be fixed in position. If we keep screw T loose and tighten screw S, then by working the nut

FIG 3.2(b). *Vernier sliding along side the main scale.*



D attached to the tangent screw, the vernier can be given a slow motion.

Principle of the vernier scale

The main scale M is divided into centimetres and millimetres. V is the vernier which can slide along the side of the main scale. On the vernier we have 10 equal divisions such that their total length is equal to the length of 9 of the smallest divisions of the main scale (*e.g.* mm) (fig. 3.3). Obviously,

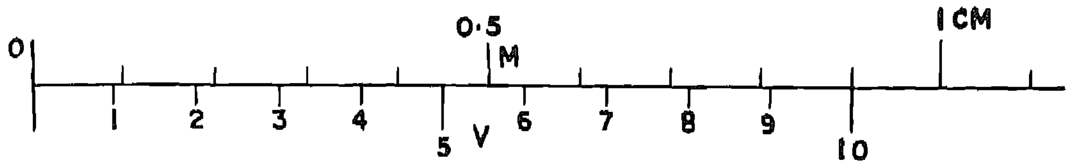


FIG. 3.3. *Vernier scale.*

10 div. on the vernier scale=9 div. of the main scale

1 div. on the vernier scale=9/10 of a division of the main scale=0·9 mm.

The difference between one division on the vernier scale and one division on the main scale=(1 – 0·9) mm=0·1 mm.

This quantity is called the least count of the vernier or the *Vernier Constant.*

General rule

Suppose there are n divisions on the vernier scale. These may be equal to n–1 divisions on the main scale. Therefore one vernier division is equal to

$$\frac{n-1}{n} \text{ or } 1-\frac{1}{n} \text{ scale division.}$$

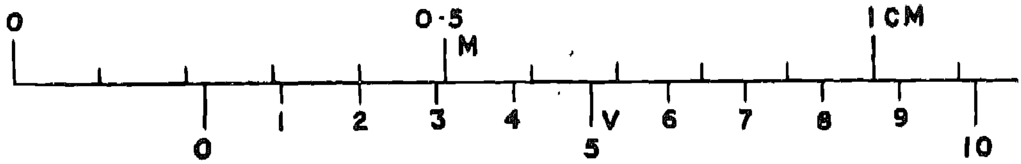
Therefore the least count is $\frac{1}{n}$ of a main scale division.

Suppose such a vernier scale has slid into the position shown in Fig. 3.4. We have to read the correct position of the zero mark of the vernier scale as indicated on the main scale The zero line lies a little beyond the 2 mm line of the main scale. This is the main scale reading

If you examine the lines, you will find that as you go away from the zero line, the lines of the two scales become closer and closer together up to a certain point and beyond it the lines separate out. Now, find the line on the vernier scale which coincides best with one of the lines on the main scale. In the figure we find that it is the second line after the zero mark on the vernier scale which coincides with one of the lines on the main scale—and that it is the only line which does so.

Now, each division of the vernier scale is shorter than the division on the main scale by 0·1 mm. We see that the second line on the vernier coincides with a line on the main scale, so the first line on the vernier will fall at a distance of 0·1 mm to the right of the nearest line on the main scale, and the zero line of the vernier will similarly fall at a distance of 0·2 mm on the right of the corresponding line of the main scale.

FIG. 3.4. *Vernier scale in a new position.*



Thus, the zero line of the vernier scale is at a distance of 0·2 mm to the right of the corresponding line on the main scale.

So the vernier reading	=0·2 mm
The main scale reading	=2·00 mm
Total reading	=2·2 mm
	=0·22 cm

In short, to take the reading on the vernier scale :

1. First find the value of the smallest division on the main scale.
2. Then find the number of divisions on the vernier scale. The value of the smallest main scale division divided by the number of divisions on the vernier scale gives the vernier constant or the least count.
3. Take the reading on the main scale.
4. Next find which line on the vernier coincides best with a line on the main scale. Suppose it is the mth line then $m \times$ *vernier constant* will give the vernier reading. Add it to the main scale reading. This will give the total reading at which the vernier scale has been set.

How to use the vernier calliper

Suppose the length of a small block of wood is to be measured. Open the jaw to allow the block to slide in it freely. Close the jaws so that the block is held between them. The jaws should just touch the end faces of the block.

Take the main scale reading and then the vernier reading in the manner described above The sum of the two readings will give the required length.

To obtain greater accuracy in determining small lengths the main scale is sometimes divided into half millimetres and the vernier carries 50 small divisions.

The smallest division of the main scale

$=\frac{1}{2}$ mm$=0\cdot 5$ mm,

vernier constant $=0\cdot 5/50=0\cdot 01$ mm,

$=0\cdot 001$ cm,

so that in this case the accuracy is 10 times as great as that of the simple vernier scale described earlier.

In some verniers the total length of the n divisions of the vernier scale may be equal to (mn–1) divisions of the main scale. *Argue and show that the value of the vernier constant is obtained by dividing the value of a main scale division by the number of divisions on the vernier scale.*

Zero error of the vernier calliper

When the two jaws of the vernier callipers are in contact, the zero mark of the vernier scale should exactly coincide with the zero line of the main scale However, in some callipers the vernier zero may not coincide with the scale zero. If the vernier zero stands to the right of the scale zero, the instrument is said to possess a zero error which may be called a positive zero error. This zero error must be determined and then it should be accounted for while taking readings with this instrument.

If the mth line of the vernier coincides with one of the lines of the main scale, then the positive zero error is m times the least count. This must be subtracted from the reading of the instrument while measuring any given length.

It is very seldom that the vernier zero falls to the left of the main scale zero when the jaws are in contact. If it is so, the zero error is called *negative.* In such a case the zero error will be equal to (n–m) × least count where n is the number of divisions of the vernier and the mth line of the vernier coincides with one of the lines on the main scale. This zero error is negative and subtracting it from the observed value amounts to adding it to the reading of the instrument

provided that the error is less than one scale division.

Hence, we may formulate that the zero error is always to be subtracted from the observed reading taking into account the sign of the zero error.

In case this negative error is more than one scale division, the jaws should be reset by adjustment till the zero line of the vernier stands within one scale division to the left of the zero of the main scale division.

3.2 The Micrometer Screw Gauge

With an ordinary vernier scale lengths can be measured accurately up to 0·1 mm. To increase the accuracy further, we use another instrument called the micrometer screw gauge It enables us to measure small thicknesses with an accuracy of 0·01 mm or even of 0·005 mm.

Principle of the micrometer screw gauge

Examine a screw on which a very accurate thread has been cut. This means that spacings between consecutive turns of the thread are the same throughout. This spacing which is equal to the distance between similar points on consecutive turns of the thread is called the *pitch of the screw.*

In the micrometer screw gauge there is a screw Y with an accurate thread cut on it

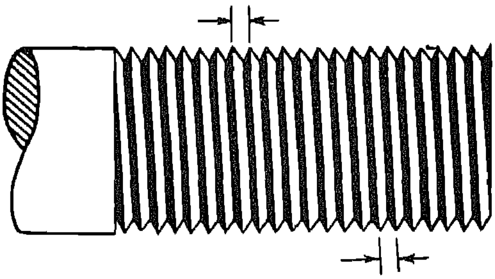
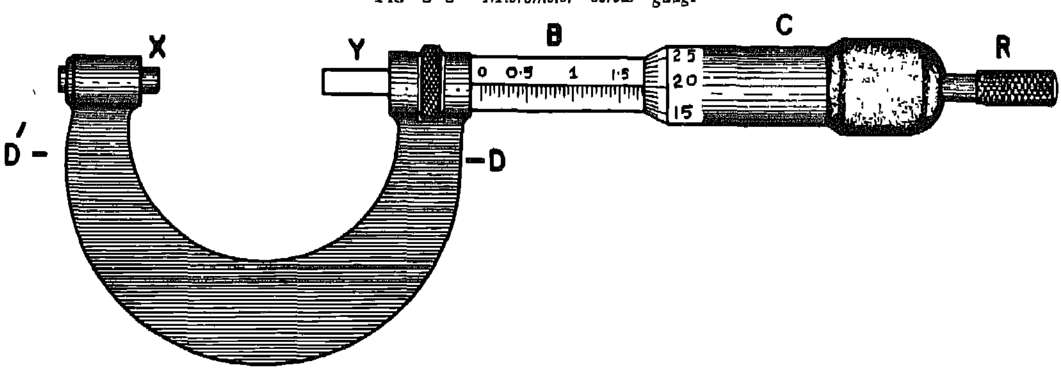


FIG. 3.5. *Pitch of the screw*

(Fig. 3 6) It can move inside a nut attached to the end D of a U-shaped metal piece. On giving the screw one full turn in the clockwise direction the tip of the screw will move forward through a distance equal to the pitch of the screw. The arm D of the U-shaped piece is attached to a hollow cylindrical shaft B on which a scale is engraved parallel to its axis showing millimetres and half millimetres. The head of the screw carries a collar C which rotates with the screw. The rim of this collar is usually divided into 100 equal divisions and these divisions are numbered from 0 to 100.

The arm D of the U piece carries the butt end X which is bevelled as shown in the figure. When the collar is rotated clockwise, the screw advances towards the butt end. When the two touch each other, the zero mark on the rim of the collar should

FIG 3 6 *Micrometer screw gauge*



coincide with axial line at the beginning of the millimetre scale.

Working of the screw gauge

The collar head is given one full rotation so that the zero mark once again coincides with the axial line. Suppose the rim has moved a distance of 1 mm on the scale, then the pitch of the screw is 1 mm. If the collar-head is rotated through one division only, the tip of the screw will move through $\frac{1}{100}$ mm. This is called the least count of the screw gauge. So that the least count

$$=\frac{\text{Pitch}}{\text{No. of divisions on the collar head.}}$$

In order to measure, say, the diameter of a piece of wire, the screw is opened out and the wire is inserted between X and Y. Now the screw is rotated till the wire is held gently between the butt end and the tip of the screw. The reading on the millimetre scale is noted. Suppose it is 2 mm. Next, we see which of the marks on the collar-head coincides with the axial line. Suppose it is the 25th mark. Then, the collar-head reading will be 25 times the least count. Thus the diameter of the wire

$$=2 \text{ mm}+25\times0\cdot01 \text{ mm}$$
$$=2\cdot25 \text{ mm.}$$

Care should be taken to avoid tightening the screw too hard on the object held between the tip of the screw and the butt end. For this purpose a free wheel device R is attached to the head which allows the head to turn freely when the pressure on the object held increases beyond a certain minimum. This device R is called the Ratchet head.

The screw should be worked in by rotating the ratchet head and the reading should be taken when the ratchet head begins to turn freely.

In the case of the micrometer also there may be a zero error as in the case of vernier callipers. Its error should be determined and the necessary correction made.

Backlash

Due to wear and tear sometimes a slackness occurs in the fit of the screw and the nut. This causes what is called the *backlash*. If such a screw is adjusted to any position by turning it, say, in the forward direction and then it is turned in the opposite direction, the collar head has to be rotated through some angle before the screw begins to actually move along the axis. To avoid this backlash, the readings should be taken by rotating the screw only in one direction.

3.3 Spherometer

This instrument is used for measuring the thickness of thin plates and the radius of curvature of surfaces. It can be used even when a part of a spherical surface is available, as in the case of a spherical mirror or that of a lens.

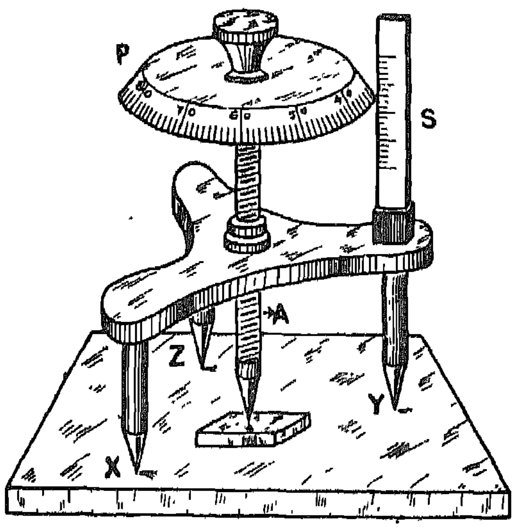


FIG. 3.7. *Spherometer.*

The instrument consists of a triangular frame supported on three legs X, Y and Z

the lower ends of which form the corners of an equilateral triangle (Fig. 3.7). At the centre of the triangular frame is fixed a nut through which passes a screw A. A large disc P is attached to the head of the screw. Its circumference is usually divided into 100 equal divisions A scale S graduated in millimetres or half millimetres is fixed vertically to the triangular frame. The rim of the disc P lies just along the side of this scale so that its position on the vertical scale can be easily read. The vertical scale is so adjusted that the zero mark on it coincides with the zero mark on the disc P when the lower tip of the screw is in the same plane as the lower tips of the three outer legs.

Working of the spherometer

The pitch of the screw and the least count of the spherometer are determined as in the case of screw gauge.

(i) *To find the thickness of a thin plate of glass*

For using the spherometer a glass plate with a plane surface is necessary Place the spherometer so that the three outer legs rest on the plate. Raise the central screw far enough to allow the given small piece of the glass plate, the thickness of which is to be measured, to come under it. Now work the central screw downwards till its tip just touches the piece of glass Take the reading of the spherometer. Next, remove the piece of glass from under the central screw and work the screw downwards till it just touches the glass plate. As before, take the reading of the spherometer.

On subtracting this reading from the first reading, we get the thickness of the given piece of glass. To avoid backlash the readings should be taken so that all the time the screw is rotated in one and the same direction.

(ii) *To find the radius of curvature of a spherical surface*

(a) Let the given surface be a convex one. Screw up the central screw and place the spherometer on the surface so that the three outer legs rest upon it. Next work the central screw down till it just touches the surface. Take the reading of the spherometer. Now place the spherometer on a plane sheet of glass and work the central screw downwards till it just touches the plate. Take the reading again.

On subtracting this reading from the first one we get a value say h

Now the formula $R = \left(\frac{l^2}{6h} + \frac{h}{2}\right)$

(derived below) gives R the radius of curvature of the surface. Here l is the average distance between the three outer legs.

(b) If the given surface is a concave one, then first take the reading on the plane sheet of glass and then on the concave surface The difference between the two gives h. Using the above formula we can find R, the radius of curvature of the concave surface Has the backlash error been avoided in this measurement? Explain.

The formula

When all the outer legs and the central screw touch the convex surface (Fig 3·8) the tip of the central screw is at a height h above the plane containing the three outer legs. Figure 3.9 represents the plane containing the lower ends of the outer legs. C is the position of the point where the central screw touches this plane when adjusted on a plane sheet of glass. Let a be the distance between the central screw and one of the three outer legs.

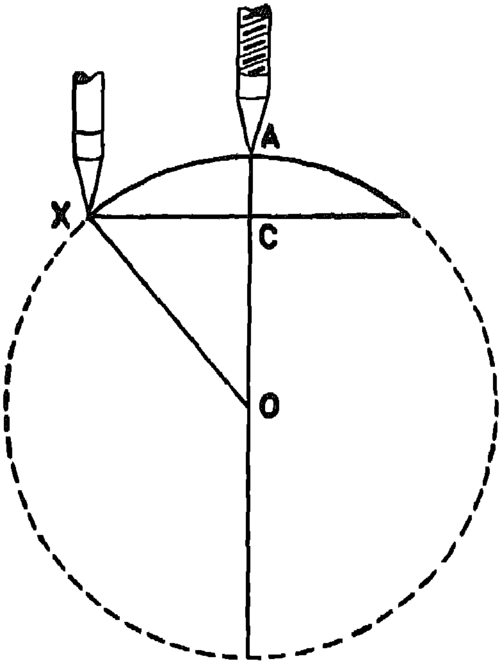


FIG 3 8 *Convex surface and the points where the outer legs and the central screw of the spherometer touch it.*

In Fig. 3.8, XC$=a$ and XO$=$R$=$the radius of curvature; also AC$=$h.

It is easy to prove that :

$$R=\frac{a^2}{2h}+\frac{h}{2}.$$

Now, from Fig. 3.9

$$a=\frac{XY}{\sqrt{3}}=\frac{l}{\sqrt{3}},$$

where X Y$=l=$the distance between two outer legs.

So we have $R=\frac{l^2}{6h}+\frac{h}{2}$.

Measurement of long distances

Suppose a distance of 5 kilometres is to be measured along the road This could be done with a chain or tape of known length. But a more convenient method would be to roll a wheel from one end to the other end of the distance and count the numbers of revolutions made. Multiplying this number with

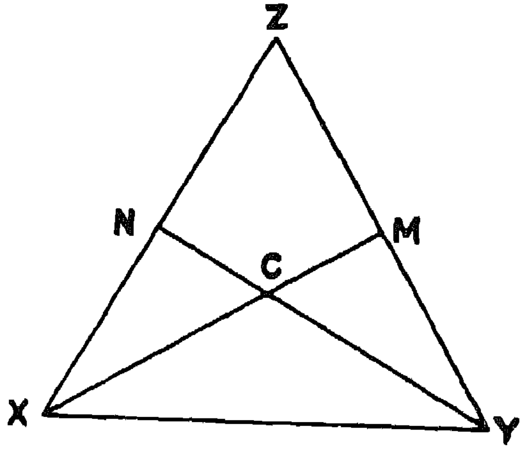


FIG 3 9.

the circumference of the wheel we will get the distance. Or better, a mechanical counter can be attached to the wheel. A similar device attached to the motor car measures directly the distance travelled.

3.4 Method of Triangulation

Suppose we have to measure the width of a river. We can do so without going across the river by a method known as the method of triangulation. The observer chooses some landmark, say a tree A, on the opposite bank.

He further selects two points B and C some distance apart along the bank on his own side of the river. Now he measures ∠ABC and ∠ACB. These angles may be measured with the help of a theodolite (described later). Finally he measures the distance BC also.

Next, on a piece of paper he constructs a triangle DEF (Fig. 3.11) such that EF

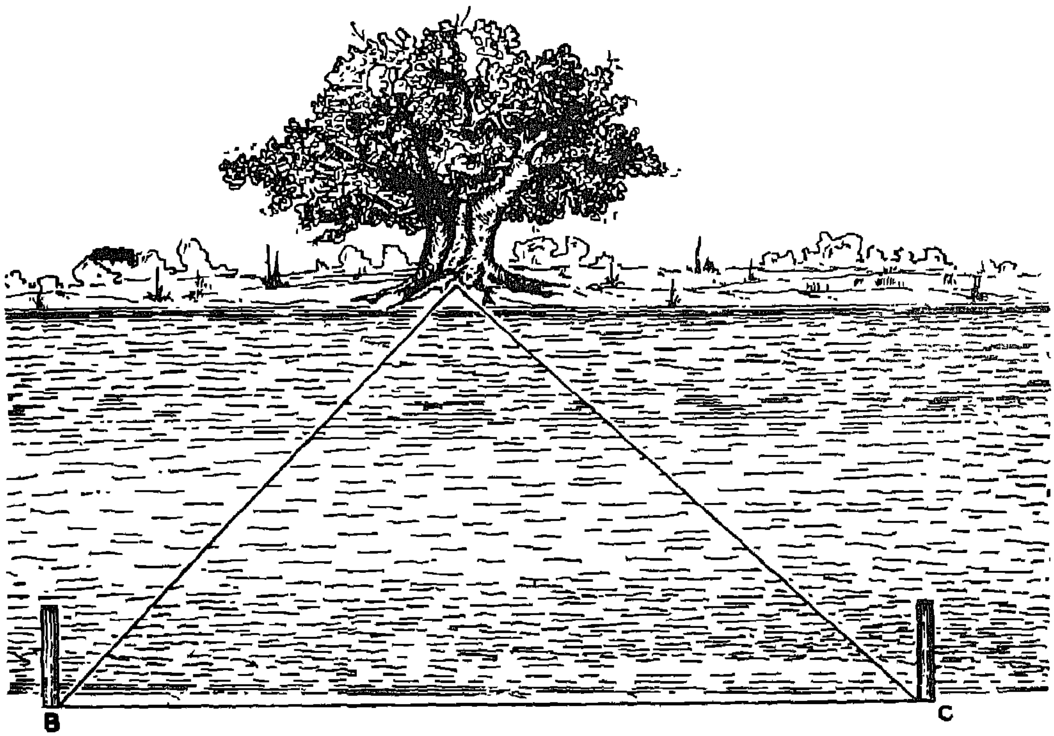


FIG. 3.10. *Measuring the width of the river by triangulation.*

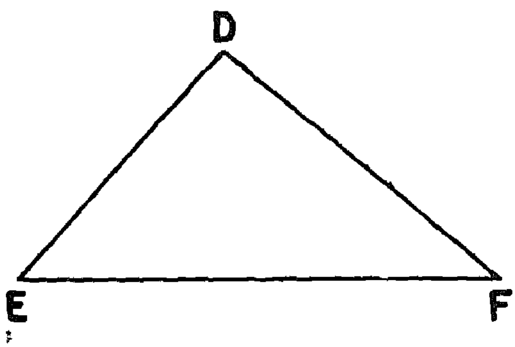


FIG. 3.11.

represents on a suitable scale, the side BC while $\angle DEF = \angle ABC$ and $\angle DFE = \angle ACB$. He measures the length of the perpendicular from D on EF It represents the *width* of the river on the same scale.

This method is called the method of triangulation. The same method is used to measure the height of a building or the height of inaccessible peaks like Mount Everest. The most recent determination of the height of Mount Everest gives its height as 8,847·7 metres (29,028 ft.).

Astronomers make use of this method for measuring the distance of the moon from the earth. Two points P and Q on the surface of the earth are selected (Fig. 3.12). The straight line joining these points is our base line corresponding to BC above. From the ends of this base line we measure the directions of a definite spot on the surface of the moon. This spot M corresponds to point A of the example given above. In actual practice the position of the moon *relative to the stars* is observed from two

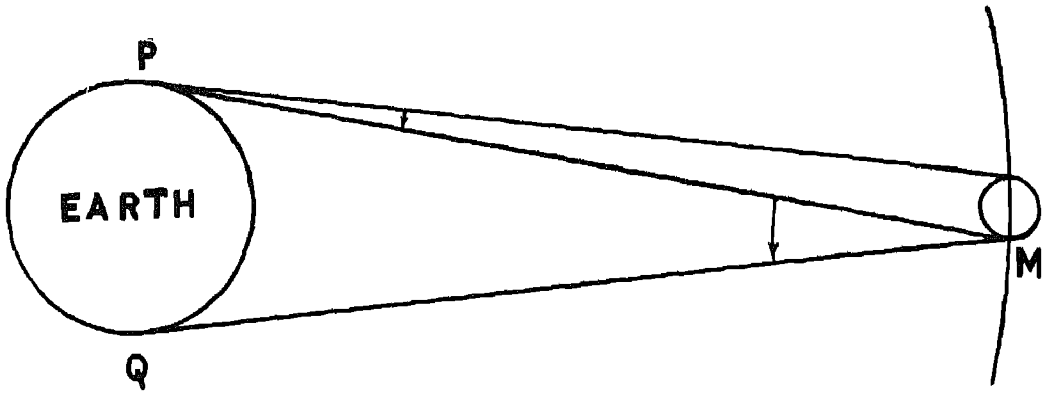


FIG 3.12. *Measuring the distance of the moon from the earth by triangulation.*

distant points on the surface of the earth to determine the angle subtended by the base line at a point on the moon. As before, the distance of the moon from the base line can be easily determined.

The distance between these points has to be a few thousand kilometres as the base line has to be suitably large in order to measure long distances accurately.

In principle, the same method can be used for determining the distance of the sun from the earth, only the base line taken in this case must be longer. Even the distance of the nearer stars can be determined by a method based on this principle. As the stars are a long way off, we have to take a base line which is very long indeed. Even the distance between the two farthest points on the surface of the earth is not sufficient for this purpose. In this case we take the base line as the line joining the two positions of the earth in its orbit round the sun six months apart (position of summer solstice and winter solstice). The apparent displacement of the nearer stars with respect to the more distant stars is determined. The angle subtended at the star by the distance of the earth from the sun is known as the parallax for the star. For details of the measurement of distances of stars you may consult some book on Astronomy. Since the stars are at a very long distance from the earth, a very large unit is employed in expressing their distances. The unit is called the *light-year* It is equal to the distance travelled by light in one year. We know that the velocity of light is 3×10^8 m per second.

Therefore 1 light-year $= (3 \times 10^8 \times 31, 556, 926)$ m $= 9 \cdot 5 \times 10^{12}$ km (approx.)

Distances of about 6,000 nearer stars have been determined by this method. The distances of the more distant stars have to be determined by optical methods. In passing we may note that the nearest star Proxima Centauri is at a distance of 4·15 light-years from the earth.

3.5 Orders of Magnitude

In science very often we like to have a very approximate value for a measurement so as to be able to compare it with other measurements. The approximate value is expressed as the power of ten nearest to the given value of the measurement. When the approximate value is so expressed it is called *the order of magnitude.*

As an example, the number 9 is nearer to 10^1 than to 10^0. So the order of magnitude of 9 is taken as 10^1.

TABLE 3.1

Distances from the earth of certain bodies

	Order in metres	Distance
Distances too large to be measured by geometrical methods	10^{26}	Farthest galaxy photographed
	10^{22}	Distance to the nearest galaxy
	10^{20}	Distance between the sun and the centre of our galaxy, the Milky way
	10^{19}	Distance to the North star
	10^{18}	Greatest distance measurable by parallax
	10^{16}	Distance to the nearest star—Proxima Contauri
	10^{13}	Distance of Neptune from the Sun
	10^{11}	Distance of the Earth from the Sun
	10^{10}	Distance of Mercury from the Sun
	10^{9}	Radius of the Sun
	10^{8}	Average distance of the Moon from the Earth
	10^{7}	Radius of the Earth
	10^{6}	Radius of the Moon
	10^{3}	One Mile
	10^{2}	Length of a football field
	10^{0}	Hockey stick
	10^{-1}	Width of your hand
	10^{-2}	Thickness of a pencil
	10^{-3}	Thickness of a window pane
	10^{-4}	Thickness of a piece of paper
	10^{-5}	Diameter of red blood corpuscle
Distances too small to be measured by geometrical methods	10^{-6}	Average distance between successive collisions of molecules of air near the surface of the earth
	10^{-7}	Thickness of an oil film on water
	10^{-8}	Average distance between molecules of air in a room
	10^{-9}	Size of a molecule of oil
	10^{-10}	Average distance between particles of a crystalline solid
	10^{-12}	Average distance between atoms in the centre of densest stars
	10^{-14}	Size of the largest atomic nuclei
	10^{-15}	Diameter of proton

NOTE . Adopted from *Physics* prepared by the Physical Science Study Committee, M.I.T.

Again 125 is closer to 100 than it is to 1,000 So that the order of magnitude in this case is 10^2

Similarly $0 \cdot 002 = \frac{2}{1,000}$ is closer to $\frac{1}{1000}$ than it is to $\frac{1}{100}$ or to $\frac{1}{10,000}$. Therefore the order of magnitude here is 10^{-3}.

Take the distance of the moon from the earth. It is about 3,80,000 km$=3 \cdot 8 \times 10^5$ km This value is nearer to 10^5 than to any other power of ten. So the order of magnitude is 10^5 km

Thus the order of magnitude gives a rough and ready estimate of the quantity under consideration.

The orders of magnitude of distances along with the methods employed for measuring them are given in Table 3.1.

Classroom Activities

1. Using two pieces of cardboard, make a vernier which can measure length correct to the nearest tenth of a centimetre.
2. How will you construct a vernier scale reading to the nearest twentieth of a centimetre ?

Questions

1. A circular scale on a spectrometer is graduated to read angles correct to one-third of a degree of arc. How will you construct a vernier so as to enable you to read angles correct to a minute of the arc ?
2. Explain the terms : Pitch of a screw and the Least Count. A micrometre screw gauge has 20 threads to a centimetre. How many divisions should be engraved on the barrel rim so that its least count be $0 \cdot 001$ cm ?
3. Derive the formula $R = \frac{a^2}{2h} + \frac{h}{2}$, where 'a' is the distance between the central leg and outer leg of the spherometer
4. The graduation on the main scale should be correct to a distance equal to the least count, otherwise the vernier will give faulty readings Explain why.
5. You are given two glass plates and a vernier calliper. Use the two plates to determine the zero error of the vernier calliper.
6. A spherometer has 100 divisions on the disc. By merely increasing the size of the disc and increasing the number of divisions to say 10,000, is it possible to improve the accuracy of measurement by the spherometer 100 times ? Discuss the reasons.
7. The spherometer has three supporting legs. Other instruments (such as camera stand, spectrometer, physical balance, etc.) also have only three supporting legs. Discuss the reason.
8. Supposing that your measuring instrument can measure angles with an accuracy of $1'$ of arc, and you have to measure the width of river which is about 1 km. What base line will you choose to achieve an accuracy of 1% in the measurement?
9. You are required to measure the distance of a star by the method of triangulation and the instrument you are supplied with can measure angles up to an accuracy of $0 \cdot 01''$ of arc. If you choose a base line on the earth 5,000 km long, what is the maximum distance of the object you can determine ? What is the maximum

distance of a star which you can measure using this method, when the base line is the diameter of the earth's orbit (3×10^8 km) with the above accuracy ?

10. The radius of curvature of a spherical surface is about 100 cm, and the least count of your spherometer is 0·001 cm. What should be the distance between the legs of the spherometer so that the radius of curvature can be determined with an accuracy of 1% ?

Further Reading

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CHAPTER IV

Measurement of Time

4.1 Unit of Time

We defined in chapter 2 the unit of time as 'second', we shall now discuss the methods of measuring intervals of time. For the earliest man the smallest unit of time was probably a day. With the advance of science it has now become necessary to measure very short intervals of time. Scientists have devised techniques which enable them to measure time with an accuracy of 1 in 10^{10}.

The earliest time-piece on record is the water clock. It worked on the principle that equal amounts of water take the same time in flowing through a small opening. In Roman times the hour-glass was used for measuring intervals of time. In this device sand ran from one portion of a dumb-bell shaped (like the shape of damaroo) container (held vertically) to another through a hole connecting them. The time taken for sand to run from one container to the other represented their unit hour. Hence the phrase, the sands of time are running out. Candle clocks were also used in those days. The entire length of the candle was marked at equal distances so that it took the same time in burning from one mark to the next The sun-dial was used to record time in the days of Alexander the Great (300 B.C.). In this device a thin plate called gnomon is mounted at the centre of a horizontal circle with the plane of the gnomon in the north-south vertical and its edge parallel to the axis of rotation of the earth, *i.e.* its edge points to the north pole in the northern hemisphere and to the south pole in the southern hemisphere. The shadow of the edge of the gnomon marks the hours on the dial.

4.2 The Pendulum

The first real advance in the science of time measurement was made by the great Italian physicist Galileo. It is said that while watching a big lamp swinging from the ceiling of a church, Galileo in 1581 noticed that the lamp appeared to take the same time to complete each swing in spite of the fact that the amplitude decreased with each swing. Since watches were not available in those days, he counted his pulse beats

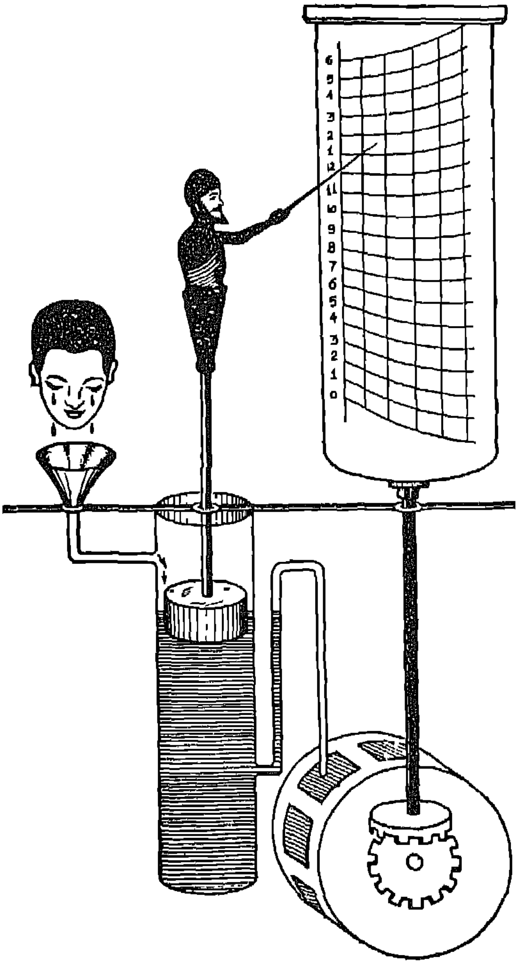


FIG. 4·1(a). *Ancient water clock.*

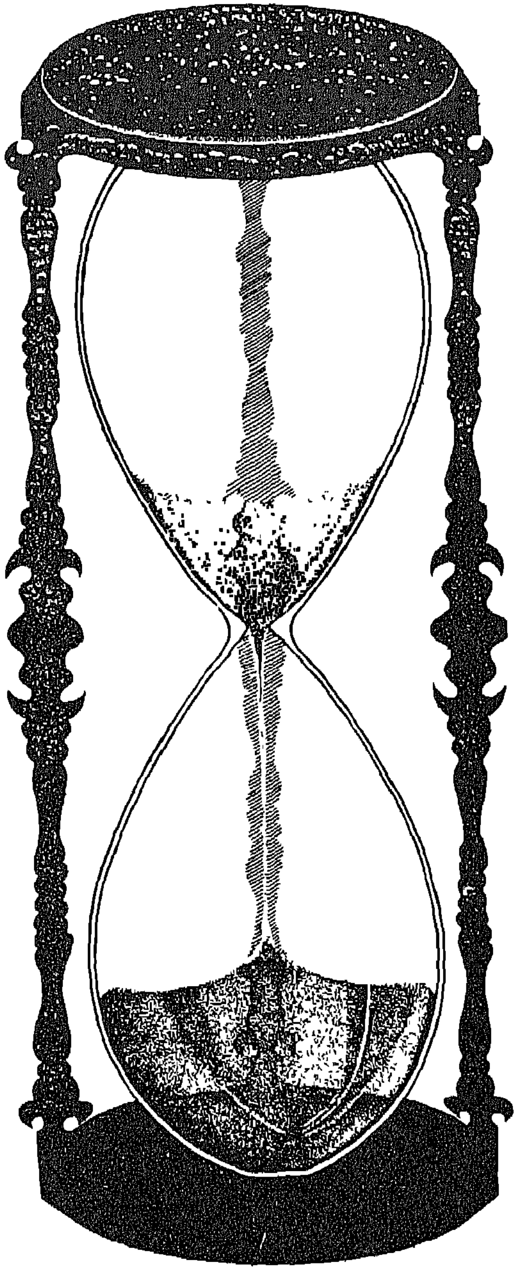


FIG. 4·1(b). *A sand clock.*

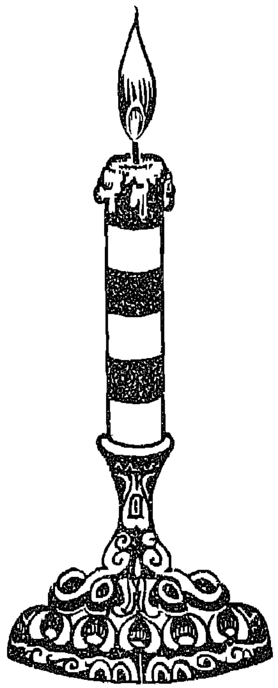


Fig. 4.1(c) *A candle clock*

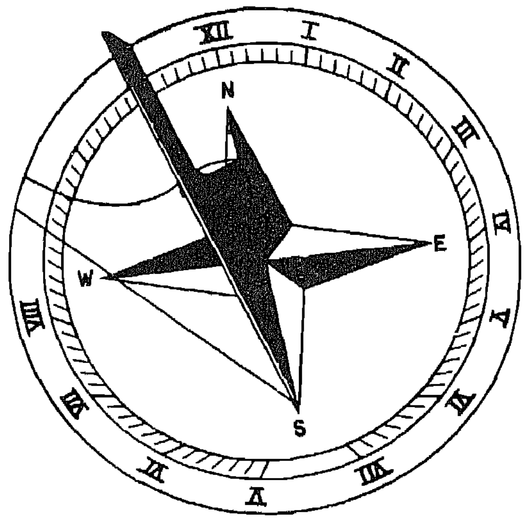


Fig. 4.1(d). *Sun dial.*

to determine the time of the swing of the lamp. He found that the same number of pulse beats marked the duration of each swing.

This observation of Galileo led to the idea that if a pendulum could be kept swinging by a spring or a slowly falling weight, it could be used for measuring time. To make a very simple pendulum a heavy piece of metal is tied to one end of a piece of light string. The other end of the string is fixed to a rigid support and the weight allowed to hang freely. Such a pendulum when allowed to oscillate, takes the same time to complete each oscillation. This time for one oscillation is called the period of the pendulum. The piece of metal, called the bob, is said to complete one oscillation when it goes from B to C, from C to D and comes back to B (Fig. 4 2).

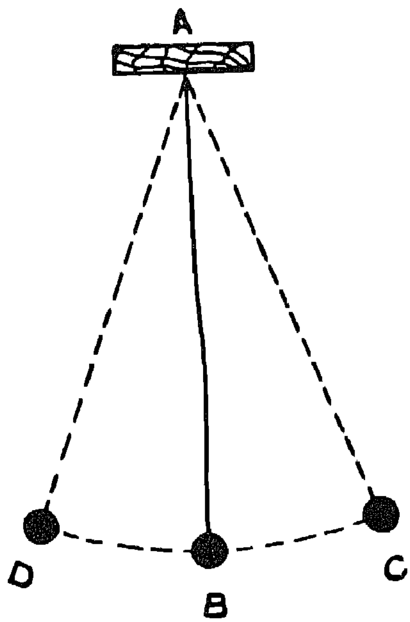


Fig. 4.2. *Simple pendulum.*

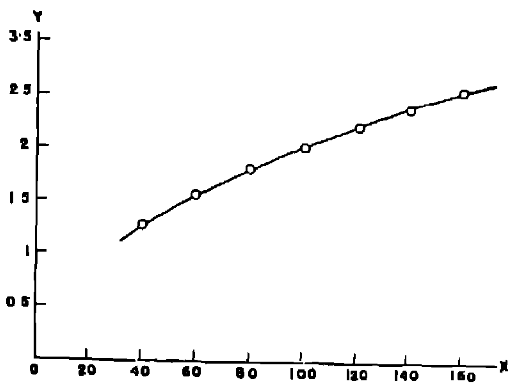
Laws of oscillations of a simple pendulum

1. Attach a mass of 50 g to one end of a piece of thin string about a metre long and suspend it from a stand placed on a table so that the pendulum hangs freely. Pull the bob to one side and then release it gently so that it oscillates in one plane.

With the help of a stop-watch find the time it takes to complete say 20 oscillations and thus obtain the time for one oscillation. Next repeat the experiment taking masses of 100 g, 150 g and 200 g as the bob, keeping the length of the string fixed and find the period in each case. You will find that the period of the pendulum is the same; it does not depend upon the mass or the material of the bob provided the length of the pendulum remains unchanged. The total length of the pendulum is the distance from the point of suspension to the centre of gravity of the bob. We conclude from this experiment that the time of oscillation of the pendulum does not depend upon the mass or the material of the bob.

2. Now take the same bob (spherical) but attach it by turns to strings of different lengths so that the lengths of the pendulums are 40 cm, 80 cm and 160 cm. It is to be kept in mind that the length of the pendulum is the distance from the point of support to the centre of gravity of the bob.

FIG. 4·3. *Graph between the time period and the length of pendulum*



Oscillate the pendulum taking care that each time the bob is pulled to one-side through a small distance only. Determine the time for 20 oscillations and find the period for each pendulum. Actual values of the period obtained from an experiment of this kind performed in the laboratory are given below in a tabular form :

Length of Pendulum	Period
40 cm	1·27 sec
60 cm	1 55 sec
80 cm	1·79 sec
100 cm	2·00 sec
120 cm	2·20 sec
140 cm	2·37 sec
160 cm	2·54 sec

Now plot a graph between the period and the length. Take the period on the Y-axis and the length on the X-axis (Fig. 4·3). The graph shows that the period T increases with the length, but it is not doubled when the length is doubled. Again when the length is trebled from 40 to 120 cm, the period also increases from 1·27 sec to 2·20 sec but it is not trebled.

Now prepare a table between the square root of the length and the corresponding period. We get the following Table :

$\sqrt{l}$ in $\sqrt{\text{cm}}$	T in sec
6·325	1·27
7·746	1·55
8·944	1·79
10·00	2·00
10·95	2·20
11·83	2·37
12·65	2·54

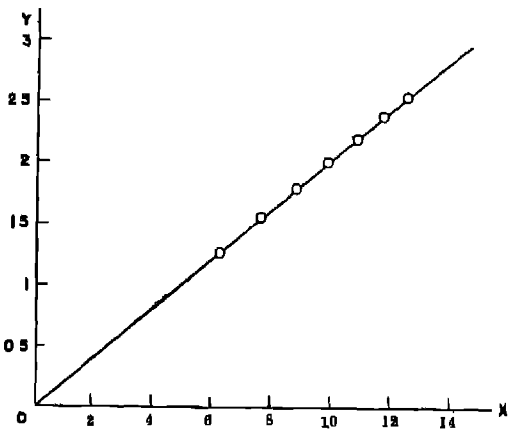


FIG 4.4. *Graph between the time period and the square root of the length.*

Plot a graph between these two quantities taking the period along the Y-axis and square root of the length along the X-axis. This time you will get a straight line showing that the period varies as the square root of the length of the pendulum (Fig. 4.4). This means that if the length is increased from 40 cm to 160 cm, the period will increase in the ratio of $\sqrt{40} : \sqrt{160}$ or in the ratio of 1 : 2. Our readings show that actually this is the case. So if the length changes by nine times then the period will be increased by three times. Therefore for a simple pendulum we conclude that $t \propto \sqrt{l}$.

3. Amplitude means the displacement from the mean position (rest position) of the bob to the extreme position. We can measure the amplitude in terms of the angle the string makes in the extreme position with its mean position because the displacement is proportional to the angle. In fig. 4.5 $\angle$BAC measures the amplitude in degrees of arc.

Set up a protractor upside down in a vertical position so that the mid-point of the diameter of the semi-circle of the protractor passes through the point of support A of the pendulum (Fig 4.5). In the rest position the line of the string will pass through the 90° division. Pull the string to one side so that now the string passes through the 85° division and then release it. This means that the amplitude is 5°. Taking the length of the pendulum as 100 cm, repeat the experiment making the amplitude 10°, 15°, 20°, and 30° and 50° and find the period in each case. Here you will notice that the period remains unchanged as long as the amplitude is small. It is only when the amplitude becomes greater than 15° or so, that the period increases slightly.

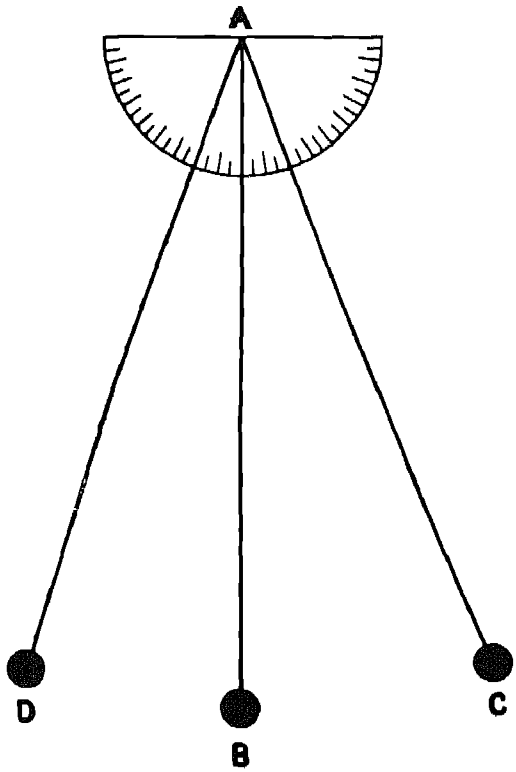


FIG. 4.5 *Amplitude of a pendulum*

Thus we arrive at the following laws of oscillations of a pendulum :

1. The period is independent of the material and mass of the pendulum bob.

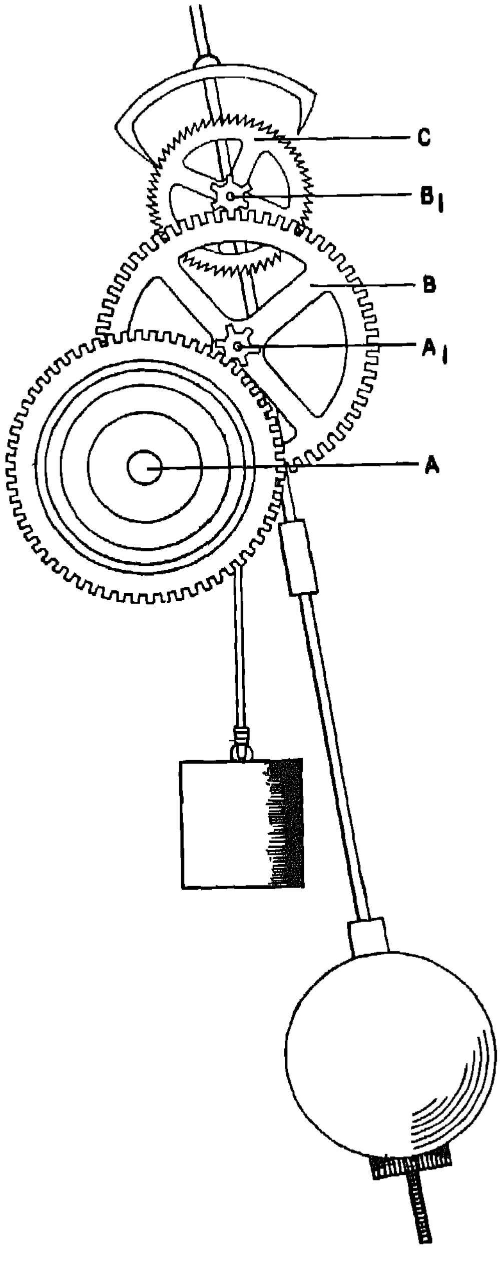


FIG. 4.6(a). *Principle of pendulum clock.*

2. The period is proportional to the square root of the length of the pendulum.
3. The period is independent of the amplitude provided the amplitude is small (less than 15° of arc).
   We shall learn later that the period also depends upon the acceleration due to gravity at the places of the experiment. This gives us the fourth law, namely.
4. The period is inversely proportional to the square root of the acceleration due to gravity. On combining these laws we get the following equation :

$T=K\sqrt{\frac{l}{g}}$ where T=period,

l =length of the pendulum,
g =acceleration due to gravity,
K =a constant.

The value of the constant K can be shown to be 2π. Hence, the above equation takes the form

$$T=2\pi\sqrt{\frac{l}{g}}.$$

Second's pendulum

A pendulum that beats seconds is called a second's pendulum It means that the bob of such a pendulum takes one second in going from one extreme position to the other, so that its period is two seconds. We can easily calculate, with the help of the above equation for the time period of a simple pendulum, the length of the second's simple pendulum.

we have,

$$2 = 2\pi\sqrt{\frac{l}{g}},$$

$$4 = \frac{4\pi^2 l}{g},$$

$$4 = \frac{4\pi^2 l}{980},$$

$$\therefore\ l = \frac{980}{\pi^2} = 99\cdot2 \text{ cm}.$$

Therefore the length of the second's pendulum=99·2 cm. The second's pendulum can be used to regulate a clock

4.3 Parts of a Clock

The main parts of a clock are a set of toothed wheels, a weight or spring which moves the wheels and a pendulum which regulates the speed of the rotation of wheels. The weight is connected to a chain or cord which is wound round a barrel or a drum which in turn is connected to the wheel A. The energy for the movement of wheels is supplied by the falling weight or the spring.

As the drum turns the wheel A rotates. The rotation of the wheel A is transferred into the rotation of the wheel C by a system of gear teeth. As is apparent from the figure 4·6(a) the wheel A rotates a smaller-wheel A_1, which is rigidly fixed at the centre of a larger wheel B. This wheel in turn rotates another small wheel which is fixed at the centre of wheel C called the escapement wheel, The net result of this system is that the rotation of wheel A is very much magnified in the rotation of wheel C. To make this clear we consider an example.

Suppose wheel A has 60 teeth and A_1 has 6 teeth. Since A_1 is fixed at the centre of B, B will make 10 rotations while A makes one rotation. Again if B had 60 teeth and C 6, then C completes 10 rotations when B completes one. Therefore, for every one rotation of A, C rotates 100 times. The advantage of having this sort of arrangement is that once the drum is wound, the clock runs for a long time before rewinding becomes necessary. Above the wheel C, there is a curved plate called an anchor. It has hooks, called pallets, on each end which fit into the escapement wheel. The anchor is rigidly attached to the top of the second's

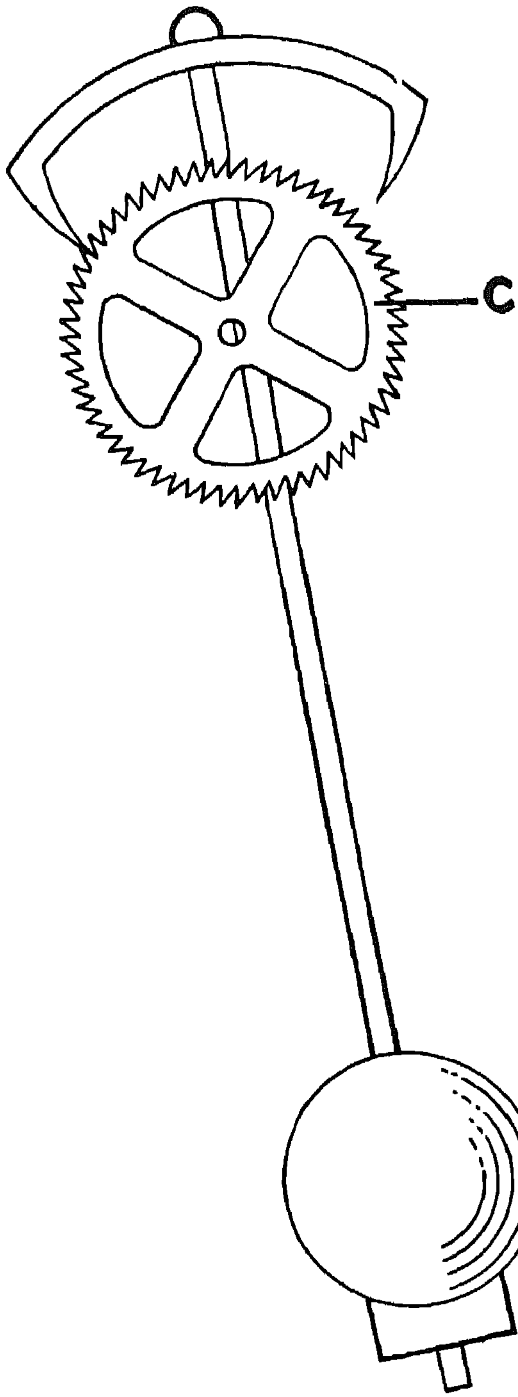


FIG. 4.6(b). *Escapement wheel with anchor.*

pendulum rod which hangs behind the system of wheels.

The pendulum permits the wheel C to move forward one tooth at a time as the device is tilted from side to side. Let us see how this system marks off seconds. Suppose that the left hook of the anchor is caught between the teeth of the escapement wheel The wheel stops for a moment. The pendulum at this moment is at extreme right position as is clear from the fig. 4 6b. As the pendulum starts moving towards left, the left hook moves up and the wheel revolves through one tooth. By the time the pendulum reaches its extreme left position, the right hook of the anchor comes down and again stops the wheel. Thus the two hooks alternately stop the wheel. In this way the pendulum's swing regulates the speed of the rotation of wheels. This goes on and on; the pendulum allows the wheel to move through one tooth only at each half oscillation. Since half oscillation of a second's pendulum takes only one second, the escapement wheel revolves through one tooth at intervals of exactly one second. Thus the second hand attached to this mechanism moves through one division on the dial indicating an interval of one second.

When the pendulum is oscillating, it possesses energy. The amplitude of the oscillation slowly decreases because the pendulum looses energy mainly due to friction of the air. To keep it oscillating this loss of energy has to be made up. The pendulum receives energy from the escapement wheel. As the hook moves up slightly, the wheel tries to rotate and in doing so gives some energy to the pendulum by pushing up the hook Thus the pendulum continues to oscillate without change in amplitude.

To indicate intervals of minutes and hours, we have to incorporate more wheels. The central idea behind the arrangement is to have a wheel which turns around once every hour. The hand attached to this wheel indicates minutes The hour hand is moved by turning a small wheel with 6 teeth against a large wheel with 72 teeth. The system is so arranged that the small wheel works in conjunction with the minute wheel, *i.e.* both take the same time for one revolution. Therefore, every time the small wheel revolves, the large wheel to which hour hand is attached moves one–twelfth of a complete turn. Thus the minute hand turns twelve times while the wheel which moves the hour hand turns round once.

Electric clocks in general depend for their power on an alternating current of electricity. This current oscillates at an even rate of fifty times per second which serves to keep the clock exactly on time.

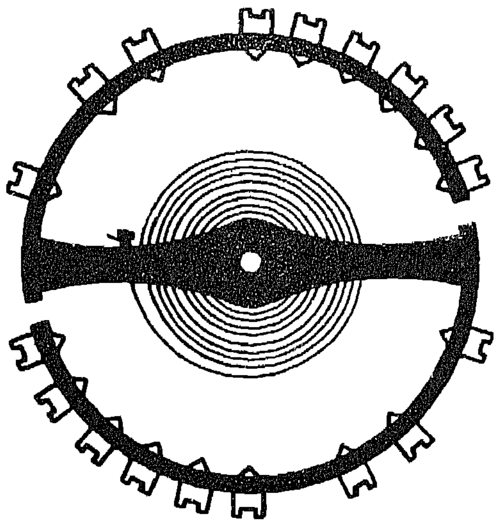


FIG. 4·7(a). *Balance wheel.*

In a watch, the time is regulated by a device called the balance wheel. A coiled hair spring oscillates sideways so that it completes half an oscillation in one second. Here too an anchor is rigidly attached to the shaft which oscillates with the hair spring. In each swing the anchor releases the

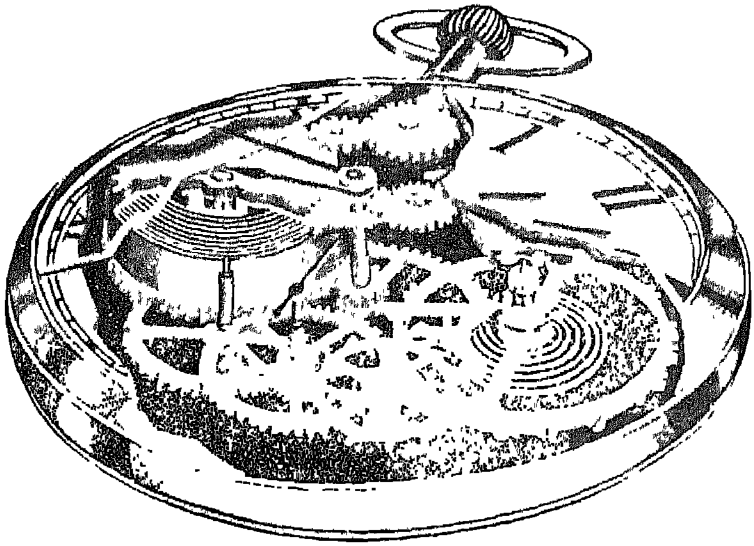
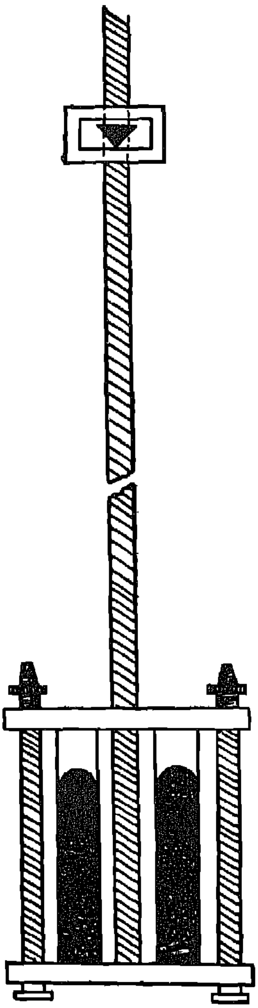


FIG. 4 7(b). *Pocket watch.*

FIG 4 8 *Compensated pendulum*



escapement wheel through one tooth and the second hand advances each time marking one second (Fig. 4.7a, 4.7b).

Electric stop-watches are powered by a tiny battery. The battery maintains the vibration of a miniature tuning fork, at frequency of 400 vibrations per second. The fork replaces the conventional balance wheel. Amplified by transistors, its vibrations drive the second, minute, and hour hands of the watch.

When time is regulated either by a pendulum or by a balance wheel, we have to keep in mind the fact that the period of the pendulum may change due to variations of temperature. An increase in temperature will cause the length of the pendulum to increase and consequently the period of the pendulum will increase and the clock will run slow—it will lose time. If the temperature decreases the clock will run fast—it will gain time. To avoid such a possibility modern clocks are fitted with what are called compensated pendulums. In such a pendulum variations of temperature do not cause any change in the effective length of the pendulum One such pendulum is shown in Fig. 4.8. The bob consists of a glass tube

containing mercury. The pendulum is so constructed that increase in the length of the pendulum rod due to rise in temperature is compensated by the rise in the level of the mercury in the tube so that the effective length of the pendulum remains unchanged Thus the clock neither loses time nor gains it.

In the balance wheel, its rim may expand due to rise in temperature so that the rate of swing is lowered, causing the watch to lose time. To overcome this defect the balance wheel is made of a compound strip of brass and steel with the brass strip on the outside (Fig 4.7a) When the temperature rises the iron strip tends to expand making the diameter of the wheel larger. But the expansion of brass being greater than that of steel, the outer rim curves inwards just enough to compensate for the effect of the expansion of the iron rim. So the rate of oscillation of the balance wheel remains unchanged.

4.4 Measurement of Small Intervals of Time

Ordinary watches and clocks produce ticks at intervals of one second. The second is a convenient unit of time. It is approximately equal to the interval betweeen heart-beats.

However, in scientific experiments we are required to measure much shorter intervals of time. For this purpose stop-watches have been devised which can measure time up to one hundredth of a second. Often we are required to measure one thousandth of a second (milli-second) or one millionth of a second (micro-second), and somtimes even smaller durations, like a billionth of second (nano-second). Physicists have invented many interesting devices to measure extremely short intervals of time. A simple device is a stroboscope.

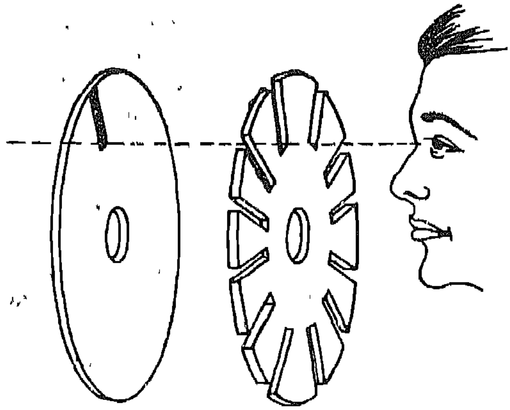


FIG. 4.9. *The stroboscope*

Suppose a disc has a black dot near the edge. Let the disc be rotated rapidly about a horizontal axis passing through its centre. We wish to measure the time of one rotation of the disc. Take another disc. Cut off 12 slits along its edge which should be equidistant from one another Set this disc in front of the experimental disc so that this disc can rotate in a vertical plane along its axis. Such a disc with slits around the edge is called a stroboscope disc

Let the first disc be stationary with the black dot in the highest position. Set the stroboscopic disc with one of its slits in the highest position so that the dot can be seen through the slit. If now the experimental disc is rotated at a uniform speed, the view of the dot through the slit of the stationary stroboscopic disc will be intermittent. Next rotate the stroboscopic disc also slowly with the help of an electric motor and attach to the disc a counter so that you are able to count the number of rotations made in one second. When both the discs are moving, the view of the black dot will, in general be interrupted.

Now increase the speed of rotation of the stroboscopic disc gradually till on looking through the top slit, the black dot on the experimental disc appears to be stationary. This means that during the interval the adjacent slit moves to the position previously occupied by the slit in the line of sight, the black dot on the experimental disc completes one full rotation and comes back to the position in the line of sight.

Clearly then, during the time the experimental disc completes one full rotation, the stroboscopic disc completes only 1/12 of its rotation. Suppose in the adjusted conditions, the counter registers 100 rotations per second. Then the stroboscopic disc must take $\frac{1}{100 \times 12}$ sec in completing 1/12 of its rotation. Therefore, we conclude that the experimental disc completes one rotation in 1/1200 sec. In short, if the stroboscopic disc has m slits on its rim and is making n rotations per second when the dot on the experimental disc appears to be stationary, then the time for one rotation of this disc will be $\frac{1}{m \times n}$ sec. By employing such a device it has been possible to measure time intervals of the order of 10^{-4} sec.

4.5 To Measure Time Intervals of the Order of 10^{-5} Sec or Less

To measure such short intervals we make use of electronic devices about which you will learn in higher classes. With the help of these devices we can measure time intervals up to 10^{-10} sec. Time intervals shorter than 10^{-10} sec are met with in subatomic phenomena only. For example the time required for an electron to make one revolution around the nucleus of the hydrogen atom is 10^{-15} sec and the oscillations of a particle inside the nucleus of an atom have a period of the order of 10^{-22} sec. These time intervals lying between the limits 10^{-10} sec and 10^{-22} sec can be measured by indirect means only.

4.6 Long Intervals of Time

With the help of our physical senses we are able to experience time intervals from one tenth of a second (10^{-1} sec.) to about 10^{9} sec., which is approximately equal to our life-span. For estimating longer intervals we employ indirect methods. For instance, the age of the earth has been estimated by methods based upon radioactive disintegration. In this way we reach time intervals of the order of several thousands of millions of years (10^{17} sec). Using other indirect methods the physicists have reached a time interval of the order 10^{18} seconds—the life-span of a typical star. The age of the universe itself has been estimated to be about 10 billion years.

Orders of Magnitude of Time

Time Interval (in sec)	Associated Event	Time Interval (in sec)	Associated Event
10^{18}	Expected total life of the Sun as a normal star	10^{11}	Time elapsed since the beginning of the Christian era
10^{17}	Age of the oldest rocks	10^{10}	Time elapsed since the fall of Vijainagar empire.
10^{16}	Time for the Sun to revolve around the galaxy		Time elapsed since the discovery of America
10^{15}	Time elapsed since dinosaurs. Birth of the Himalayas		Earliest European settlements in India
		10^{9}	Human life-span
10^{14}	Remaining life of Niagara falls	10^{8}	Time elapsed since you began school
10^{13}	Time elapsed since earliest man	10^{7}	Time for the earth to revolve around the sun (year)
10^{12}	______	10^{6}	One month.

10^{5}	Time for the earth to rotate once on the axis (day)	10^{-1}	Time for bullet (30 caliber) to cover the length of a football field	10^{-6}	Time for high speed bullet to cross a letter of type	10^{-14}	________
10^{4}	Duration of average hockey game	10^{-2}	Time for electric fan to complete one rotation	10^{-7}	Time for the electron beam to go from source to screen in a television tube	10^{-15}	Time for electron to revolve around proton in hydrogen atom
	Train journey from Delhi to Agra	10^{-3}	Time for a fly to beat its wings once Time that a fired bullet is in the barrel of a rifle	10^{-8}	Time for light to cross a room	10^{-16}	________
10^{3}	Time for light from the Sun to reach the Earth	10^{-4}	Time for one vibration of the highest pitched audible sound	10^{-9}	Time during which an atom emits visible light	10^{-17}	________
10^{2}	One minute	10^{-5}	Time during which a fire cracker is exploding	10^{-10}	________	10^{-18}	________
10^{1}	________		________	10^{-11}	Time for light to penetrate a window pane	10^{-19}	
10^{0}	Time between heart beats (1 second)			10^{-12}	Time for air molecule to spin once	10^{-20}	Time for inner most electron to revolve around nucleus in heaviest atom
				10^{-13}	________	10^{-21}	________
						10^{-22}	Time for proton to revolve once in nucleus

NOTE : The above Table has been taken from *Physics* by Physical Science Study Committee, USA.

Classroom Activities

1. Make a sun dial.
2. Take a piece of carboard $8'' \times 8''$, suspend it by a corner. Find the effective length of the string pendulum.
3. Find the time of the free swing of your arm (a) without any thing in the hand, (b) holding 1 kg in the hand.
4. Estimate the time period of the swing of your arms when you are walking normally. Next ask one of your friends to measure the time of the swing with the help of the stop-watch. Compare the two values.

Questions

1. In modern times, time must be measured accurately in fractions of a second. Give some examples where you think such measurements are necessary.
2. Find the length of a simple pendulum which beats seconds at a place where the acceleration due to gravity is $9 \cdot 701$ metres/sec^2.
3. Photographers while working in the dark room estimate time intervals by counting aloud. Can you explain how this can be done without committing an appreciable error ?
4. Express a year in seconds using powers of ten notation.
5. Give the number of revolutions made by the electron in a hydrogen atom during the time the earth takes in rotating about its axis once (use power of ten rotation.)

6. How can a stroboscopic disc be used to determine the time taken by a tuning fork to complete one oscillation ?
7. A stroboscopic disc has 20 equidistant slits cut along its rim. It is adjusted so that when seen through one of the slits, a mark on the rotating shaft of a motor appears to be stationary. If the shaft takes 1/50 sec in completing one rotation, find the rate of rotation of the stroboscopic disc.
8. In a snap shot photograph with an ordinary camera a fast moving object gives a blurred photograph. Can you account for this ?
9. A camera has a shutter speed of 1/1000 sec. How far will a rifle bullet move during the interval the shutter remains open at the shutter speed 1/1000 sec (the bullet has a speed 1200 m/sec) ?
10. A stroboscopic disc has 10 slits around the rim. It can be rotated 50 times per second. What fraction of a second can be measured accurately with this Stroboscope ?
11. Sometimes on a movie screen the wheels of a car appear to turn backwards though the car is actually moving forward. How can this be possible ?

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CHAPTER V

Motion

5.1 Motion in a Straight Line

We become aware of the fact that a body is in motion when we find that the body is changing its position with respect to the positions of other objects. Hence motion may be defined as the continuous process of change of position.

Suppose a particle is initially at the point O. After some time it reaches the point A (Fig. 5.1). Then OA is the change in its position which is called its displacement. This displacement is a result of motion. In other words motion is a process by which an object gets a displacement

Fig. 5.1.

In the example given above the displacement is the distance OA in the direction obtained by joining O to A. So displacement carries an idea of the magnitude of distance as well as the idea of direction.

5.2 Vectors

Take another example. Let a person travel one km from his house. This datum alone does not give us complete information about his location. To locate him we must also specify the direction in which he travelled. Therefore, to know his final position, we must specify two quantities ·

(i) The magnitude of the distance travelled, and

(ii) The Direction in which he travelled.

All quantities which possess magnitude as well as direction are called vectors. Thus displacement is a vector. You will come across other examples of vectors later.

5.3 Scalars

Quantities which possess magnitude only but no direction are called scalars. For example volume is a scalar quantity. When we say that the volume of a body is 10 cc the statement gives complete information about the volume of the body. On the other hand when we say that a body has been given a displacement of 10 cm the statement remains incomplete unless we also indicate

the direction in which the displacement has taken place. So to describe a vector completely we must give its magnitude as well as its direction. For scalars, only the magnitude is required. Other examples of scalar quantities with which you are familiar are length and time.

5.4 Representation of a Vector

A vector quantity may, therefore, be represented by a straight line such that its length is proportional to the magnitude of the quantity and its direction, shown by an arrow head, giving the direction of the vector. The vector may have any direction in space. Suppose we wish to represent a a displacement of 20 km in the east direction. We shall draw a straight line AB of length 2 cm from west to east and we shall put an arrow head on the line pointing towards east (Fig. 5·2). Here the scale is 1 cm=10 km.

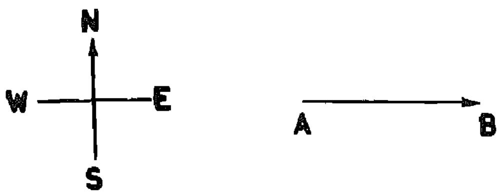


Fig. 5.2.

5.5 Speed

Suppose a car is moving along a road due north and then it takes a turn to the east All the time its speedometer may show the speed to be 30 km per hour. What does this reading indicate? It means that the car is moving through a distance of 30 km in one hour. The distance covered may be due north or due east or in any other direction. So speed does not carry any sense of direction, it is a scalar quantity. It possesses only magnitude but no direction.

$$\text{Thus, speed} = \frac{\text{distance travelled}}{\text{time taken}}.$$

Hence, the unit of speed is the unit of distance divided by the unit of time. It is therefore expressed as m/sec, km/hour, or miles/hour.

When the car starts from rest its speed increases gradually, *i.e.* its speed is different at different instants of time. Even when the car has been in motion for some time, its speed may be constantly changing, if it has to move through a crowded street or has to negotiate sharp bends. If the speed of the car is constant and we draw a graph between the distance travelled (along Y-axis) and the time (along X-axis), we will get a straight line. However, when the speed is variable, the distance time graph will be a curved line. If we now take a small portion of the graph and draw it on a magnified scale, the graph will appear to be almost a straight line, *i.e.* during this small interval of time the speed is almost constant. If we choose two points on the graph, very near to each other and if the car has travelled up to distance d_1 at the time t_1 and up to the distance d_2 at the time t_2, the speed of the car during the interval t_2-t_1 is given by

$$\text{speed} = \frac{d_2-d_1}{t_2-t_1} = \frac{\Delta d}{\Delta t}.$$

This is the speed of the car at time $t = \frac{t_2+t_1}{2}$. When the interval t_2-t_1 is made very small, we can speak of the speed at the instant t. This is the quantity.

$$v = \lim_{t_2 \to t_1} \frac{d_2-d_1}{t_2-t_1} = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}.$$

The speed at a given instant is called the instantaneous speed.

5.6 Velocity

By velocity we mean the distance travelled per unit time in a ***particular direction.*** In the example given above, as long as the car is travelling on the straight road due north, its velocity remains 30 km/hour due

north. When the car is travelling due east, its velocity becomes 30 km/hour due east. That is, its velocity has changed though the speed remains the same.

Therefore velocity is a vector quantity. Mathematically velocity can be defined

$$\text{velocity} = \frac{\text{distance in a particular direction}}{\text{time taken}},$$

=rate of motion in a particular direction or in other words,

$$\text{velocity} = \frac{\text{displacement}}{\text{time}},$$

=rate of displacement.

Since displacement possesses magnitude as well as direction, velocity too possesses magnitude as well as direction. Hence the unit of velocity is the same as that of speed but to specify the velocity of a particle, we have to specify its direction in addition to the unit. Velocity also can be represented by a straight line such that its length is proportional to the magnitude of the velocity and the arrow-head on it gives the direction of the velocity.

Uniform and variable velocity

If in a particular direction, the body travels equal distances in equal intervals of time, howsoever small these intervals be, then the velocity is said to be uniform. If the body in a particular direction does not travel equal distances in equal intervals of time, then the velocity is said to be variable.

When the velocity is uniform, we say that the body is ***in uniform motion***. Thus, uniform motion implies that the body is moving in a straight line and with constant speed.

Now in a variable velocity only the magnitude may change but the direction may remain unchanged. When brakes are applied to a car speeding along a straight level road, the magnitude of its velocity decreases but the direction of motion remains unchanged so the car travels with a variable velocity before it comes to rest. Again the magnitude of the velocity of a body may remain unchanged but its direction may change; here too the velocity will be variable. Take an example. Tie a piece of stone to one end of a piece of string and hold the free end in your hand. Swirl the stone about you in a horizontal circle; the length of the string forms the radius of the circle. Now the stone will be travelling along the circumference of the circle such that it covers equal distances in equal intervals of time. Thus the magnitude of the velocity is constant, but its direction is continually changing. Hence the velocity of the stone here too is changing. We can therefore say that the speed of the stone in motion is constant but its velocity is variable.

3.7 Acceleration

In a sports event when the athlete runs a straight course, his velocity along the track is certainly not constant. It is variable. At the start his velocity is zero because he starts running from the position of rest. After 1 sec his velocity may be 3 metres/sec and at the end of 2 sec the velocity may be 6 metres/sec at the end of 3 sec the velocity may be 9 metres/sec.

So this is a case of variable velocity in which the magnitude of the velocity is increasing steadily. What is the rate of increase of velocity? In the first second, the velocity increased from 0 metres/sec to 3 metres/sec, in the next second the velocity increased from 3 metres/sec to 6 metres/sec and in the third second, it increased from 6 metres/sec to 9 metres/sec. Hence the rate of increase of velocity is 3 metres/sec in every second or the rate of increase of velocity is 3 metres/sec per sec along the track. This can also be expressed as 3 metres per sec per sec or 3 metres/sec^2.

The rate of increase of velocity is called acceleration. It is a vector quantity.

Uniform and variable acceleration

If the rate of change of velocity remains the same throughout the motion, then the acceleration is said to be uniform. If the rate of change of velocity is different at different instants, then the acceleration is variable. For example, when a railway train goes from one station to the next, the acceleration of the train is variable. In the beginning the acceleration is low, then it increases. After the train has run for a few kilometres, the train attains its maximum velocity. If the track is straight, the velocity is no more increasing, and the acceleration is nil. As the train approaches the next station, its velocity decreases, that is the acceleration is now negative and ultimately the velocity is reduced to zero, and the train comes to a stop. Negative acceleration is the rate of decrease of velocity. It is some times called deceleration.

Metres per second is abbreviated to $\frac{m}{sec}$.
For convenience of printing and typing it is often put as m/sec. In scientific term this is some times expressed as m sec^{-1}. Thus acceleration which is expressed as metre per second per second is written as m sec^{-2} and density as kg m^{-3}.

5.8 Velocity—Time Graph

We can plot a graph between the velocity (on Y-axis) and time (on X-axis) for the motion and obtain a velocity—time graph.

1. *For uniform motion.* Suppose the velocity is 10 metres/sec which remains unchanged throughout. Initially the velocity is 10 metres/sec, at the end of 1 sec again the velocity is 10 metres/sec, at the end of 2 sec also the velocity is 10 metres/sec

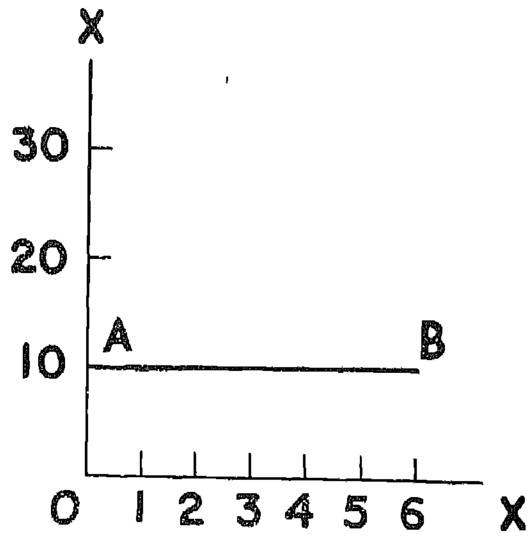


Fig. 5.3.

and so on. Thus we get a graph as shown in fig. 5·3. If the body was in motion for 6 sec then the graph will be the straight line AB parallel to the X-axis.

2. *For uniformly accelerated motion.* Suppose the body starts from rest and the acceleration is 3 metres/sec^2 throughout the motion. We can plot a graph between its velocity and time for the first 6 seconds of its motion.

Plotting the graph between the two quantities, we get the straight line OP (Fig. 5·4) which is inclined to the X-axis. Since the graph is a straight line inclined to the X-axis, we conclude that the velocity is proportional to time.

3. *For motion with a variable acceleration.* Consider a body which starts from rest and has an acceleration of 3 metres/sec^2 in the first second, 4 metres/sec^2 in the second, 5 metres/sec^2 in the third, 6 metres/sec^2 in the fourth second and so on.

Then we have its velocities at different instants as given in the table below.

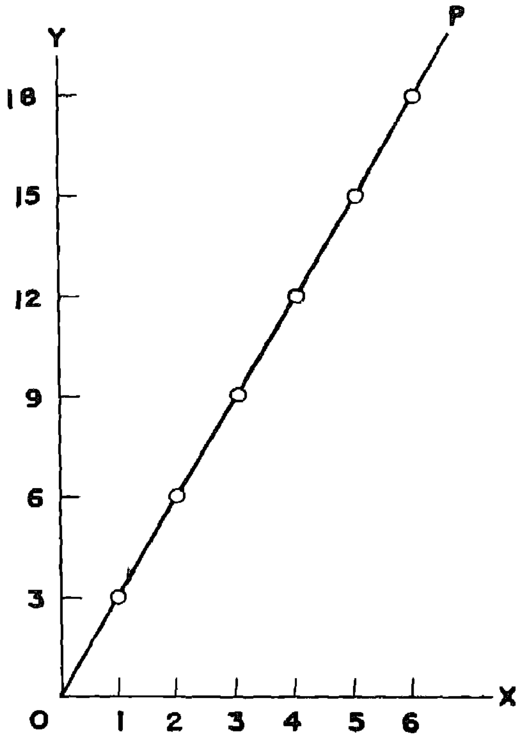
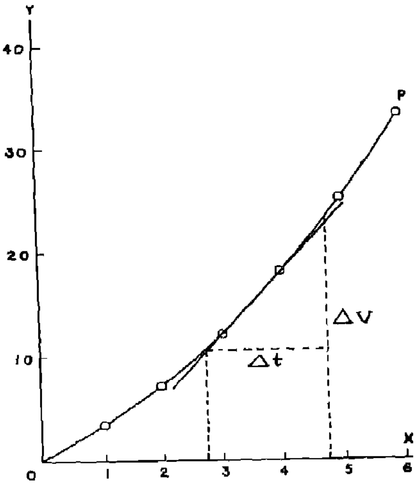


FIG. 5.4 *Graph between velocity and time (Motion with a uniform acceleration).*

Velocity metres/sec	Time in sec
0	0
3	1
7	2
12	3
18	4
25	5
33	6

on plotting the graph we see that the graph is no more a straight line.

FIG. 5.5. *Graph between velocity and time (Motion with a variable acceleration)*



From the graph we can now define the instantaneous acceleration as

$$a = \lim_{t_2 \to t_1} \frac{v_2 - v_1}{t_2 - t_1} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}.^*$$

5.9 Motion of Freely Falling Bodies

Experiments show that if a body is allowed to fall freely from rest, its velocity downwards will be 9·8 metres/sec after 1 sec, 19·6 mteres/sec after 2 sec, 29·4 metres/sec at the end of 3 sec and so on. Thus the rate of increase of velocity, acceleration is uniform and its value velocity is 9·8 metres/sec². It is independent of the material or the size of the body.

Similarly, if a body is thrown upwards vertically, its motion is uniformly retarded. The negative acceleration or the deceleration is 9·8 metres/sec².

* In vector notation this equation can be written as follows :

$$\vec{a} = \lim_{t_2 \to t_1} \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

This acceleration directed downwards or towards the centre of the earth is called acceleration due to gravity and is commonly represented by the letter 'g'.

5.10 Equations for Uniformly Accelerated Motion

It was the Italian scientist Galileo Galilei (1564–1642) who first made experimental study of bodies moving under uniform acceleration. He derived equations which establish the relations between the initial velocity, the final velocity, the time for which the body was in motion and the distance travelled.

We shall derive these equations by mathematical reasoning. Suppose a body has already an initial velocity u and has uniform acceleration a. We wish to find its velocity after time t.

Let the velocity after time t be v. By definition,

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}},$$

$$\text{or,} \quad a = \frac{v-u}{t},$$

$$\text{or,} \quad at = v-u,$$

$$v = u+at. \qquad \ldots\ldots\ldots \text{(i)}$$

That is, the final velocity = initial velocity + acceleration multiplied by time.

To find the distance travelled in time t, we proceed as follows.

Average velocity during the first second is $\frac{u+a}{2}$ and that during the last second is $\frac{v-a}{2}$. Hence the average for these two intervals is $\frac{v+u}{2}$. Similarly taking intervals from the beginning and the end we can prove that for the whole journey, the average velocity is $(v+u)/2$.

So the distance travelled = average velocity × time,

$$\text{or} \quad s = \frac{v+u}{2} \times t.$$

Substituting the value of v from equation (i),

$$s = \frac{u+at+u}{2} \times t,$$

$$s = ut + \tfrac{1}{2}at^2. \qquad \ldots\ldots\ldots \text{(ii)}$$

If we know the initial velocity u and the acceleration a we can, with the help of equation (ii), easily find the distance s travelled in time t

By eliminating t from equations (i) and (ii) we get a third equation connecting the distance travelled with the final and initial velocities. It can easily be proved that

$$v^2 = u^2 + 2as. \qquad \ldots\ldots\ldots\ldots \text{(iii)}$$

Thus we have three equations of motion for bodies with uniform acceleration.

$v = u + at$ (distance not included)
$s = ut + \frac{1}{2}at^2$ (final velocity not included)
$v^2 = u^2 + 2as$ (time not included)

Sometimes bodies start moving with uniform acceleration from a position of rest. In such cases the initial velocity $u=0$ and the equations of motion take the form

$$v = at,$$
$$s = \tfrac{1}{2}at^2,$$
$$v^2 = 2as$$

For a freely falling body the uniform acceleration is the acceleration due to gravity, g. In this case the equations of motion take the form

$$v = u \pm gt,$$
$$s = ut \pm \tfrac{1}{2}gt^2,$$
$$v^2 = u^2 \pm 2gs.$$

The negative sign is used when the body is projected vertically upwards, for in this case the acceleration due to gravity is in a direction opposite to that in which the body is moving.

To find the distance travelled in a given second, say the nth second, we proceed as follows :

The velocity at the beginning of the nth second, *i.e.* at the end of (n–1) seconds is $u+(n-1)a$. The velocity at the end of n seconds is $u+na$ Hence the average velocity during the nth second is $u+(n-\frac{1}{2})a$ and this is the distance travelled in the nth second. This can also be determined by finding the difference (S_n-S_{n-1}) where S_n and S_{n-1} are the distances travelled in n seconds and (n–1) seconds respectively.

5.11 Use of Graph for Determining the Distance Travelled by a Body in Motion with a Uniform Acceleration

Let the body have an initial velocity u. Its acceleration is a. We wish to determine the distance travelled in time t

Plot a graph between the velocity and time as below. We get a straight line as the velocity—time graph (fig. 5·6), the area under this graph is given by the area of the figure ACEOB

$=$area of the rectangle BCEO $+$area of the $\triangle$ACB,

$=BO\times OE+\frac{1}{2}AC\times BC,$

$=ut+\frac{1}{2}at\times t,$

$=ut+\frac{1}{2}at^2.$

This gives the distance travelled by the body in time t [See equation (ii) for uniformly accelerated motion]. Thus, we arrive at the conclusion *that the area under the velocity—time graph gives the distance travelled.*

Examples :

1. A motor car is travelling at 30 km/hour. After 10 sec the velocity becomes 60 km/hour. Assuming the velocity change took place at a uniform rate, find the acceleration.

$v=u+at,$

60 km/hour $=$ 30 km/hour $+a\times10$ sec,

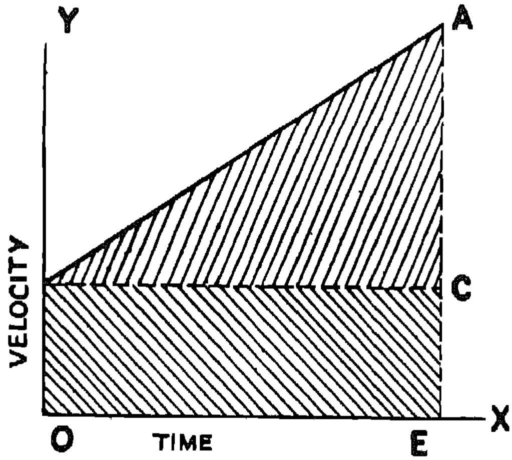


FIG. 5 6 *The area under a velocity-time curve represents distance travelled by the body during the time t.*

or, $a=\frac{30}{10}$ km/hour per sec,

$=\frac{3\times10^3}{3600}$ m/sec per sec.

$\therefore$ Acceleration $=0\cdot8333$ m sec^{-2}.

2. The velocity of a sprinter is 9 metre/sec. After 4 sec, his velocity becomes zero. If the acceleration is taken to be uniform, find its value.

$v=u+at,$

or, $a=\frac{-9}{4}$metre/sec^2,

$\therefore$ Acceleration $=-2\cdot25$ metre/sec^2.

3. A body has a velocity 300 cm/sec. If it has a uniform acceleration of 10 cm/sec^2, find the distance it travels in 3 sec.

$s=ut+\frac{1}{2}at^2,$

$=300$ cm/sec $\times 3$ sec $+5$ cm/sec$^2\times9$ sec^2,

$=(900+45)$ cm,

$=9\cdot45$ metres.

4. If a body has a velocity 50 cm/sec and the uniform acceleration is 5 cm/sec^2, find the time it takes in travelling a distance of 240 cm.

Let the time taken be t sec. Then

240 cm $=$ 50 cm/sec $\times$ t sec

$+\frac{1}{2}$ 5 cm/sec^2 $\times$ t^2 sec^2,

or, $48 = 10t + \frac{t^2}{2}$,

or, $t^2 + 20t - 96 = 0$,

$\therefore t = 4$ or $t = -24$.

Time cannot be negative, hence $t=4$ is the correct solution.

$\therefore$ Time taken $=4$ sec.

5. A car is moving with a velocity of 25 metres/sec. Brakes are applied so that the car comes to a stop after travelling a distance of 80 metres. Find the acceleration.

$$v^2 = u^2 + 2\,as,$$

$$0 = (25\text{m/sec})^2 + 2a \times 80 \text{ metre},$$

$$a = -\frac{625}{160}\text{metres/sec}^2.$$

$\therefore$ Acceleration $= -3 \cdot 9$ metres/sec^2.

5.12 Addition of Velocities

If on a particle two velocities are imposed simultaneously, the particle will have a velocity which will be different from either of the two velocities. The velocity that it takes up as a result of the two velocities imposed is called the resultant velocity. The process of combining the two velocities so imposed is known as addition of velocities.

Three cases may arise :

1. The two velocities are in the same direction.

2. The two velocities are in opposite directions.

3. The two velocities have their directions inclined to each other.

Case 1. Suppose a man is able to row a boat on still water with a velocity of 4 km/hour. He rows the boat downstream in a river which flows with a velocity of 2 km/hour. Now the boat has two velocities imposed on it; one is 4 km/hour and the other is 2 km/hour. Both are in the same direction.

The resultant velocity $=4$ km/hour $+2$ km/hour,

$=6$ km/hour downstream.

Thus the two velocities will be added to give the resultant velocity.

Case 2. In the above example, suppose the man rows upstream. Here the two velocities are in opposite directions.

Obviously the resultant velocity will be the difference between the two velocities.

Resultant velocity $=4$ km/hour-2 km/hour,

$=2$ km/hour upstream.

Thus, the resultant velocity will be equal to the difference of the two velocities and its direction will be the same as that of the greater velocity.

Case 3. The third case will be considered later.

Classroom Activities

Take a hollow tube about a metre long and closed at one end and fitted with an air tight stopcock at the other end. Before fitting the stop cock put a feather and a small piece of stone in it. Hold the tube vertically and let the feather and the piece of stone reach the bottom. Now invert the tube suddenly so that the feather and stone piece start to fall at the same time. Which of the two falls faster ? Now with the help of a vacuum pump evacuate the tube and hold it vertically. Again invert the tube suddenly and observe their fall. Try to explain any change in their fall which you now observe.

Questions

1 What is motion ? How do we detect motion ?

2 If you were the only object in the Universe, could you detect your motion ?

3. Justify that acceleration is a vector quantity.

4. Is it possible for a body to have a constant speed but variable velocity ? Can a body have a constant velocity but variable speed ? Illustrate your answer with examples.

5. Is it possible for a body in motion to be under a uniform acceleration directed away from the line of motion ? Give reasons for your answer

6 Is it necessary that a change in the speed must produce a change in the velocity of the body ? Does a change in velocity necessarily mean a change in the speed of the body ?

7. A car has a velocity of 50 m/sec when a uniform acceleration of 5 m/sec^2 is imposed on it in the line of motion. Determine the distance travelled in 10 sec by drawing the velocity—time graph.

8(*a*)A car starting from rest acquires a velocity of 40 km/hour in 5 sec. Taking the acceleration as uniform, find its value in m/sec^2. Also find the total distance travelled during this time.

8(*b*) Find the instantaneous acceleration for the motion shown in Fig. 5.5 at t=2 sec, t=5 sec.

9 A sprinter running with a velocity of 9 m/sec stops in 5 sec Find his acceleration and the distance travelled while stopping.

10. If brakes applied to a car produce a negative acceleration of 60 cm/sec^2, what distance will the car travel further if it takes 40 sec to stop ?

11. A body is moving with a velocity of 25 m/sec. A uniform negative acceleration acts on it so that the body travels 57 metres in the next 5 sec. Find the magnitude of the negative acceleration.

12. A piece of stone is thrown vertically upwards with a velocity of 19·6 m/sec. How high will it rise ? After how many seconds will it come back to the starting point ?

13. A body starts from rest with an acceleration of 6 m/sec^2. What will be its velocity when it has travelled a distance of 300 metres ? How many seconds did it take to cover this distance ?

14. The brakes of a car can produce an acceleration of –3 m/sec^2. How much time will be needed to reduce the velocity of the car from 100 km/hr to 50 km/hr ? How many metres will the car travel during this interval ?

15. A body starts moving from rest with an acceleration of 20 cm/sec^2. What distance will it travel in the tenth second?
16. A train is travelling with a velocity of 40 km/hr. What should be the acceleration on it so that it may reach a point 10 km ahead in 8 minutes ? What will be its velocity on reaching that point ?
17. An electron travelling at 5×10^6 cm/sec passes through an electric field that gives it an acceleration of 10^{17} cm/sec^2 acting in the direction of travel.
    (a) How much time will it take for the electron to double its initial velocity?
    (b) Through what distance will the electron travel in this time (Physics by Gamow & Cleveland)?

Further Reading

ABBOTT, A. F. *Ordinary Level Physics.* London : Heinemann Educational Books Ltd., 1963.

JOG, D. S. *Intermediate Physics* (Vol 1). Bombay : Popular Book Depot, 1959.

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CHAPTER VI

Vectors

6.1 Trips and Vectors; Addition and Subtraction of Vectors

A car moving along a road travels on a predetermined path, but there are many motions in nature for which the path or road is not fixed in advance. If you drive a motorboat on a lake, pilot an airplane, or want to find out about the motion of a satellite moving across the sky, you are facing a new situation. There are no roads or paths on the surface of the water or up in space.

To describe motion in general, we must know more than the speed We must extend the ideas of chapter V to include the direction of motion. We therefore introduce quantities which have both magnitude and a direction. We shall represent these quantities by straight line segments. The length of the line gives the magnitude, and its direction specifies the direction in space. As already discussed in the last chapter we call these quantities vectors.

The simplest situation involving vectors arises when we consider motion along a straight line Suppose you take a trip in a car along a straight road. Fig 6.1 shows a line representing the road; the points A,B,C. . on the road are equidistant—for example, one km apart. If you start at C and stop at H, this trip is evidently represented by the arrow labelled CH in the figure. It is 5 km long Now, if you turn around and ride back to F, this second trip is represented by the arrow labelled HF. It is 2 km long and in the opposite direction The result of these two trips is the same as if you made a single trip from C to F, represented by the arrow CF, 3 km long. This says that

$$\text{Trip CH} + \text{Trip HF} = \text{Trip CF}$$

If we try to use the length of each trip to represent it in this equation, we will be led to an incorrect conclusion, since $5 \text{ km} + 2 \text{ km} \neq 3 \text{ km}$. On the other hand, from the figure you see that we can represent the trip HF by the negative number (–2 km). Then we get the correct result ·

$$5 \text{ km} + (-2 \text{ km}) = 3 \text{ km}.$$

As this example shows, we can represent the two directions possible for trips along a straight line by positive and

negative numbers; for example, positive numbers for arrows pointing to the right and negative numbers for arrows pointing to the left. Addition and subtraction of such vectors is the same as addition and subtraction of positive and negative numbers.

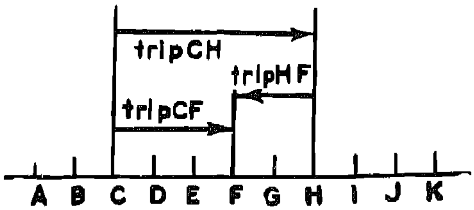


Fig. 6.1. *Trip CH followed by trip HF leads to the same results as trip CF.*

By considering trips in any direction on a flat surface instead of restricting the trips to a straight line, we meet a new problem. Suppose we start at a point A. We make a trip 2·7 km north to B and follow that by a trip 2·7 km east to C as shown in Fig. 6.2 (a). A trip directly from A to C gives the same result as these two trips separately, as shown in Fig. 6.2 (b). Therefore we can write

Trip AB + Trip BC = Trip AC.

We can add trips at any angle, such as those in Fig. 6.3. Notice that adding trips that are not along the same line is a different kind of addition from adding numbers. In particular, the distance we would walk from A to C via B is longer than the distance going straight from A to C.

In order to indicate a trip from a point A to a point B, we have drawn an arrow-head at B in the direction from A to B. The direction of the arrow-head tells us the direction of the trip and the length of the arrow tells us the distance we have travelled. When we want to discuss this trip from A to B, instead of writing "Trip AB" we write the vector symbol $\overrightarrow{AB}$. Thus in Fig. 6.4(a) we conclude that $\overrightarrow{LM}+\overrightarrow{MN}=\overrightarrow{LN}$. Usually it is more convenient to use a single letter to denote a vector. The trips from L to M can be represented either by the symbol $\overrightarrow{LM}$ or by the new symbol $\vec{R}$, and the description of Fig. 6.4(b) is $\vec{R}+\vec{S}=\vec{T}$.

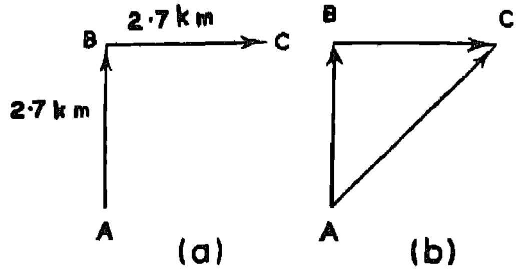


Fig. 6.2. *Trips on any surface, such as that of this page or that of the earth, may be added by the use of scale drawings.*

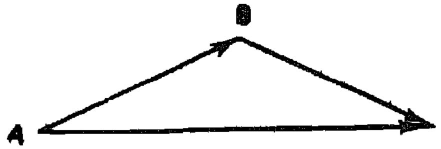


Fig. 6.3. *Trip AB + Trip BC = Trip AC.*

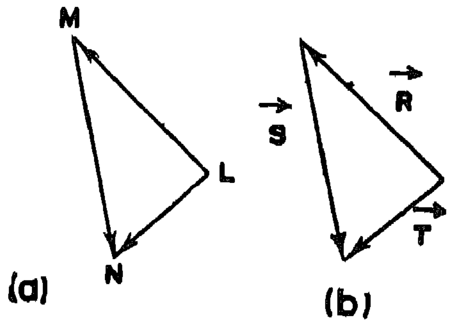


Fig. 6.4. *The same addition is shown in (a) and (b), although the vector notations are different.*

To describe a straight line trip completely, you need to know its length, its direction, and either its starting or end point. However, we shall often want to think of two such trips as being equivalent if they cover the same distance in the same direction, even though they start

from different points. When we are interested only in the distance and direction, we shall use the name displacement instead of trip. The statement that you have travelled 1·4 km in a direction 15° west of north describes your displacement completely. Two vectors of the same length and the same direction represent the same displacement. When you displace a book one metre east and one metre north, it ends up $\sqrt{2}$ metres north-east regardless of where it started.

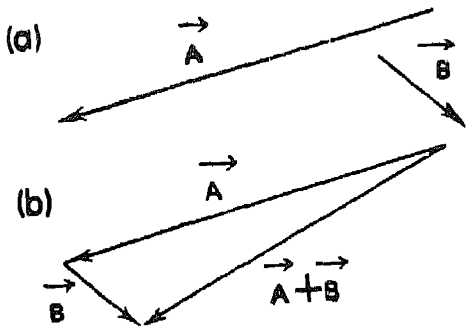


FIG 6.5. *Addition of two vectors after moving one parallel to itself.*

In general we shall define a vector without specifying where either end is, and we shall add vectors by putting them head to tail as in Fig 6.4. Thus, if we want to find the sum of the two vectors shown in Fig. 6.5 (a), we can move one of them "parallel to itself" until its tail coincides with the head of the other, as shown in Fig. 6.5 (b). This means that two vectors are the same if their magnitude (length) and direction are the same, no matter where we draw them. If the position of one end of a vector is important we shall add that information when we use it.

Now we know how to add vectors. Is there a corresponding process for subtracting them? Recall that when we deal with positive and negative numbers the answer to the problem

$$6-8=?$$

is the same as the answer to the problem

$$6+(-8)=?$$

or to the problem

$$6=8+?$$

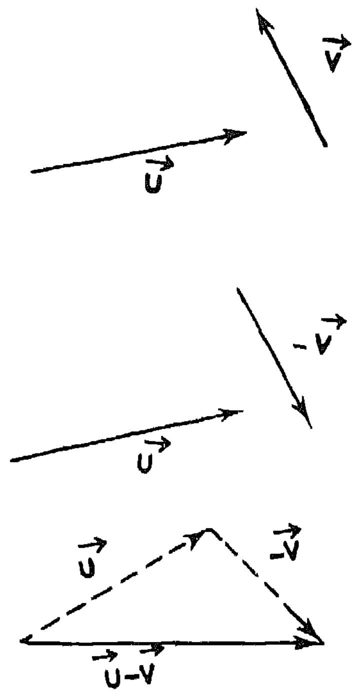


FIG 6 6. *To find the difference of two vectors we add the negative of the second to the first*

There are similar procedures for subtracting vectors. To see why, let us go back to trips for a moment. If you add the trip $\vec{BA}$ to the trip $\vec{AB}$, you get back to your starting point. The result is a "zero trip". We therefore say that AB and BA are opposite trips, whose sum is zero, and we write :

$$\vec{AB}=-\vec{BA} \text{ or } -\vec{AB}=\vec{BA}$$

In general, adding the trip $\vec{BA}$ is the same as taking away the trip $\vec{AB}$, and $\vec{CB}-\vec{AB}$ is the same as $\vec{CB}+\vec{BA}$ or as $\vec{CB}+(-\vec{AB})$.

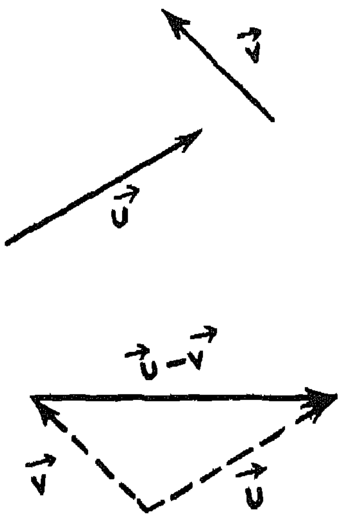


FIG. 6 7. *We can also find $\vec{U}-\vec{V}$ by drawing $\vec{U}$ and $\vec{V}$ tail to tail then drawing a vector from the head of $\vec{V}$ to the head of $\vec{U}$. You can check that you have drawn this vector in the right direction by remembering that $\vec{V}+(\vec{U}-\vec{V})=\vec{U}$.*

For vectors the end points do not matter. The opposite of the vector $\vec{V}$ is any vector $-\vec{V}$ that is equal to $\vec{V}$ in length and opposite in direction. We can therefore subtract $\vec{V}$ from $\vec{U}$ by adding $-\vec{V}$ to $\vec{U}$. The procedure is illustrated in fig. 6·6 We can also subtract $\vec{V}$ from $\vec{U}$ by answering the question $\vec{U}=\vec{V}+$? In Fig. 6.7, we show the vector $\vec{U}$, the vector $\vec{V}$, and the difference $\vec{U}-\vec{V}$. Notice that this vector in Fig. 6·7 is parallel to the corresponding vector of the same length and direction in Fig. 6·6. Therefore they are the same. Both methods give the same answer; use the method most convenient for you.

We have used displacements and the addition of displacements as examples of the new quantities we call vectors. We can represent these quantities by straight line segments, each with an arrow-head to indicate the direction and the length to give us the magnitude This gives us a standard way of representing vectors. But what do we really mean by a vector? Vectors are any quantities which add and subtract in the same way as displacements. Notice that vectors are really new quantities and cannot be described by single numbers. The physical magnitudes which can be given with single numbers we shall call scalars. The temperature of a room is a good example of a scalar physical quantity just as a displacement 3 kilometers north-east is a good example of a vector quantity. We shall meet several types of vectors in physics.

6.2 Velocity Vectors

If an airplane has a speed of 250 km/hr through the surrounding air and flies in a wind of 50 km/hr, how fast does the airplane go over the ground? This question cannot be answered without further information, such as whether the wind is a head wind (the wind against you) a tail wind (the wind from behind and helping you) or a cross wind. Because the direction of motion as well as the speed is important in problems like this, we need vectors to represent motions, just as we needed them to represent displacements.

In chapter V we saw how to find the instantaneous speed of an object, such as a car, moving along a fixed path. Now we shall represent the direction of motion of a body by the direction of a vector, and represent the instantaneous speed (regardless of direction) by the magnitude of the vector. We call this vector the instantaneous velocity.

When we determine the speed of a car by reference to kilometre-posts along a road, we find the speed relative to the road. To take into account the direction of the road,

we specify it with respect to some real or imaginary fixed line on the ground, such as a line pointing north. When the speed and direction are both described, we have specified the velocity of the car relative to the ground.

It is important to recognize that we are using two words "velocity" and "speed" in a technical sense which is different from everyday usage. When we are interested only in how fast a body is moving, we shall speak of speed. When we are concerned with both the magnitude and the direction, we shall use velocity. Thus, velocity is a vector and speed is a scalar

Let us turn back to the example of the airplane mentioned at the beginning of this section. How can we find out how fast the airplane moves relative to the ground? To see what is involved, let us start with a simpler situation. Suppose we are in a free balloon, a few hundred feet in the air. If the air is perfectly still, the balloon will not move at all. If the whole mass of air is moving at 10 km/hr—in other words, if there is a 10 km/hr wind—the balloon moves right along with it Only when the air mass changes velocity does the air "blow against" the balloon, speeding it up or slowing it down until it again moves at the same speed as the air.

Now let us imagine ourselves in a blimp, a large balloon with engines. With the engines off, the blimp moves only as the wind moves. But when the engines are going, the propellers pull us through the air at a certain velocity—a velocity with respect to the whole mass of air. Our motion relative to the ground now results from the combined effect of the motion of the air relative to the ground and of the motion of the blimp through the air.

In an airplane, the same thing happens. The airplane is supported by the air. Its engines give it a velocity through the air which must be added to the wind velocity to obtain the airplane velocity relative to the ground. In particular, let us go back to the example of the airplane headed north, travelling through the air at 250 km/hr. Suppose the 50 km/hr wind is blowing toward the east. By "headed" north we mean that the nose of the airplane is due north of the tail. A few minutes later the nose is still due north of the tail, but the airplane is not due north of its former position. As well as going north, the airplane has moved east along with the whole mass of air. In fig. 6 8(a) the airplane is shown headed north at A. If there were no wind, the airplane would move in the direction in which it is pointed and would later be located at B. But during the time t, while the plane would advance the distance t × 250 km/hr from A to B, the air has moved east by an amount

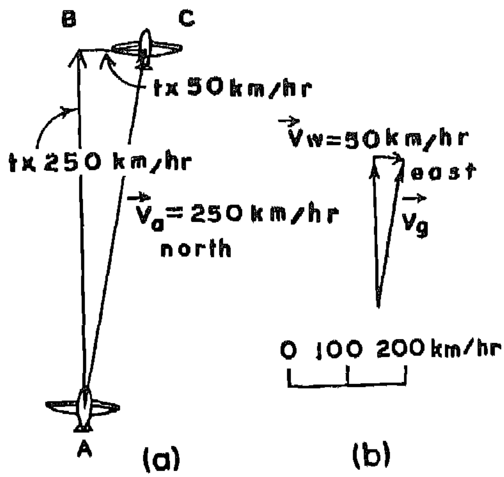


FIG. 6.8. *Part (a) shows the addition of the displacement vector $\vec{AB}$, which represents the airplane's motion through the air and $\vec{BC}$, which represents the displacement of the air. Their sum is the vector $\vec{AC}$, the displacement of the airplane relative to the ground. On dividing by the elapsed time, the velocity vectors are found as shown in part (b). The vector sum of its velocity relative to the air and the velocity of the wind relative to the ground.*

$t \times 50$ km/hr, and so the plane arrives at the position C

The addition of the two displacement vectors $\vec{AB}$ and $\vec{BC}$ leads to the vector $\vec{AC}$, the displacement of the plane relative to the ground These three vectors form a triangle. Each vector is proportional to a speed in a particular direction; $\vec{AB}$ to the speed north, $\vec{BC}$ to the speed east, and $\vec{AC}$ to the speed of the plane in the direction of motion with respect to the ground On dividing each of these displacements by the elapsed time, we get a similar triangle representing the corresponding velocities Since we now see that these velocities add exactly as the displacements, we have proved that velocity is a vector In fig. 6·8(b) the velocity vector diagram is shown The velocity $\vec{v}_a$ of the plane relative to the air is plotted in the same direction as $\vec{AB}$. It is a line north of length proportional to 250 km/hr. To this velocity we must add the velocity of the wind relative to the earth $\vec{v}_w = 50$ km/hr pointed east. We then obtain the velocity $\vec{v}_g$ of the plane relative to the ground. Both its direction and its magnitude can be obtained directly from its graphical representation.

Because $\vec{v}_a$ and $\vec{v}_w$ are at right angles in this example, it is easy to calculate the magnitude of the velocity relative to the earth (called the ground speed). Using the Pythagorean relationship, we see that

$$v_g = \sqrt{v_a^2 + v_w^2}$$

$$v_g = \sqrt{(250 \text{ km/hr})^2 + (50 \text{km/hr})^2}$$
$$= 255 \text{ km/hr}.$$

Notice that the various speeds are indicated by v_g, v_a and v_w, the same symbols that are used for the velocities, except that they do not carry the vector sign over them. This is a very convenient and commonly used notation. You can practice up on vectors and in particular on relative velocities by working through the practical navigation problem discussed in the boxed material. It is more complicated than the problem we just discussed, but the basic ideas are the same.

A NAVIGATION PROBLEM

As a practical example of the relations that we have found, suppose that the navigator of an airplane wishes to go from a city C to another city D which is 900 km from C in a direction 30° east of north. The meteorologist tells him that there is a wind blowing from west to east with a speed of 50 km/hr, and he knows that the pilot plans to maintain an air speed of 240 km/hr. To get to D, the airplane must move over the earth in a direction 30° east of north. It is the navigator's problem to give the pilot his heading—to tell him the direction in which the airplane must be pointed.

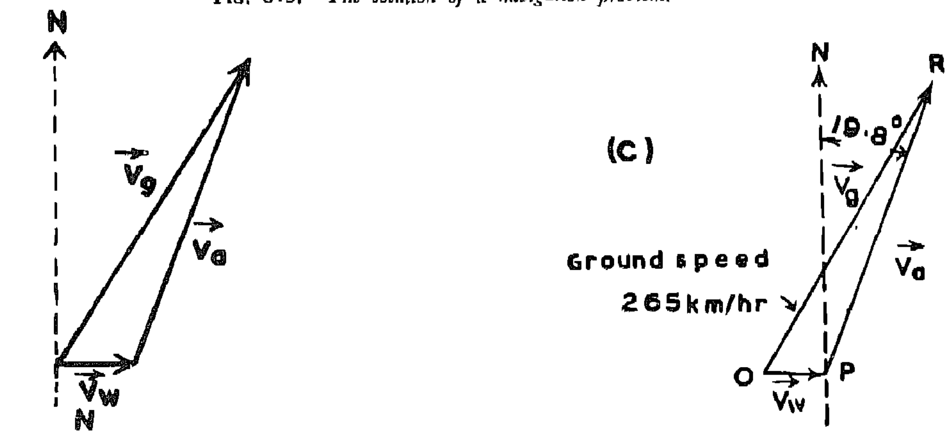
The navigator solves his problem by constructing a velocity vector diagram like that shown in Fig. 6.9(a).

Here the velocity $\vec{v}_g$ of the airplane

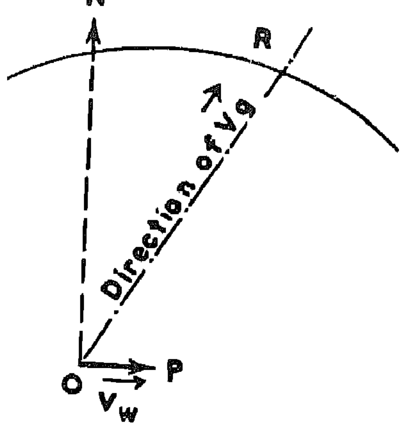
over the ground is made up as the vector sum of the wind velocity $\vec{v}_w$ and the airplane's velocity relative to the air $\vec{v}_a$. The navigator starts drawing his diagram from any point O, as in Fig. 6.9(b). He knows the direction in which the airplane must go, that is, the direction of $\vec{v}_g$, and he lays this direction out from O. Also he knows the wind velocity completely so he lays out that vector It runs directly east from O and its length OP represents the wind speed of 50 km per hour. Now to get the velocity vector $\vec{v}_g$ of the airplane with respect to the ground, the navigator must add the velocity $\vec{v}_a$ of the airplane with respect to the air. The vector $\vec{v}_a$ must therefore start from the point P, and its length (given by the speed of the airplane) must represent 240 km per hour. But the navigator does not know the direction of $\vec{v}_a$. All he can do, therefore is to draw a circle with its centre at P and its radius the right length to represent the speed of 240 km per

FIG. 6.9. *The solution of a navigation problem.*

(a)



(b)



hour. In Fig 6.9(b) we see the direction of $\vec{v}_g$ drawn 30° east of north, we see the vector $\vec{v}_w$ representing the wind velocity; and we see the circle showing the possible end points of the vector $\vec{v}_a$ drawn from the centre P Notice that this circle intersects the direction $\vec{v}_g$ of the airplane with respect to the ground in only one point, the point R on the

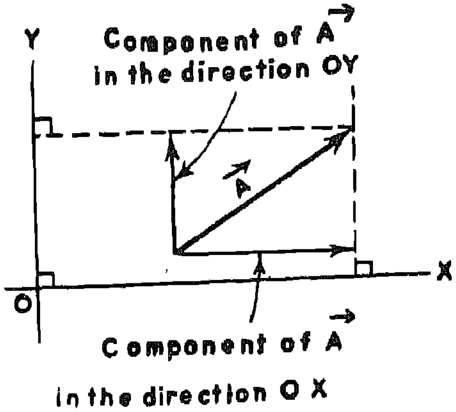


FIG. 6.12 *How to construct the rectangular components of a vector in a plane.*

magnitude and direction of velocity with this pair of components. Fig. 6.14 illustrates the point. As we see, the vector sum of the north and east components is the velocity. Notice the similarity between locating points on a graph by giving the coordinates and specifying a vector by giving its east and north components. When we give the coordinates of points on a graph, we are telling how far across the graph paper and how far up the graph paper we must go from the origin to locate the point.

We could represent a vector, using reference directions that are not at right angles. But rectangular components have an important advantage over any other choice. As we saw in the case of the swimmer, they are independent of each other All sets of rectangular reference directions have this property of independence. If a vector $\vec{R}$ is perpendicular to a reference line AB, what is the component of $\vec{R}$ along AB? Does the component of $\vec{R}$ along AB change when the component perpendicular to AB is doubled?

We have needed only two components to specify a vector because we have been

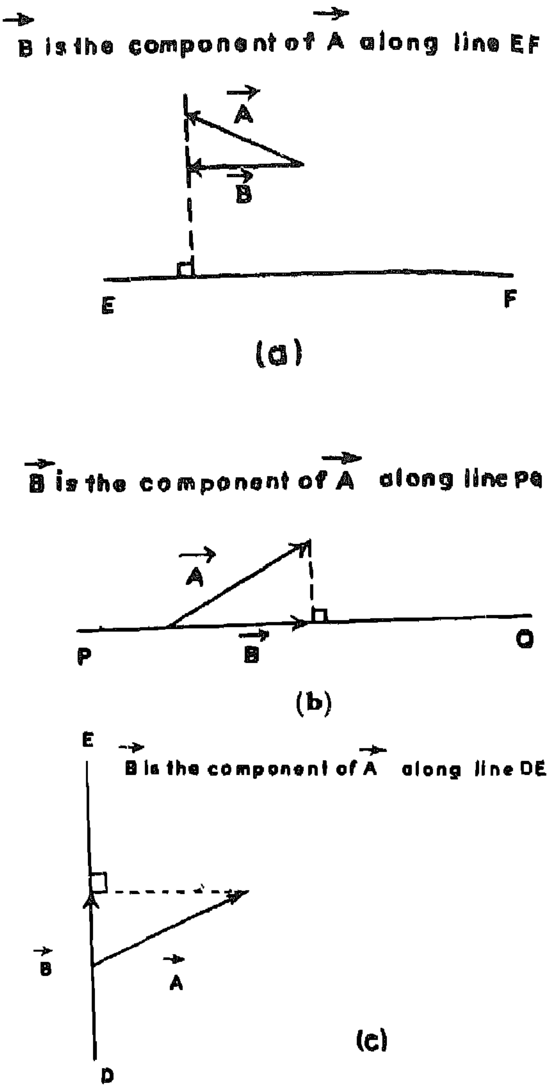


FIG. 6 13 *Examples of components of vectors along various directions.*

considering vectors in a plane—that is, in two dimensions. In three dimensions we can specify a vector by giving its components along three mutually perpendicular directions. As illustrated in Fig. 6.15, these three rectangular components added together give the vector.

6.4 Multiplying Vectors by Numbers and Scalars

Suppose that you have a vector $\vec{A}$, as shown in Fig. 6·16, what do you think is meant by $2\vec{A}$? Because multiplying a quantity by two means adding the quantity to itself, we conclude that $2\vec{A}=\vec{A}+\vec{A}$ and

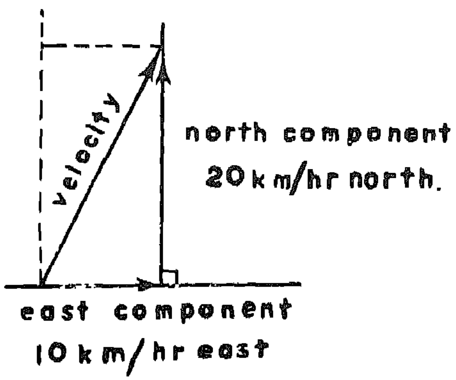


FIG. 6.14. *Finding a vector whose components are known. The velocity is 22 km/hr 27° east of north.*

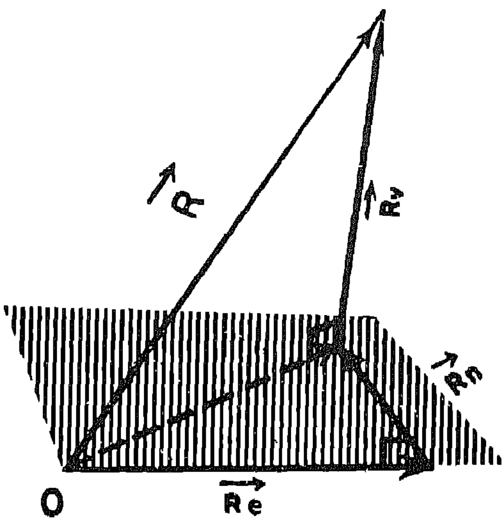


FIG. 6 15 *$\vec{R}_e$, $\vec{R}_n$, and $\vec{R}_v$ are the east, north, and vertical components of vector $\vec{R}$*

we see that $2\vec{A}$ is a vector twice as long as $\vec{A}$ pointing in the same direction. In general $k\vec{A}$, where k is any positive number, means a vector parallel to $\vec{A}$ and k times as long. What would you expect if k is negative?

What do we mean by $k\vec{A}$ when, for instance, $k=\frac{1}{4}$? In algebra $\frac{1}{4}x$ means a number which added four times over gives x. Here by $\frac{1}{4}\vec{A}$ we mean a vector which added four times over gives $\vec{A}$ The vector $\frac{1}{4}$ as long and in the same direction as $\vec{A}$ will do this job. You can show that no other vector will do, since any other vector differs in direction or in magnitude.

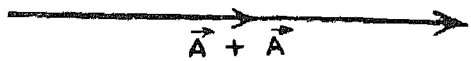


FIG. 6 16. *Multiplication of a vector by the number 2*

Briefly then, multiplying or dividing a vector by an ordinary number means multiplying or dividing the magnitudes while leaving the direction unchanged.

The rule that we have just found for multiplying a vector by a number such as 2 works just as well when a vector is multiplied by any scalar quantity such as a time interval. There is, however, one important difference between multiplication by a simple number and multiplication by a scalar that has units. When a vector is multiplied by a number, the new vector has the same units as before. It can be drawn on the same diagram as the original vector and to the same scale. When a vector is multiplied by a

scalar having units, the units of the product are different from those of the original vector. To avoid confusion, it is best to represent the new vector on a new diagram

Suppose, for example, that an airplane is flying north-east at 300 km/hr. We can represent its velocity vector as in Fig. 6.17. If the plane flies for $\frac{1}{2}$ hour it will undergo a displacement, $\vec{R}=\vec{v}t$, of (300 km/hr, north-east) $\times$ ($\frac{1}{2}$hr) = 150 km north-east. We can represent this displacement vector on another vector diagram, as in Fig. 6 18 (a), with a different scale to represent a distance instead

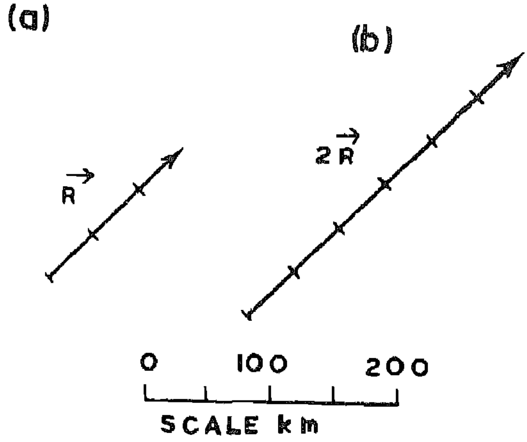


FIG 6.18. *Displacement vectors for $\frac{1}{2}$ hour (a) and for two $\frac{1}{2}$ hours (b). The vector in (a) is obtained from that in Fig. 6.17 by multiplying $\frac{1}{2}$ hour by the velocity, a quantity with dimensions. The vector in (b) is obtained from that in (a) by multiplying by the pure number 2.*

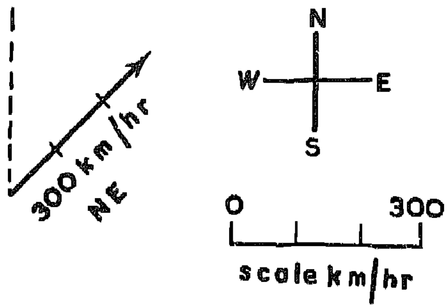


FIG. 6 17. *The velocity vector of an airplane flying north-east at 300 km/hr.*

of a rate of motion. If the airplane now continues for another $\frac{1}{2}$ hour, it undergoes a second equal displacement, and the sum of the two displacements must be just two times the orginal one. The total displacement is then found by multiplying the original displacement vector by the number 2. The total displacement of 300 km north-east, as in Fig. 6.18 (b), can usefully be represented on the same vector diagram as the original displacement. It should not be placed on the same diagram as the velocity vector of 300 km/hr because they are two different things, different physical quantities with different units.

6.5 Velocity Changes and Constant Vector Acceleration

Fig. 6.19 is a multiple-flash photograph of two balls. The ball at the left is falling straight down. Let us analyse its motion by finding the velocity vectors at successive intervals as the ball falls. We can get the average velocity vector for a given time interval by measuring the distance between two images of the ball and dividing by the time between the flashes which made those images This gives us the length of the velocity vector; its direction is the direction of motion of the ball from one image to the other. In Fig. 6.20 we have measured off the successive displacements and put them side by side. The black strings in Fig. 6.19 are 6 inches apart and the interval between flashes is $\frac{1}{30}$ of a second. Using these facts, we have computed a scale so that we can read these vectors directly as the average velocities.

A glance at Fig. 6.20 shows that the velocity vector changes steadily. In each

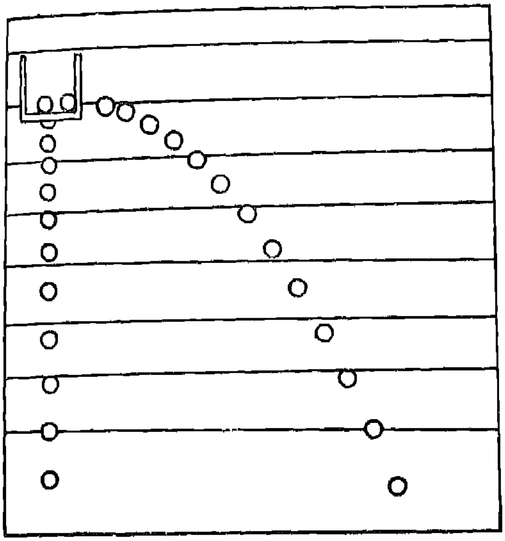


FIG. 6.19. *A flash photograph of two golf balls released simultaneously from the mechanism shown. One of the balls was allowed to drop freely, and the other was projected horizontally with an initial velocity of 2 m/sec. The light flashes were 1/30 of a second apart. The lines in the figure are a series of parallel strings placed behind the golf balls, six inches apart*

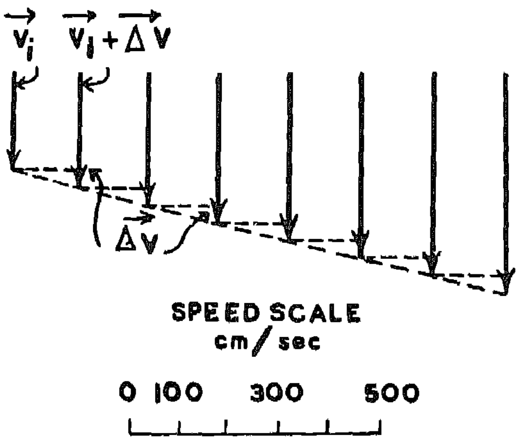


FIG. 6.20 *The length of the arrows is equal to 2½ times the displacements of the left-hand ball in* Fig. 6.19 *during the last eight successive intervals of* 1/30 *of a second. Because we know the actual separations of the white lines in* Fig 6.19 *and the time intervals, we can turn the magnitudes of these displacements into speeds. The scale enables you to read the lengths of the arrows directly as speeds in cm/sec.*

successive interval it increases by the same amount. Consequently, we can find the velocity as $\vec{v}_n = \vec{v}_i + n\vec{\Delta v}$. Here $\vec{v}_i$ is the velocity vector with which we start. $\vec{\Delta v}$ is the constant change in velocity that occurs in each interval. By adding n of these changes to the original velocity, we get the velocity n intervals further along.

We can rewrite the last equations so that it more closely resembles the equations we developed for the description of motion along a pre-assigned path—(see chapter V). There we defined $a = \Delta v/\Delta t$, that is, the acceleration along the path. Here by dividing $\vec{\Delta v}$ by Δt we shall introduce the vector acceleration $\vec{a} = \vec{\Delta v}/\Delta t$. Using it, our last equation becomes

$$\vec{v}_n = \vec{v}_i + n\,\Delta t\,\frac{\vec{\Delta v}}{\Delta t},$$

$$= \vec{v}_i + \vec{a}t.$$

In the last line of this equation we have replaced $n\Delta t$ by t, the time during which the velocity has changed from its initial value $\vec{v}_i$ to its value in the nth time interval.

The velocity of the falling ball really increases steadily, as we can show by taking flash pictures at smaller and smaller time intervals. Therefore, after any time interval t it is $\vec{v}_t = \vec{v}_i + \vec{a}t$.

We have just described the motion of a falling ball in vector language. But since the ball moves on a predictable straight downward path, we could have done the analysis equally easily. We would only need to add a statement that the motion is always straight down. The vector language, however, becomes far more useful when we analyse a more complicated motion. To

see this let us get back to Fig 6·19 and study the motion of the other ball, the one which moves out to the right in the figure.

The second ball in Fig. 6 19 moves both to the right and down From the fact that the distance between the positions of the ball at successive flashes of the strong light is greater for the later pictures, we see that the speed is increasing Since the path is not a straight line, the direction of the velocity is changing too. We can analyse Fig. 6·19 to get the instantaneous velocity of the ball at various points along the path*. The results of such an analysis are shown in Fig. 6.22.

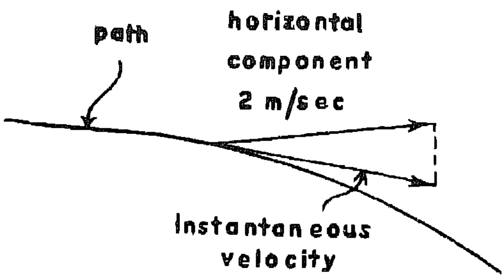


Fig. 6 21 *How to find the instantaneous velocity vector It is tangent to the path and of such length that its horizontal component is equal to the initial horizontal velocity of the projectile.*

There both the position of the ball and its instantaneous velocity at 0·10 sec intervals are shown in the same graph Note the two scales, one for distance and other for velocity

Fig. 6·23 shows only the sequence of velocity vectors of Fig 6·22. Here, however, we have drawn the velocity vectors from the same starting point. Examination of this figure shows us that the successive vectors

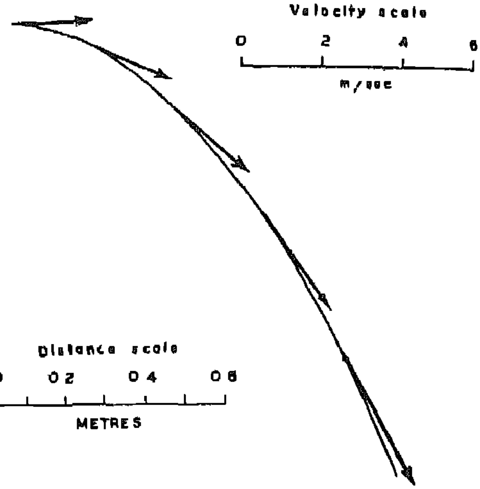


Fig 6·22. *The position and velocity of the "thrown" golf ball in Fig 6 19 are shown here on a single graph.*

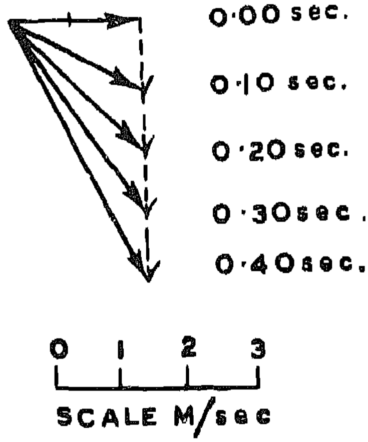


Fig 6 23 *A sequence that shows only the velocity vectors of Fig 6·22 Successive vectors are found by adding a constant vector directed vertically downward.*

are obtained by adding a velocity vector of about 1 m/sec (actually it is 0·98 m/sec) directed vertically downward. We can express this rule in equation form. To do this we first express the components of the velocity. The horizontal component of velocity.

$\vec{v}_h = 2\cdot00$ m/sec, to the right stays constant throughout the flight. On the other hand,

*One way of finding the instantaneous velocity in this case is to note that the horizontal component of the velocity is constant This follows from the fact that the horizontal displacement is the same in each time interval We then get $\vec{v}$ from this fact and the fact that the instantaneous velocity vector always points in the direction of the path (see Fig. 6 21). Other methods of analysis may give more precise results.

the vertical component is zero at $t=0\cdot00$ sec and increases by $0\cdot98$ m/sec during each $0\cdot10$ sec. This is uniform increase at the rate of $9\cdot8$ m/sec², so the vertical component of velocity at any time t is $\vec{v}_v=(9\cdot8 \text{ m/sec}^2)$ t, downward, where t is the time in seconds.

Now, combining the two rectangular components to give the vector velocity at any time t, we get

$$\vec{v}_t = (2\cdot00 \text{ m/sec}), \text{ to the right} + (9\cdot8 \text{ m/sec}^2)t, \text{ downward}$$

The downward component of this vector is the product of a time and an acceleration. Since the time is measured in seconds and the acceleration in m/sec², the product has the units of m/sec, appropriate to a component of velocity. This is another illustration of the fact stated in section 6·4, that the multiplication of a vector by a scalar gives a new vector, having the same direction, but of magnitude equal to the product of the scalar times the original vector.

We can put our equation for the motion of the ball thrown to the right into just the same form as we did for the ball falling straight down. Here (2 00 m/sec, to the right) is the initial velocity $\vec{v}_i$, and (9·8 m/sec², down) is the constant acceleration $\vec{a}$. So we get $\vec{v}_t = \vec{v}_i+\vec{a}t$ again. Notice that the acceleration of both balls is down. Also the downward motion is the same for both, as you can see by looking across the picture to check that one moves down the same amount as the other in each time interval. The only difference in describing the motion of the two balls is the value of $\vec{v}_i$. The acceleration vector $\vec{a}$ is the same for both.

In chapter V we found that $v_t=v_i+at$, described the speed at any time when there was constant acceleration along the path The equation $\vec{v}_t=\vec{v}_i+\vec{a}t$, describes the velocity vector at any time as long as the acceleration vector is constant. Notice that the acceleration vector need not point along the path, as in the example of the right-hand ball in Fig. 6 19 Furthermore, $\vec{a}$ can be any constant vector of the right units The 9·8 m/sec², down, which we found for the balls, is just a special value that occurs when balls move freely near the surface of the earth. In studying other motion we shall find other constant values of $\vec{a}$ and also acceleration vector that change as time goes on. For instance, by pushing a ball we can get any $\vec{a}$ we wish.

6·6 Changing Acceleration and the Instantaneous Acceleration Vector

In the last section we described motion with constant vector acceleration. We introduced the vector acceleration to describe how the velocity changes. Even when the acceleration vector is not itself constant, we can introduce it in just the same way. We define it by $\vec{a}=\frac{\Delta\vec{v}}{\Delta t}$ where $\Delta\vec{v}$ is the vector change in v during the time interval Δt. Notice that this vector acceleration has the same direction as the change $\Delta\vec{v}$ of the velocity since this change need not be in the same direction as $\vec{v}$, the acceleration $\vec{a}$ may point in any direction with respect to the motion. As we saw in the case of the right-hand ball in the last section, it need not be along the motion.

The vector acceleration we have just defined is the average acceleration over the time interval Δt. If the acceleration is itself changing as time goes on, $\vec{a}$ will depend on the time interval we choose. Let us take an example. Suppose a speedboat moves along the path shown in Fig 6.24(a). At time t_1 it is moving with vector velocity $\vec{v}_1$, and at the later time t_2, it is moving with vector velocity $\vec{v}_2$. What is the average acceleration in the time interval $\Delta t = t_2 - t_1$? To find out we must determine the vector changes $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ in the velocity. Then the average acceleration is $\vec{a} = \Delta\vec{v}/\Delta t$. The procedure is indicated in Fig. 6 24(b). We take the vector difference between $\vec{v}_2$ and $\vec{v}_1$, divide $\Delta\vec{v}$ by Δt, and plot the average acceleration vector $\vec{a}$ for the time Δt on a new diagram with the appropriate scale

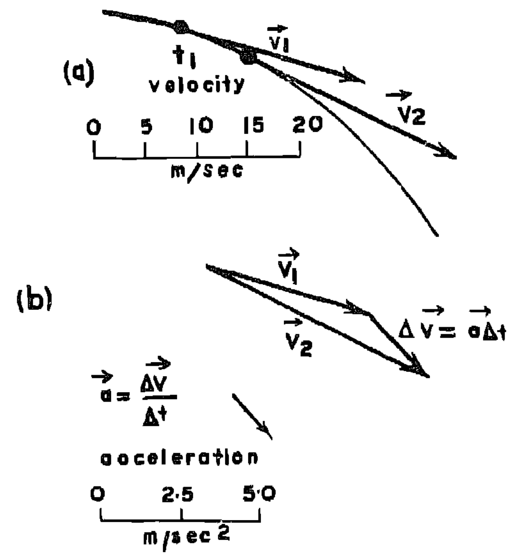


FIG. 6.24. *To find the average acceleration in the interval $\Delta t = t_2 - t_1$: First find $\Delta\vec{v} = \Delta\vec{v}_2 - \Delta\vec{v}_1$ and divide the vector difference by Δt The result is the average acceleration vector a, which can now be plotted with an appropriate scale as shown above.*

As the speedboat moves on, its acceleration may change. In fact we do not expect that it has remained constant even over the time interval from t_1 to t_2. In such situations we need to know the instantaneous vector acceleration at various times t rather than only the average acceleration vector over various intervals.

To find the instantaneous acceleration vector at any particular time t, we determine the average acceleration vector for shorter and shorter time intervals which include the time t. We define the instantaneous acceleration at a given time as the limit of the average acceleration $\Delta\vec{v}/\Delta t$ as the interval Δt becomes smaller and smaller. Usually this limiting vector has a definite size and direction.

Suppose, for example, that Fig. 6.25 shows the successive velocity vectors of our speedboat at time intervals of 10 seconds. If we make a composite picture in which the tails of these vectors coincide, we get Fig. 6.26. The changes in velocity during each successive time interval are also shown in this figure. We see that the velocity changes $\Delta\vec{v}$ in each of the four intervals differ both in magnitude and in direction. Therefore,

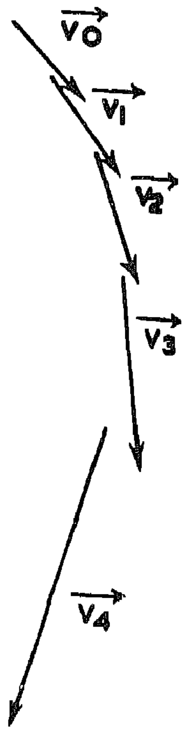
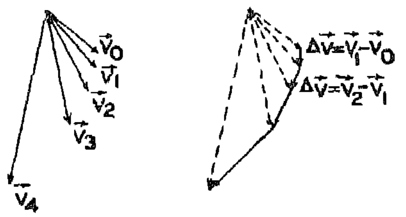


FIG. 6.25 *Successive velocity vectors of the speedboat at 10-second time intervals.*

since the time intervals are the same, the average accelerations are different.

If, instead of using 10-second intervals, we had used 2-second intervals, we would have got the pictures shown in Fig. 6.27. Since it is difficult to see the details of Fig. 6.27, a magnified portion of the first-10 seconds is shown in Fig 6.28, using five fold magnification. Notice that all five of the velocity changes have more nearly the same magnitude and direction than those of Fig. 6.26(b).



(a) (b)

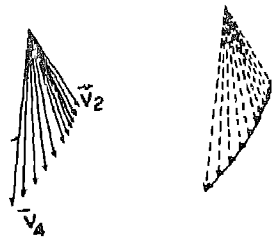
Fig. 6.26. *The vectors of Fig 6.25 are shown here drawn from the same origin. The acceleration is not constant in direction or magnitude, as shown in (b).*

If we had chosen a time interval shorter than 2 seconds, the successive velocity changes would have been even more alike. Usually we can choose time intervals so short that the acceleration does not change appreciably either in magnitude or direction in going from one interval to the next. With this understanding of "very short" we see that the instantaneous vector acceleration at time t is the average acceleration for a very short time interval that includes the time t. We can therefore say that the instantaneous acceleration is the limit of the ratio $\Delta \vec{v} / \Delta t$ as Δt becomes very small. Just as the instantaneous acceleration along the path is

$$a_p = \lim_{\Delta t \to 0} \Delta \vec{v} / \Delta t$$

so the instantaneous vector acceleration is

$$\vec{a} = \lim_{\Delta t \to 0} \Delta \vec{v} / \Delta t$$



(b)

(a)

Fig. 6.27. *If we take 2-sec time intervals, the motion described in Fig. 6.25 looks like this when the vectors are drawn from the same origin. By taking a shorter time interval, the changes of velocity become more nearly equal.*

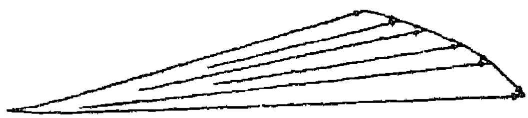


Fig. 6.28. *When the time intervals become very short, the successive velocity changes are nearly equal in magnitude and direction. Here are the first 10 secs from Fig. 6.27. The vectors are magnified 5 times and rotated to fit the page.*

The really new thing that we learn by considering vector acceleration as compared with acceleration along the path is that changes in the direction of the velocity give rise to an acceleration. Even if a body moves along a curved path at constant speed (see section 5.6), it is accelerated. The simplest and most important example of this is a body moving in a circular path at constant speed. This is accelerated motion and the acceleration is a changing one. In Fig. 6.29 the circular path is shown and on it a sequence of velocity vectors at equal time intervals. These vectors are all of the same length, but each points in a direction different from any other. If we construct a velocity diagram, as in Fig. 6.29 (b), in which the velocity vectors are plotted from

a common origin, we see that the successive changes $\overrightarrow{\Delta v}$ in velocity are also different in direction one from the other. Because the successive changes in velocity are not parallel to each other, the vector acceleration which measures the change of velocity with time cannot be constant. By taking smaller and smaller time intervals, as in Fig 6 29(c), we can see that the instantaneous acceleration vector is directed perpendicular to the velocity vector at each instant of time. The fact that the angle between the acceleration and velocity vectors stays constant (it is always 90°) means that they rotate at the same uniform rate.

This example of constant-speed motion shows that acceleration may result from a change in direction without any change in speed. For motion along a straight line, on the other hand, the acceleration is the rate of change of speed In general, if a body moves on a curved path, its acceleration vector stands at an angle to the motion. Then the component of acceleration along the path gives the rate of change in the speed (Fig. 6.30), while the component perpendicular is related to the change in direction of the velocity vector (Fig. 6.31). Can you use this fact to prove that the acceleration in uniform circular motion (Fig. 6.30) must be directed along the radius of the circle?

We can make an object move with constant speed along any path. Because the speed does not change, the instantaneous acceleration then can have no component along the path. The acceleration vector is perpendicular to the direction of motion, that is, perpendicular to the path at all points. Motion at constant speed on a circular path is therefore only one of many motions in which the velocity vector and the acceleration vector are perpendicular to each other. Circular motion is the special case in which

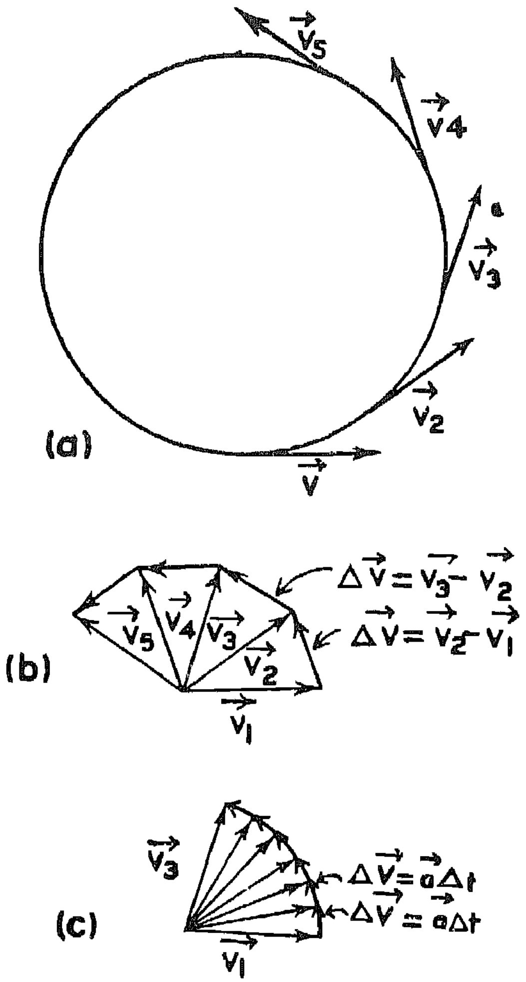


FIG. 6 29 *Part (a) shows the velocity of a body moving at constant speed in a circular path. In (b) the velocity vectors are drawn from a common origin, showing that the changes in velocity are in different directions. By taking shorter time intervals, as in (c), the instantaneous acceleration vector is seen to be perpendicular to the velocity vector.*

the magnitude of the acceleration stays constant. When on the other hand, a body moves on a more complicated path at constant speed, the magnitude of the perpendicular acceleration changes. The acceleration is greater where the path curves more sharply.

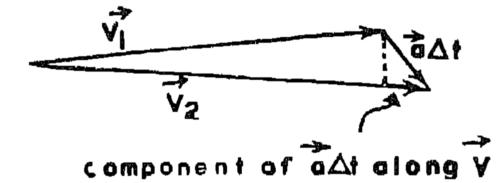


FIG. 6 30 *The component of $\vec{a}\Delta t$ along $\vec{v}$ is the change in magnitude of $\vec{v}$ When the component is zero there is no change in speed.*

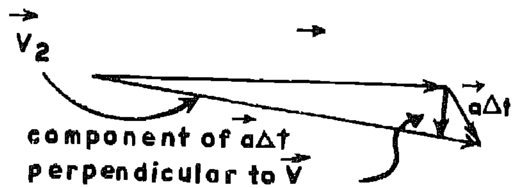


FIG 6.31. *The component of $\vec{a}\Delta t$ perpendicular to $\vec{v}$ changes the direction of $\vec{v}$. When the component is zero there is no change in direction.*

6 7 The Description of Motion; Frames of Reference

We have studied vectors largely in order to describe motion, to describe the successive positive positions of an object in space and how fast it moves through them. The graceful motions of a waltz (a type of western dance) can be described fully, as a sequence of positions of the hands, the feet, and the rest of the bodies of the dancers, an appropriate position for each instant of time. In fact, dancing to music implies a manner of time measurement. For each note, there is a correct figure of the dance; and to each note, a proper instant of time.

The simplest and most fundamental motion is the motion of a single object, whose parts we do not find it necessary to distinguish, either because we cannot observe them or because they remain fixed relative to one another A planet, such as Jupiter, moves in the sky We can represent its position at each instant of time by a vector which we imagine drawn from the earth to the planet Jupiter An airplane drones on its course for some destination; its positions can also be represented by vectors (Fig. 6.32). Neither the planet nor the airplane is a tiny point; and, if we wish to know what happens within them, we would need much more information than is given by these position vectors. But for many of the needs of astronomy or of navigation, it is sufficient to think of each as a point at the tip of a vector. As the planet or airplane moves, the vector moves, changing in magnitude and direction.

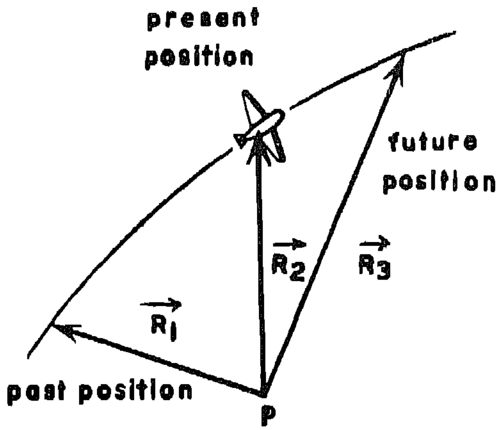


FIG. 6 32. *The positions of an airplane with respect to a point may be shown by position vectors giving distance and direction.*

In each of our examples the position vector extends from some identifiable point called the origin to the moving object, from the earth to Jupiter, from the control tower to the airplane. Furthermore, motion, as we noticed in section 6.2, is always measured with respect to some frame of reference, and so are positions. The airplane's motion is described with respect to the earth's surface, for example, and so is the motion of the falling balls in Fig. 6.19

The motion of the airplane with respect to another airplane may be very different

from its motion with respect to the ground The motion of the thrown ball in Fig. 6.19 is just motion with a constant velocity with respect to the other ball falling on the left of the figure. The motion of Jupiter with respect to the sun is simpler than its motion with respect to the earth. To the driver of a moving car a raindrop falling vertically with respect to the earth rushes almost horizontally towards him Motion is described differently depending on the frame of reference with respect to which we give the description.

In general, we wish to view motion in such a way as to make it appear simple. We therefore place ourselves mentally in a frame of reference in which the motions are easy to describe, and draw the position vectors locating the object from the most convenient point Another example will show what we mean. Suppose we stand on the earth again and look at the motion of a point on the edge of a slowly moving wheel of a car. The point moves through the curve illustrated in

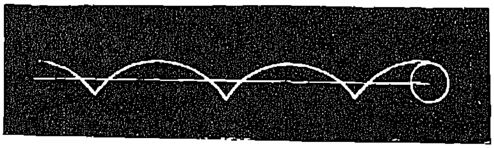


FIG. 6.33 *Path of a point on the rim of a rolling wheel. This curve a cycloid, shows the path as it appears to an observer standing alongside. One small light was mounted on the rim of a wheel and another light of the centre The camera shutter was held open while the wheel rolled*

Fig. 6.33, a complicated curve known as a cycloid A position vector from us as origin to a point on the wheel performs an extremely complicated motion. But a point on the earth is not always the most convenient origin for position vectors We are far better off if we get in the car and hang out of the window to look at the wheel The point on the wheel then moves steadily around a circle, and the motion of the position vector looks far simpler.

In describing motion we try to put the origin in the most convenient place even though we are sometimes physically unable to go there The motions of the planets, for example, look very complicated when we describe them with position vectors whose origin is on the earth. Copernicus pointed out that the planetary motions are much simpler to describe when we assume that the sun is the centre of the solar system, and move the origin of the position vectors to the sun. Ever since Copernicus pointed out the importance of placing the origin in a convenient place, the proper choice of origin has been an important technique used by physicists to describe motion in simple terms.

6.8 Kinematics and Dynamics

What we have been discussing is the branch of physics called kinematics after the Greek word *kinema*, meaning motion. It is that part of our subject which treats with the description of motion, without taking into account what it is that moves or what causes the motion. The science of mechanics, indeed much of physics, is dominated by the study of motion; but that study is complete only when we extend it to what is called dynamics (after the Greek word *dynamis*, meaning power). In dynamics one discusses the causes of motion, what is moving, and how its nature affects the motion. In our study of kinematics, we needed to measure only positions and times; in dynamics, the pushes and pulls which cause and resist and determine motions must be taken into account as well.

Motion can be simple or complex. The method of physics is to analyse first the simpler cases, to extract what we can from them, and to go forward to more and

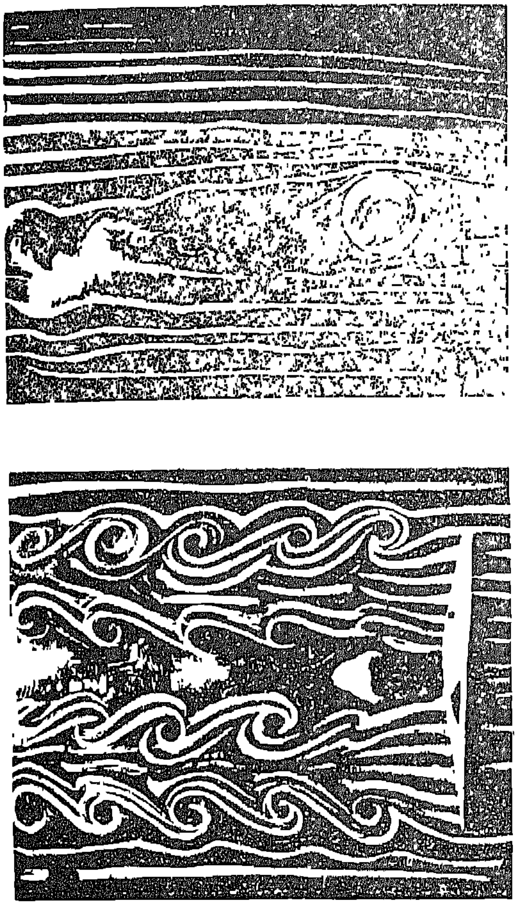


FIG. 6.34 (*Above*) *The motion imparted to air particles by a rapidly spinning ball, such as a pitcher throws for a curve, is an example of a very complex form of motion.* (*Below*) *Eddies formed behind a propeller spinning at* 4,080 *rpm. This is another example of a highly complex motion* (*Photos courtesy. F.N.M. Brown*).

more complicated cases. It would be a mistake to think that a quick elementary study of physics can explain the breaking of the surf on a shore, the path of a jet plane in the sky, or the intricate rhythms of a diesel engine. But these motions and many more can be handled by pursuing the same methods of analysis that we have outlined here in simple cases. When, instead of a single separate object like a pebble or a car, a whole fluid like the air or the water is in motion, even the kinematics may become difficult The eddies behind a spinning propeller and a flying baseball (Fig 6.34) are complex indeed Their study is a specialized kind of unraveling, requiring enormous "book-keeping" operations to keep track of every small bit of the moving air Powerful mathematical methods are needed, but the basic ideas used in solving such problems flow out of the discussion we have given; the underlying laws are not different. One may well be wary, though, of too simple an explanation of what is obviously kinematically complex You cannot design airplanes or understand the weather without checking theories with experiments or without interpreting experimental results in terms of theories We must go back and forth between theory and the experimental study of the real complex motions. Nevertheless, great progress has been and will be made with such a process. Nowhere in the whole domain of the motions of the natural world, here on earth or in the skies, in our large-scale machines or in our means of transport, have we yet found any case to which the analysis of mechanics does not apply. Where the kinematics is hard, progress is slow, where the kinematics is less demanding, progress is fast. In this course, where we are trying to learn fundamentals, we do not want to produce the false impression that there are no complex cases of interest; there are, especially in technology. In engineering and technology, the aim is to get something done for people to use; in a science such as physics, the aim is to understand and hence to predict and to control. In physics we seek out the simple and examine it, even if it is costly and difficult to attain In engineering the orderly methods developed in this research are painstakingly and

often brilliantly applied to more and more complex cases, which have clear usefulness for men The complicated trial-and-error extension always rests on the foundation of the simple and the well-understood.

6.9 The Speed of Light

We have pointed out earlier that from everyday experiences we can develop a wide extension or extrapolation. This is the kind of thing we have just been doing, basing our study of motion on the ideas of space and time we have built up from experience But if such schemes are pushed too far, they may go wrong. They need to be tested. For fifty years now we have been finding that our usual notions of space and time, extremely reliable for most of the motions that we notice daily, do not work for extremely fast motions. For speeds which begin to be of the order of magnitude of that of light, the kinematic we have just discussed begins to go wrong. It turns out that the speed scale cannot be extended indefinitely; if you add velocities that are too big, you reach a region where the rules of addition go wrong There arises a natural limit to speed, which cannot be surpassed. This universal speed limit is the speed of light in free space, which is known to be very close to 3×10^8 m/sec. The speed of the fastest rockets does not exceed about 2×10^4 m/sec. Speeds up to 10^5 m/sec span the whole range of speeds of every large object in the solar system, from planets and meteors to the engineering devices of men. Only small particles, electrons, and their kin, move appreciably faster. These closely approach the speed of light. For their study, the ideas of relativity kinematics are needed. But for everything else—solar-system astronomy, engineering, or any large-scale laboratory physics—the kinematics we have studied, that of Newton, is accurate enough. This whole topic is an example of how you can begin with familiar ideas, which hold well over a wide range, and reform them entirely when you reach another order of magnitude. The fact that familiar ideas may be modified to fit extreme conditions does not end their meaning and use in the domain for which they were originally built up and in which they have been amply tested. Newtonian kinematics is a good approximation to relativity kinematics whenever the speeds are small compared with the speed of light.

Questions

1. A vector has zero magnitude. Is it necessary to specify its direction ?
2. Several vectors, which are not all in the same plane, add together to give a zero resultant What is the minimum number of vectors which can satisfy this requirement ?
3. Rain-drops are falling vertically at 20 km/hr At what angle should you tilt your umbrella if you are walking at 10 km/hr ?
4. A racing pigeon can fly with a speed of 30 m/sec. The wind is blowing from the west with a speed of 15m/sec. In which direction must the bird fly in order to reach a destination north-east of his starting point ? If the distance to be covered is 25 kilometres, how long will it take ?
5. A balloon is rising with a vertical velocity of 2 m/sec. If, in addition, the wind is blowing with a velocity of 5 m/sec., in which direction is the balloon moving ?
6. A train has a velocity of 50 km/hr If I walk at 10 km/hr from one side of the train to the point directly opposite on the other side, what are the magnitude and direction of my velocity relative to the tracks ?
7. A rifle is fired from a jeep travelling with a speed of 10 m/sec. The target is stationary 100 m away in a direction at right angles to the velocity of the jeep The muzzle velocity of the bullet is 200 m/sec If the rifle is aimed directly at the centre of the target, how far to one side will the bullet strike ? How far to one side must it be aimed in order to hit the centre ?
8. A missile is fired toward the north-west with a speed of 5,000 m/sec. at 45° to the horizontal. Find the three rectangular components of its velocity if the X-axis points towards the west, the Y-axis towards the north and the Z-axis vertically upward.
9. A man follows this route : From his house he travels four blocks east, three blocks north, three blocks east, six blocks south, three blocks west, three blocks south, two blocks east, two blocks south, eight blocks west, six blocks north, and two blocks east. How far and in what direction will he be from home ?
10. A man walks along a path which is in the shape of a regular hexagon; each side is 10 m long. Two of the sides run east and west.
    (a) Describe his successive displacements.
    (b) What is his total displacement ?
11. A horizontal vector 10·00 cm long is added to another vector 10·00 cm long pointing 70·0° up from the horizontal. Make as accurate a scale drawing as possible to determine the magnitude and direction of their sum. In your answer try for 0·1 mm precision in length and 0·1° precision in angle.
12. A plane flies south-east at 600 km/hr. Draw a diagram of its displacement from its starting point after 0·50 hr, 1·00 hr, 1·50 hr, and 2·00 hr.
13. The air speed of a plane is 480 km/hr. What is the ground speed of the plane
    (a) with a 32 km/hr head wind ?
    (b) with a 32 km/hr tail wind ?

14. An airplane maintains a heading of due south at an airspeed of 540 km/hr. It is flying through a jet stream which is moving east at 250 km/hr.
    (a) What direction is the plane moving with respect to the ground ?
    (b) What is the plane's speed with respect to the ground ?
    (c) What distance over the ground does the plane travel in 15 min?
15. The pilot of an airplane which flies at an air speed of 300 km/hr wishes to travel to a city 600 km due north. There is a 40 km/hr. wind from the west.
    (a) What heading should the plane fly ?
    (b) How long does it take to make the trip ?
16. A steamer is sailing directly south at 25 km/hr in an area where the wind is from the south-west at 18 km/hr. What is the angle from true north of the smoke trail from the stack ?
17. A man rows a boat "across" a river at $1 \cdot 0$ km/hr (*i.e.*, the boat is always kept headed at right angles to the stream). The river is flowing at $6 \cdot 0$ km/hr and is $0 \cdot 20$ km across.
    (a) What direction does his boat actually go relative to the shore ?
    (b) How long does it take him to cross the river ?
    (c) How far is his landing point downstream from his starting point ?
    (d) How long would it take him to cross the river if there were no current ?
18. What is the magnitude of a displacement whose components along the perpendicular X, Y and Z axes are respectively $4 \cdot 00$ km, $2 \cdot 50$ km, and $8 \cdot 50$ km ?
19. At what range will a radar set show a plane flying at 10 km above the ground and at a distance on the map of 18 km from the radar station ?
20. A plane flying north at 320 km/hr passes directly under another plane flying east at 260 km/hr.
    (a) What is the horizontal component of the displacement of the second plane relative to the first 20 minutes after they pass each other ? 50 minutes after they pass ?
    (b) What is the horizontal component of the velocity of the plane flying east relative to the plane flying north ?
    (c) Does the direction of this velocity vector (relative to the earth) change ?
21. An ocean liner is travelling at 18 km/hr. A passenger on deck walks toward the rear of the ship at rate of $4 \cdot 0$ m/sec. After walking 30 metres he turns right and walks at the same rate to the rail which is 12 metres from his turning point.
    (a) What is his velocity relative to the water surface while walking to the rear ? While walking toward the rail ?
    (b) Draw the displacement vectors relative to the water surface for his stroll What was the total displacement from his starting point ?
22. A watch has a second hand $2 \cdot 0$ cm long.
    (a) Compute the speed of the tip of the second hand.
    (b) What is the velocity of the tip of the second hand at $0 \cdot 0$ second ? at 15 seconds ?
    (c) Compute its change in velocity between $0 \cdot 0$ and 15 seconds.
    (d) Compute its average vector acceleration between $0 \cdot 0$ and 15 seconds.

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CHAPTER VII

Newton's Laws of Motion

7.1 Introduction

In chapter V we studied the motion of bodies in a straight line and learnt that with the help of three equations of motion we can solve problems involving initial velocity, duration of motion, acceleration and the distance travelled. We however did not bother about the cause of motion or cause of acceleration responsible for producing the increase in the velocity of the moving body.

Aristotle (384–322 B.C.) had taught that the "natural state" of all bodies is a state of rest and that an external influence is always required to move a body or to keep a body moving with a constant speed along a straight line.

People believed in Aristotle's ideas for almost two thousand years until Galileo (1564–1642) made the startling discovery that a uniform motion in a straight line did not require any force to maintain it. So that if a body is in motion, it will continue to move with the same speed and along the same straight line provided it is left to itself and no external influence acts on it.

In nature we do not seem to observe this. If we slide a book on the table it stops or if we throw a ball up in the air it also stops after attaining a certain height and returns back. In these examples the motion changes because the condition "no external influence acts" is not satisfied. In the case of the book, it rubs against the table, whereas in the other case the earth "pulls" the ball down.

How did Galileo make this discovery? In order to study this aspect of motion Galileo rolled a marble ball on an inclined plane. This plane could be tilted up or down the horizontal at any angle. He noted that the ball when moving down the plane was accelerated and when moving up the plane it was retarded. From this he argued that when the plane slopes neither downwards nor upwards, the ball will neither have acceleration nor deceleration. So motion along the horizontal plane should be of constant velocity; it will be a uniform motion.

Galileo arranged his apparatus as shown in fig. 7.1. Two smooth planes

AB and CD are inclined to the smooth horizontal surface BC. The inclination of AB was kept fixed throughout, but that of CD can be changed.

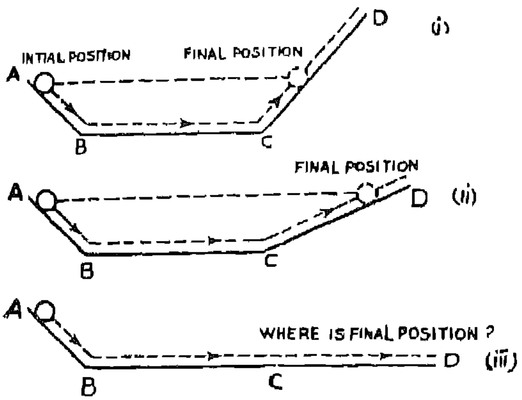

FIG. 7·1 *Galileo observed that a ball tends to rise to its original height regardless of the slope of the incline. With zero slope, the original height can never be reached. Therefore motion along a horizontal plane should be perpetual.*

When the marble ball was released from A on the plane AB, it climbed up the plane CD almost to the same height as that of A. If the surfaces of the plane and the ball were polished more carefully so that there is less friction, the ball would rise more nearly to the height of A before coming to rest. When the inclination of the plane CD to the horizontal is gradually decreased, it was found that the ball would again climb almost to the same height, but in doing so it has to travel over a longer distance on the plane CD [fig. 7·1 (i) (ii) (iii)].

From this he concluded that eventually when the plane CD becomes horizontal and surfaces were ideally smooth, the ball would roll on to infinity for it will not have to rise or climb to the height of the point of release while travelling on the horizontal plane.

This means that the ball should proceed to infinity with uniform speed along a straight line on the horizontal plane. Thus Galileo argued correctly that in the absence of friction the ball would continue to move with constant velocity for ever. In this way he came to the important principle that a body already in motion would continue to move in a straight line with the same velocity in the absence of an external influence.

We may note here that Galileo combined experimental result with logical reasoning in order to arrive at this revolutionary conclusion.

7.2 Newton's First Law of Motion

Newton put Galileo's ideas in the form of a law of motion, called the first law of motion. It states: "If a body is at rest it will remain at rest, or of it is in motion it will continue to move in a straight line with a constant velocity, unless compelled by some external agency to alter its initial state of rest or of uniform motion".

Thus the work of Galileo and Newton established that an external influence is necessary to *take the body out of the state of rest* or *to take it out of the state of uniform motion.*

Newton's first law of motion may be split up into two parts: (a) If a body is at rest, it continues to remain in the state of rest unless an external force acts on it. This part of the law agrees with the ideas of Aristotle.

(b) If a body is in motion, it continues to move with uniform velocity on the same straight line unless an external force acts on it.

As for the state of rest, our common experience completely agrees with this law. We can cite a number of examples to illustrate this law. The law states that a body at rest tends to stay at rest.

Place a smooth card on a glass tumbler. Place a coin on this card (fig. 7·2). Flick the card suddenly by striking at it horizontally with your finger. The card slides

off but the coin being left there falls into the tumbler. As there is almost no friction between the card and the coin, the flicking causes no force to act on the coin Thus the coin is left in the position of rest and since the card is no more there to support it, the coin falls into the tumbler

FIG. 7·2. *When we flick the card suddenly, the card slides off but the coin is left there and falls into the tumbler.*

When a bus starts moving forward suddenly, passengers inside experience a jerk backwards because their bodies tend to remain at rest unless a force acts on them This force is applied to them by the *seat-back* which of course moves forward with the rest of the bus and pushes the passengers forward. Similarly when a horse suddenly gallops forward, the man sitting on his back may be thrown back unless he presses hard his thighs against the horse.

This tendency of a body at rest to stay at rest is called inertia. (Other aspects of inertia will be discussed later).

The second part of Newton's first law that if a body is in motion it continues to move with uniform velocity in the same straight line if no external force acts on it, appears at first sight to be more difficult to understand. It is very difficult to so arrange that no external force acts on a moving body However in our daily life we come across many experiments which exemplify this law. When a car in motion is suddenly stopped, the passengers inside are thrown forward because their bodies continue in the state of uniform motion in a straight line Similarly, if a cyclist, when in speed, applies brakes suddenly, he is thrown forward because of the tendency of his body to continue in the uniform motion. It is on account of this law that people generally tumble down when they alight from a train in motion. On jumping from the moving train, the feet come in contact with platform but the rest of the body tends to continue moving with the velocity of the train, so the upper part of the body goes forward and the person tumbles down.

Take another example. Fasten a lump of stone at one end of a piece of string. Hold the other end in your hand By applying a pull on the string you can swirl the stone in a horizontal circle about you. Suppose the string breaks. Now the force causing the circular motion vanishes, but the stone does not cease to move. What is the direction of motion of the stone now? Some people may think that the stone will fly off radially. But actually this is not so At

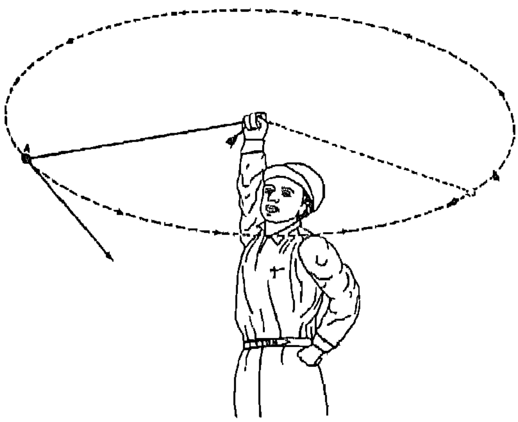

FIG 7.3 *At the instant the string breaks, the stone would fly off along the tangent of the circle described by the swirling string.*

the instant the string broke, the stone was moving on the circle and its instantaneous velocity was along the tangent to the circle. So after the string breaks, the stone would fly off along the tangent to move in the same straight line with uniform speed.

Now this tendency of bodies in motion to continue moving in a straight line with a uniform speed is also called *inertia*. We may call the tendency to remain at rest the *inertia of rest* and the tendency to keep moving with uniform speed in a straight line may be called the *inertia of motion*. Hence Newton's first law of motion is also sometimes called the *law of inertia* or the principle of inertia.

The first law of motion tells us of a certain property of all matter—inertia. The quantitative measure of inertia we shall call "mass". How is "mass" measured will be discussed later.

7.3 Newton's Second Law of Motion

The external influence which changes the state of motion of a body is known as force. When we push a book across the table we apply a force to it. When we throw a piece of stone in the air we apply a force and the greater the force with which we throw the same piece of stone the farther it goes, *i.e.*, the greater the force the greater the change in motion of a particular body. When we throw the stone, we become aware of the magnitude of the force by our own muscular effort. When the muscular effort is great, the force is large and when the effort is small, the force is small. But in physics we deal with precise measurements and our conception of the muscular effort does not give an exact idea of the magnitude of a force. When we are tired, a little effort may appear to be large.

Now we ask the question, when we apply some external influence which we call *force*—how does the motion of the body change. To establish the connection between the force and the way the motion of a body changes, Newton formulated his second law of motion. There are two alternative ways of stating the second law. According to one, "the body undergoes an acceleration in the direction of the applied force and the amount or magnitude of acceleration is directly proportional to the force and inversely proportional to its mass" (so far we have not said how to measure mass; and it is only a property possessed by every body (Newton's first law).

An alternative way of stating Newton's second law is this "The rate of change of momentum with time is proportional to the force". Here momentum is a quantity defined as the product of mass and velocity. Mathematically one may write this as :

$$\vec{F} \propto \frac{\Delta (m\vec{v})}{\Delta t} \propto \frac{m \Delta \vec{v}}{\Delta t} \propto m\vec{a}$$

where $\vec{F}$ is the applied force, m the mass, $\vec{v}$ the velocity, t the time and $\vec{a}$ the acceleration.

Or, $$\vec{F} = K\, m\vec{a} \qquad (1)$$

The magnitude of the proportionality constant K depends upon the units of $\vec{F}$, m and $\vec{a}$. We have already fixed the units of mass and acceleration. We can now choose the unit of force in such a way that K becomes unity. Hence the unit of force is that force which acting on a mass of one kilogram produces an acceleration of one metre per second per second. This unit of force we call the *newton*.

We have arbitrarily chosen the units of length, time and mass and called them three fundamental units. If any physical quantity can be expressed in terms of these

fundamental units, this expression is known as the dimension of that physical quantity. The dimensions of the three fundamental units of length, time and mass are expressed as (L), (T) and (M) respectively. Thus the dimensions of area are L^2, that of velocity LT^{-1} and that of acceleration LT^{-2}. It is evident that the dimensions of force are (MLT^{-2}), K being a dimensionless constant equal to unity. However if we fix the unit of force arbitrarily along with those of mass, length and time, the constant K will not be dimensionless and may not be equal to unity. Its dimension will be $(FT^2\ M^{-1}\ L^{-1})$ and its value will depend upon the magnitude of the additional fundamental unit F.

It must also be evident by now that it is not necessary to choose the units of mass, length and time as the fundamental units, we could have as well chosen the units of force, length and time as the fundamental units and if we wanted to make K in the above equation a dimensionless constant equal to unity, the unit of mass would depend upon the units of force, length and time. The important idea is that the dimensions of the physical quantities on the two sides of an equation must be the same. For instance, from equation (i) we can express acceleration as *newtons per kilogram* which is the same thing as *metre per second per second*. We are familiar with this idea in ordinary life. We do not equate length with area or volume. In physics we will deal with quantities which are more complex than these but the dimensions of the quantities on the two sides of an equation must always be the same. When we introduce a new physical quantity, we fix its dimension by reference to its relation with other physical quantities whose dimensions are known.

But it is not necessary to fix the dimension of a new physical quantity in terms of the old physical quantities. Sometimes it is more convenient to give the new physical quantity the status of a fundamental unit. Then the constant occurring in the equation expressing the relationship of the new physical quantity with the older quantities will not be dimensionless. We have already discussed this case in the above equation when we took the unit of force as an additional fundamental unit along with those of mass, length and time. It is not necessary that this constant should be different from unity because the value of the constant depends, as already pointed out, on the magnitude of the fundamental units.

A number of problems in dynamics can be worked out just from our knowledge of these two laws of Newton, provided we know something of the nature of the force.

Newton knew for example that the force due to gravity is mg, where g is a constant called the acceleration due to gravity and has a numerical value of 9·80 newtons per kilogram. Substituting this value of $\vec{F}$ in (i) we obtain,

$$\vec{F} = m\vec{g} = m\frac{\Delta\vec{v}}{\Delta t}$$

$$\text{or, } \frac{\Delta\vec{v}}{\Delta t} = \vec{g}$$

This is the equation of motion of a body falling freely under gravity The acceleration here is independent of the mass or of the nature of the substance.

Let us investigate the properties of the inertial mass.

1. How does the inertial mass depend upon the size of a substance of homogeneous composition?

Take a brass disc and apply a given force $\vec{F}$ to it so that it slides on a smooth glass plate without friction. Find

the acceleration $\vec{a}$. The ratio F/a gives its inertial mass, where F and a are the magnitudes of force and acceleration respectively. Next take an identical disc using the same force. You will find that the acceleration is the same. Thus the inertial mass of this disc too is the same as that of the first disc.

Now put one disc over the other and again apply the same force $\vec{F}$. This time you will find that the acceleration is half the previous value, that is, the acceleration is $\vec{a}/2$.

Hence, the inertial mass is $F/\frac{a}{2} = \frac{2F}{a}$.

So we conclude that the inertial mass is proportional to the volume of the substance of *uniform* composition. That is, the inertial mass is proportional to the amount of matter present in the body.

2. How does the inertial mass depend upon the composition of the substance?

Take two discs, one of brass and the other of copper and select their sizes such that a given force produces the same acceleration in each of them. Thus their inertial masses are equal, but their sizes are unequal.

Put one disc over the other and again apply the same force. The acceleration now will be found to be halved proving that the inertial masses are additive. In fact if we take two bodies of inertial masses m_1 and m_2 respectively and place one over the other and then determine the acceleration produced by a given force, we shall see that the ratio of the force to the acceleration produced comes out to be (m_1+m_2) confirming the above conclusion. Thus mass is a scalar quantity.

Similarly, it can also be proved that the inertial mass does not depend upon the shape of the body but only on the quantity of matter present.

7.4 Gravitational Mass and Inertial Mass

Ordinary mass is also termed as gravitational mass because these masses are compared with the help of the gravitational pull exerted on them when placed on the pans of an equal arm balance.

From every day experience we know that gravitational mass also possesses the properties shown to be present in the case of the inertial mass. That is, the gravitational mass is proportional to the amount of substance present in the body and it can be added together.

Thus the inertial mass and the gravitational mass both are equivalent to each other. Hence the same standard unit, the kilogram, is used to measure both. Therefore, in order to measure the inertial mass m of a body we can apply the same force to the given body and also to the standard 1 kilogram piece If in the first case the acceleration produced is a_1 and in the second case a_2, then

$$\frac{m \text{ kg}}{1 \text{ kg}} = \frac{a_2}{a_1}.$$

Thus m can be determined.

7.5 The Inertial Mass at High Velocities

In ordinary experiments and in problems of engineering, the velocities involved do not exceed a few thousand metres per second. Up to these velocities Newton's second law of motion as defined above is found to hold good. However it has been discovered that for bodies travelling at extremely high speeds (10^5 km/sec or more) the experimental results for the value of mass show some deviations from the values obtained for Newton's second law. At these high velocities the acceleration produced is found to be a

bit less than the value given by Newton's second law as defined above as if the inertial mass has increased? Hence we cannot say that inertial mass is strictly proportional to the quantity of matter present in the body. For the quantity of matter, that is, the number of atoms in the body remains the same all the time, but the inertial mass increases at very high velocities.

Einstein and others have modified Newton's second law so as to include the case of bodies moving with extremely high velocities. You will learn later in this book that the inertial mass of a body moving with a high velocity increases according to the equation given below:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where mo=mass at rest,
m=mass when moving with velocity v,
c=velocity of light in vacuum.

Suppose we have two bodies and suppose one body exerts some force (action) on the other, then according to Newton's third law the second body also exerts an equal but oppositely directed force (reaction) on the first body. Both forces act along the same line but are opposite in direction. The first two laws are concerned with a single body but the third law is concerned with the interaction of two bodies.

To illustrate the law, the following experiment may be performed in the laboratory. Attach a spring-balance to a nail in the wall. Take a second spring balance in your hand and engage its hook into the hook of the other spring balance (Fig. 7.4).

Apply a pull on the spring balance which is in your hand. You will find the same reading on both the balances. Your spring balance pulls the other balance in your direction while the other balance pulls yours in the opposite direction. The two forces, action and reaction, are exactly equal in magnitude but opposite in direction.

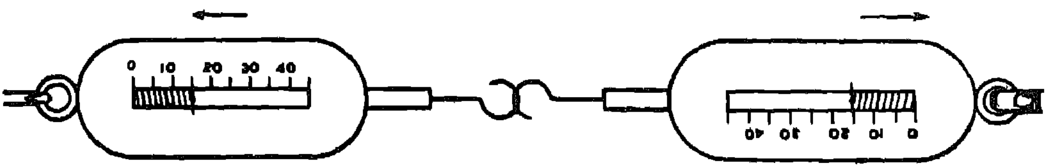

FIG. 7.4. *The spring balance in your hand pulls the other balance in your direction while the other balance pulls it in the opposite direction*

7.6 Newton's Third Law of Motion

In the first two laws of motion we deal with the motion of a single body. In Newton's third law we deal with the mutual forces between two bodies. These forces can be of many types. We have already considered above two different types of forces—gravitational and muscular. We can further have electrical force, magnetic force, nuclear force and others. Newton discovered a general property possessed by all forces. This he stated as his third law :

'Action is equal and opposite to reaction'

So the acting and reacting forces always act on different bodies, hence they can never neutralize each other. Two equal and opposite forces can neutralize each other only when they act on the same body.

When you jump from a boat to the river bank, you apply a push to the boat in doing so and in its turn the boat pushes you with an equal force in the opposite direction.

FIG. 7.5. *When you jump from a boat to the river bank you apply a push to the boat whereas the boat pushes you with an equal force in the opposite direction.*

Thus, you reach the river bank by virtue of the reaction due to the boat and the boat moves away into the river by virtue of the action you applied to it.

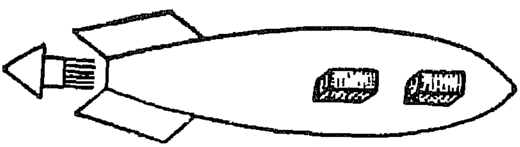

FIG. 7.6 *Rocket applies a force on the gases giving them a backward velocity and the gases apply the reacting forces sending the rocket forward.*

When a gun is fired, the exploding gases apply a force on the bullet in the forward direction and the bullet applies a reaction on the gases in the opposite direction, so the gun recoils. In fact jet planes and rockets are able to acquire such high velocities in accordance with this very law. In the jet plane or the rocket the gases are ejected backwards with a high velocity, that is, the jet plane or the rocket applies a force on the gases giving them a backward velocity and the gases apply the reacting forces sending the jet plane or the rocket forward (Fig. 7·6).

The following interesting experiment can be performed in this connection showing that out of the two forces, action or reaction, each determines the state of motion of the body on which it acts.

The train and track experiment

Place a circular track on a smooth horizontal table so that it can rotate about a vertical axis passing through the centre of the circle (Fig. 7·7).

Mount a toy railway engine which can be set in motion by a clock work on the track. As the engine moves forward, the track moves in the opposite direction. The wheels exert a force (action) in the backward

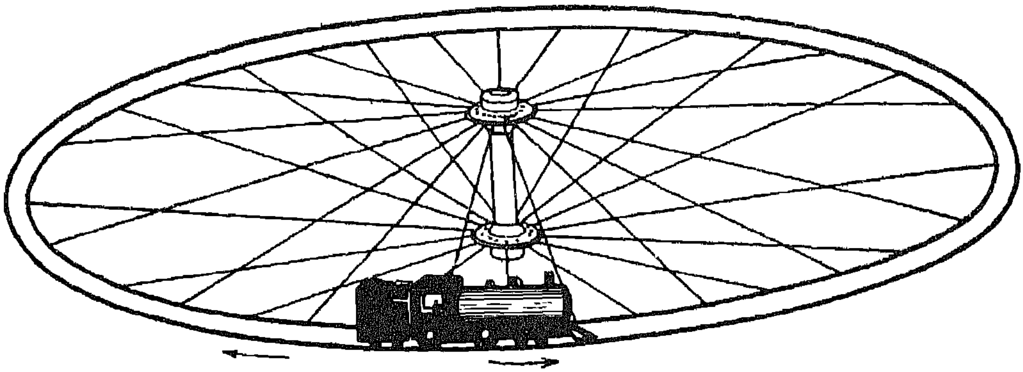

FIG 7.7 *As the engine moves forward, the track moves in the opposite directions.*

direction on the track, and the track exerts an equal and opposite force (reaction) in the forward direction on the engine

Let us consider some consequences of the third law. Suppose we have two particles interacting with each other. For instance, these may be two balls colliding with each other. They may have different masses. We will number them 1 and 2 By Newton's third law of motion the forces acting on them are equal and opposite. It then follows from Newton's second law that

$$\frac{\Delta(m_1\vec{v}_1)}{\Delta t}=\frac{-\Delta(m_2\vec{v}_2)}{\Delta t},$$

$$\text{or}\quad \frac{\Delta(m_1\vec{v}_1+m_2\vec{v}_2)}{\Delta t}=0,$$

i.e., the rate of change of total momentum with time is zero. Thus if there is no other external force acting, then the motion of these two particles will be such that the sum of their momenta will not alter with the passage of time This is known as the "law of conservation of momentum". As we shall learn later, this is true for all physical processes including nuclear reactions and is one of the most important laws in physics.

We have derived this in the case of two particles only. But the law holds good for any number of particles. Let there be three or more particles. Considering any pair of two particles, the forces between them are equal and opposite and this is true whatever the pair may be. Hence the result is that no resultant force is acting on all the particles taken together, *i.e.*, on the system of particles, on account of their mutual interactions. Hence, if no external force is acting, their total momentum remains constant.

7.7 Newton's Third Law and Measurement of Mass

We can use Newton's third law of motion to compare the masses of different bodies. With this end in view, we can perform the following experiment.

Let us take two trolleys which can move with as little friction as possible. The ideal experiment would be to take a trolley which has no friction at all. But this is impossible. To one of the trolleys, we fix a spring. In the other we have a series of holes at the top so that by means of a hook fixed to the first trolley, the two trolleys can be joined together producing different compressions in the spring. In this situation each trolley is

exerting a force on the other trolley in opposite directions through the spring, *i.e.*, the spring is experiencing at its two ends two forces in opposite directions exerted by the trolleys. The spring also exerts on the trolleys the same force at its two ends in opposite directions. If now the catch, keeping the two trolleys bound together, is removed, the spring will be exerting equal forces on the two trolleys in opposite directions so long as the spring is compressed. These forces will produce changes in the momentum of the trolleys which are equal in magnitude but opposite in direction.

Hence we have $\vec{P}_1 = -\vec{P}_2$,

where $\vec{P}_1$ and $\vec{P}_2$ are the momenta of the two trolleys

$$\text{Or} \quad m_1\vec{v}_1 = -m_2\vec{v}_2$$

where m_1, m_2 are the masses and $\vec{v}_1$ and $\vec{v}_2$ the velocities of the first and second trolley respectively. If we now consider only the magnitude of v_1 and v_2, we have

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$

Hence the mass of the second trolley can be determined in terms of the first. Now loading the second trolley by different bodies, we can determine their masses in terms of the mass of the first trolley. We can now choose the mass of one of these bodies as the standard mass and determine the masses of the other bodies in terms of the mass of this body. Since we have already chosen the kilogram as our unit of mass, we can perform the above experiment with that as the standard and determine other masses in terms of this.

The above comparison of the masses of different bodies assumes that the mass of a body does not depend upon its velocity although we have said previously that the mass depends upon the velocity. But the dependence of mass on velocity arises only at high velocities and we may assume that at ordinary velocities involved in our experiment, the mass is independent of the velocity.

For instance, if a person of mass of 100 kilograms is travelling in a supersonic aeroplane moving with a speed of 2000 km/hr, the change in mass is less than a microgram.

We should note that the third law holds good not only in the case of bodies which are in contact but also when they are separated by a distance. For example, when a magnet attracts a piece of iron towards itself, then the piece of iron too attracts the magnet with an equal force though they are separated by a distance. Similarly, the sun attracts the earth, and the earth too attracts the sun with an equal and opposite force. Here the earth and the sun are separated by an enormous distance.

Further, the law holds good equally whether the two bodies are at rest or in motion. We know that the earth is revolving round the sun and all the time the mutual attraction is present between the two. Similarly, an artificial satellite revolving round the earth is all the time being attracted by the earth. Simultaneously the satellite too attracts the earth with an equal force.

7.8 Impulse of a Force

From the second law we have the equation, force=mass×acceleration. But by definition $\vec{a} = \dfrac{\vec{v}-\vec{u}}{t}$ where $\vec{v}$ is the final velocity and $\vec{u}$ the initial velocity and t is the time for which the constant force $\vec{F}$ acts. Substituting this value of $\vec{a}$ in $\vec{F} = m \times \vec{a}$, we get

$$\frac{m \times \vec{v} - \vec{u}}{} = \vec{F} = \frac{m\vec{v} - m\vec{u}}{t}.$$

From the equation $\vec{F} = \frac{m\vec{v} - m\vec{u}}{t}$, we

get $\vec{F}t = (m\vec{v} - m\vec{u})$ the quantity on the left $\vec{F}t$ expresses the product of the force and the time for which it acts This product is known as the impulse of the force.

So we have

Impulse = Change of momentum.

We should note that this equation holds good under all circumstances. That is, the time for which the force acts may be small or large, the change in the momentum of the body will always be equal to the impulse of the force acting on the body The equation will hold good whether the body was initially at rest or it was moving with a uniform velocity in a straight line or whether it was moving with an accelerated velocity. The change in the momentum will be equal to the impulse of the force applied regardless of what comes before or after in the motion of the body. It is clear that impulse is a vector quantity.

We shall now consider the physical significance of momentum and impulse. Suppose a body at rest is acted upon by a constant force $\vec{F}$ for time t, so that the body acquires a velocity $\vec{u}$,

then $\vec{F}t = m\vec{u}$.

In such a case the impulse given to the body is equal to its momentum. So the momentum tells us how much impulse is necessary to set the body, initially at rest, moving with velocity $\vec{u}$. Or again the momentum tells us how much impulse must be imparted to the body in the opposite direction so as to bring it to a stop.

Since $\vec{F}t = m\vec{u}$, it follows that a body of mass m moving with a velocity $\vec{u}$, can be brought to rest if a force $\vec{F}$ in the opposite direction is applied to it for time t such that the impulse $\vec{F}t$ is equal to the momentum $m\vec{u}$. If the force $\vec{F}$ is large then time t will be small and vice versa The only condition is that the product of force and time must be equal to the product of mass and the velocity

The equation $\vec{F} = \frac{m\vec{u}}{t}$ explains why in catching a cricket ball one must not hold the hand rigid. For in this case the ball is brought to rest in a very short time, so the force $= \frac{m\vec{u}}{t}$ becomes large and the ball hits the hand very hard. An experienced player usually relaxes his hands as he catches the ball allowing them to "go" with the ball, thus increasing the time during which the velocity of the ball is being reduced to zero. So that a small force now acts for a longer time and the hands do not feel the impact so strongly. The same principle explains why, in falling, one should relax his body and limbs instead of stiffening them. For in the latter case the speed of fall is reduced to zero within a very short time thus increasing the force of impact.

Similarly in imparting a certain velocity to a given mass (at rest) within a very short time we have to apply a large force. On the other hand, if we wish to impart the same velocity to the body within a larger period of time, we have to apply a smaller force. This is why cricketers allow "follow through" with their drive if they want their ball to

acquire a high velocity. The follow through enables the force applied to act on the ball for a longer time—the impulse given is greater With guns too we know that a long barrelled gun upto a certain length, has a higher muzzle velocity than a short barrelled gun. The greater the length of the barrel, the longer is the time for which the force acts on the bullet. So the impulse imparted is greater in this case In shot-put or javelin throw, the velocity can be increased by proper follow through The follow through increases the duration for which the force applied by the athlete continues to act and thus the impulse too is increased

Examples

1 A mass of 1 kg which can slide on a frictionless surface is to be imparted an acceleration of 10 cm/sec² What is the magnitude of the force required?

$$\text{Formula} \quad \vec{F} = m \times \vec{a}.$$

$$\text{Now } m = 1 \text{ kg},$$

$$\vec{a} = 10 \text{ cm/sec}^2 = 0 \cdot 1 \text{ m/sec}^2,$$

$$\therefore F = 1 \text{ kg} \times 0 \cdot 1 \text{ m sec}^{-2},$$

$$= 0 \cdot 1 \text{ newton}.$$

It should be noted that the force required to accelerate the body is on account of its inertial mass only and would have been the same even on the surface of the moon

2 A 4,000 kg truck is moving with a velocity of 40 kilometres per hour. It is stopped 25m beyond the point at which the brakes are applied. Find the retarding force. How long does it travel after the brakes have been applied?

$$\text{Initial velocity} = \vec{u} = 40 \text{ km/hour}$$

$$= \frac{40 \times 10^3}{3600} = \frac{100}{9} \text{ m/sec},$$

$$\text{final velocity} = v = 0.$$

Distance travelled during retardation $= 25$ m

Now $v^2 - u^2 = 2\, a\, s$,

$$\therefore a = -\frac{100 \times 100}{81 \times 2 \times 25} = -\frac{200}{81} \text{ m/sec}^2.$$

$$\therefore \text{Retarding force} = m \times a = 4000 \times \frac{200}{81},$$

$$= 9\ 9 \times 10^3 \text{ newtons}.$$

$$\text{Again } v = u + at,$$

$$0 = \frac{100}{9} - \frac{200}{81} t,$$

$$\text{or } t = \frac{100}{9} \times \frac{81}{200},$$

$$= 4 \cdot 5 \text{ sec}.$$

Classroom Activities

1. Push a ball on a smooth floor and let it roll. Again push the rolling ball at an angle to its motion. Examine the change in its direction of motion.
2. Inflate a toy balloon with air and then suddenly release it. See the effect of action and reaction.

Questions

1. What is force ? How is it related with motion ?
2. What is meant by an unbalanced force ? Give examples.
3. What type of motion does a body have when in equilibrium ?
4. What do you mean by the term 'inertia' ? How can the inertia of a body be measured ?
5. What is the law of inertia ? Give examples from daily life.
6. Aristotle taught that a constant force was required to produce a constant velocity and from this he concluded that in the absence of force bodies would come to rest.
   (a) Name several situations where a constant force seems to produce a constant velocity.
   (b) How do you explain each of the situations in the light of Newton's law of motion ? (PSSC)
7. Why is it dangerous to drive a car fast on slippery roads ?
8. What is Newton's second law of motion ? Explain how Newton's first law gives us a qualitative definition of force and the second law gives a quantitative measure for defining the unit of force.
9. A mass of 2 kg is to be given an acceleration of 2m/sec^2. Find the magnitude of the force required.
10. What will be the force (dynes) required to give an acceleration of 10 cm/sec^2 to a mass of 25 g ? (1 dyne$=10^{-5}$ newton).
11. A car having a mass of 2,000 kg and travelling at 32 km/hr has to be accelerated so that within 5 seconds its speed may become 50 km/hr. What will be the force required ?
12. A mass of 1 kg is suspended from a spring balance and the pointer of the spring balance reads 8 divisions. Another mass of $4\frac{1}{2}$ kg is attached to a similar balance. The two systems are attached to the two ends of a string passing over a pulley. Calculate the acceleration of the system and the readings of the spring balances. (Assume spring balances to be weightless and take g$=$9.8 m/sec^2).
13. A constant horizontal force of 2 newtons acts on a body placed on a smooth table so that the body starting from rest travels a distance of 100 m in 10 sec. Find the mass of the body.
14. A force of 5 newtons is exerted on a block of wood and it acquires an acceleration of 2m/sec^2. Assuming that no other force acts on the block, find its mass.
15. A force of 4 newtons gives a block of copper, an acceleration on 5 m/sec^2 and the same force gives an acceleration of 20 m/sec^2 to a block of wood. What will be the acceleration when the same force is applied to the two blocks fastened together ?

16. A train has a mass of 10^6 kg and its engine can exert a maximum force of 3×10^5 newtons Find the velocity of the train after it has travelled 500 metres. The train is supposed to start from the position of rest.

17 A body can slide on a smooth horizontal table. When it is pulled by a constant force, the velocity of the body increases from 1 m/sec to 2 m/sec in 0·4 sec. Next time another force is applied for 0·5 sec when the speed changes from 2 m/sec to 5 m/sec.
   (a) Find the ratio of the second force to the first.
   (b) Find the acceleration produced by the second force on the given body.

18. In a circus-show a man weighing 60 kg jumps from a height of 20 m and he is caught in a net spread below which sags down 2 m under his weight. If g= 9 8 m/sec², find the average force exerted by the net in breaking his fall.

19. "A rocket acquires a thrust upwards because the gases ejected from the rear push against the air of the atmosphere." Comment upon this statement and explain the principle of rocket flight.

20. Suppose a powerful blower is installed on a sail boat Is it possible to propel the boat by blowing air from the blower against the sails ? Give reasons in support of your answer.

21. How are you able to walk on the ground ? Explain in terms of Newton's third law and thus show why it is difficult to walk on perfectly smooth ice.

22. While a train is in motion, a passenger inside the train pushes hard on the wall of the bogey he is occupying. Will he be able to change the speed of the train by the force he thus applies ? Give reasons for your answer.

23. Explain the meaning of action and reaction with reference to Newton's third law of motion Can a body exert a force upon two different bodies at the same time ?

24. A car of mass 1,000 kg travelling at 70 km/hr collides with a 3,000 kg truck going in the same direction with a velocity of 40 km/hr. In collision they get locked together. Find the velocity of the two moving together immediately after the collision.

25. A shell of mass 4 kg travelling at 1,000 m/sec strikes a mound of earth and penetrates inside through a distance of 1 metre. Find the average retarding force and also determine the time for which it remains in moton inside the mound

26. The barrel of a gun is 1 metre long and it fires a bullet of mass 40 g with a velocity of 900 m/sec. Find (a) the acceleration, (b) the force; (c) the impulse given to the bullet.

27. A railway wagon weighs 3,000 kg Three men when pushing the wagon are just able to move it, and the wagon acquires an acceleration of 0 5 m/sec². Find the force exerted by one man.

28. A bullet is fired from a heavy rifle and next the same bullet is fired from a lighter rifle. Which rifle will give a greater recoil and why ?

29. A rocket engine burns 2 kg of fuel per second and the gases formed escape from the rear end with a velocity of 2,500 m/sec. What is the thrust exerted on the rocket ?

30. A garden hose with an internal diameter of 2 cm throws a stream of water with

a speed of 10 m/sec in a horizontal direction What will be the force required to hold the hose in position ?

31. A bird of mass 400 g is flying horizontally towards the hunter with a speed of 20 m/sec. The hunter fires a bullet at the approaching bird so that the dead bird with the bullet embedded drops down immediately. What was the velocity of the bullet at the time of impact? (Mass of the bullet is 20 g)
32. A man throws a ball of mass 150 g with a speed of 20 m/sec. Assuming that while throwing he exerts a constant force over a distance of 1 metre, calculate (a) the magnitude of the force and (b) the time for which the force acts.
33. A goods-wagon weighing 3,650 kg is moving with a speed of 3 km/hr on a level track when it hits a bumper and is brought to a standstill within 0·3 sec. Calculate the average force exerted on the bumper.
34. A machine-gun fires 500 bullets per minute. The bullets weigh 50 g each. The bullets hit the target with a velocity of 450 m/sec and drop dead. Find the force necessary to keep the target in position
35. A block of mass 2 0 kg is pulled on a frictionless table by a constant force of 6·0 newtons. The block starts from rest.
    (a) What is the acceleration of the block in m/sec^2 ?
    (b) What is the speed of the block 3·0 sec after the force starts acting ?
    (c) How far does the block travel in 2·0 sec?
    (d) If at the end of 3·0 sec, the block splits up into two equal pieces—one piece still being pulled by the force of 6 0 newtons, and the other free—how far apart will the two pieces be 2·0 sec after the break occurs? (PSSC)
36. A bullet of mass 30 g moving with a constant horizontal velocity strikes a 1 kg block of wood placed on a smooth table After collision, the block with the bullet embedded slides forward with a speed of 2 m/sec. Find the initial velocity of the bullet.

Further Reading

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MARBURGER, W G. and HOFFMAN, C. W. *Physics for Our Times* New York: McGraw-Hill Book Co., Inc., 1958

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CHAPTER VIII

Principle of Moments

8.1 Introduction

Suppose you have to tighten a nut with the help of a wrench. Will you choose a wrench with a small handle or one with a long handle ? You will find that using a wrench with a long handle, one can tighten the nut with a much smaller effort than would be needed if one used a wrench with a short handle

Consider a see-saw so often seen in the children's park. Let the see-saw be perfectly balanced when no one is sitting on it. Now let a stout boy sit on the side of the see-saw half way up A younger boy can balance the see-saw horizontally, if he sits on the other side at a point near the end.

Consider the case of a man trying to push open a hinged door. Fig. 8.1 shows the top view of the man applying a force on the door which is hinged at O. It will be found that the door turns more easily if the force is applied at a point A, far removed from O. The turning effect is reduced if the same force is applied at a point B or C nearer O. Again the turning effect is greatest when the line of action of the

FIG 8 1. *Pushing a door.*

force is perpendicular to the door. When the force is applied in an oblique direction, the turning effect is reduced.

Of course in all these cases the greater the force applied the greater is the turning effect. From such experiments we can verify that the turning effect depends upon two factors; (i) the magnitude of the force and (ii) the perpendicular distance between the line of action of the force and the axis about which the body can turn This perpendicular distance is called the *moment arm*.

Now this tendency which a force can have to make a body turn is known as the *moment of the force* or the torque The turning effect or the moment is measured by the product of the magnitude of the force and the moment arm.

Moment=the moment arm × force.

Unit of moment :—If the force is expressed in newtons and the moment arm in metres, the moment will be expressed in newton-metres in the MKS system of units.

8.2 Clockwise and Anti-clockwise Moments

Suspend a sheet of cardboard on a horizontal nail which passes through the point O of the board (Fig. 8.2) P and Q are two nails fixed in the board from where weights W and W_2 are suspended as shown.

Now the force W has a tendency to turn the board clockwise while the force W_2 tends to turn the board anti-clockwise.

The moment arm for W=OP

and the moment arm for W_2=OQ.

Clockwise moment =W×OP

and anti-clockwise moment=W_2×OQ

By convention the clockwise moment is taken as negative and the anti-clockwise moment is taken as positive The clockwise or anti-clockwise direction of the moment of a force does not depend upon the direction of the force alone. It also depends upon the position of the axis about which the body is capable of rotation.

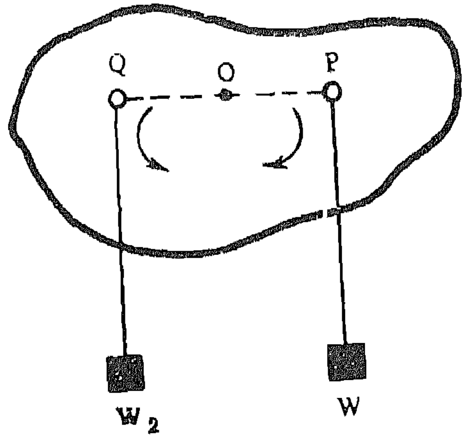


FIG. 8 2. *The force W has a tendency to turn the board clockwise while the force W_2 tends to turn the board anti-clockwise*

For instance in Fig 8 3, a force W acts vertically downwards on a nail K driven through the board. If the board is suspended at O, this force W will have a clockwise moment=W×OK. On the other hand if the board were suspended at P, then the same force W will have an anti-clockwise moment=W×PK.

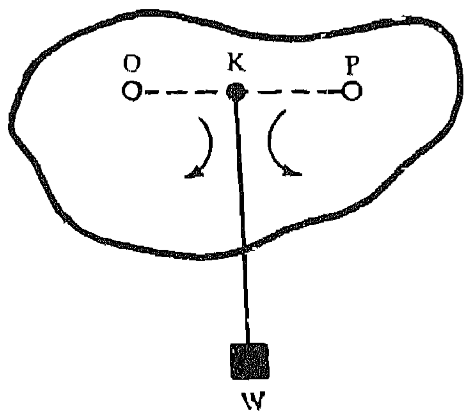


FIG. 8.3. *If the board is suspended at O, the force W will have a clockwise moment and if the board is suspended at P then the force W will have an anti-clockwise moment*

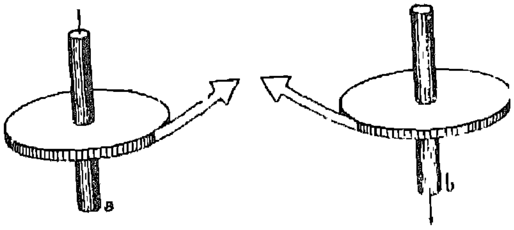


FIG. 8 4

8.3 Moment (Torque) is a Vector

From the above discussion it is clear that moment is not a scalar quantity, for, to specify it, we must state whether the moment is clockwise or anti-clockwise Moment or torque is a vector quantity. This vector is drawn along the axis of rotation and the sense of the torque vector is given by the right hand rule as shown in Fig. 8.4 (a) and (b) The rule states: 'Encircle the axis with the fingers of the right hand which point towards the direction in which the body tends to rotate, then the thumb points along the axis in the direction of the torque vector '.

Applying the rule it is clear that the direction of the axis of a clockwise moment is into the paper, *i e.*, downwards and the direction of the axis of the anti-clockwise moment is outward from the plane of the paper, *i.e.*, upwards. Hence, clockwise moment is taken to be negative and the anti-clockwise moment is taken to be positive. This can be expressed mathematically as

$\vec{M}=\vec{r}\times\vec{F}$.

This is a vector product of the two vectors $\vec{r}$ and $\vec{F}$. Its magnitude is $rF\sin\theta$ where θ is the angle between these vectors. In the vector product the order of the vectors is important If the order is changed the product becomes negative, *i.e.*, $\vec{r}\times\vec{F}=-\vec{F}\times\vec{r}$.

The moment of a number of forces acting on a rigid body

For the sake of simplicity we shall assume that all the forces act in the same plane (Fig. 8.5). Here the vertical scale OG is pivoted at O. Three forces have been applied on the point P of the scale as shown and the scale is kept vertical under the action of these forces.

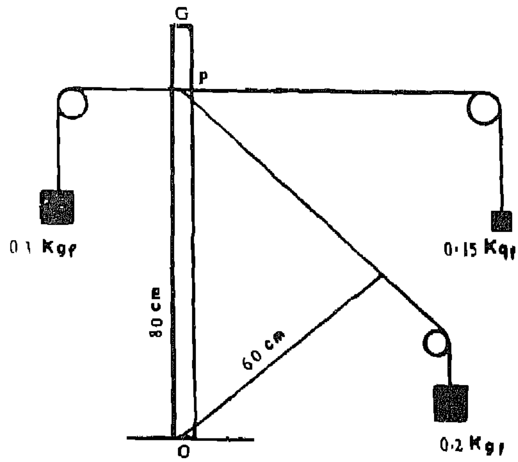


FIG. 8 5.

As is clear from the figure.

The force 0·2 kgf has a moment

$= -0·2\ \text{kgf}^* \times 0·6\ \text{m} = -0·12\ \text{kgfm},$

The force 0·15 kgf has a clockwise moment

$= -0·15\ \text{kgf} \times 0·8\ \text{m} = -0·12\ \text{kgfm},$

The force 0 3 kgf has a moment

$= 0·3\ \text{kgf} \times 0·8\ \text{m} = 0·24\ \text{kgfm}.$

Vector sum of moments=

$(-0·12 - 0·12 + 0·24)\ \text{kgfm} = 0.$

Thus the resultant moment is zero, hence the scale has no tendency to turn either clockwise or anti-clockwise. We say that the scale is in rotational equilibrium.

*A force of 1 kgf means the force with which the earth pulls a mass of 1 kg towards it. Thus a force of 1 kgf = 9 8 newtons

8.4 Rotational Equilibrium

A body is said to be in rotational equilibrium if the forces acting on it do not produce a net rotating effect on it. *It follows therefore that the rotational equilibrium will be achieved if the vector sum of moments of forces acting on the body is zero.* In other words a body will be in rotational equilibrium if the clockwise moment on it is equal to the anti-clockwise moment. This statement is known as the *Principle of Moments.*

The following example further illustrates the above principle. Consider a wheel (Fig. 8.6) mounted on a horizontal axis.

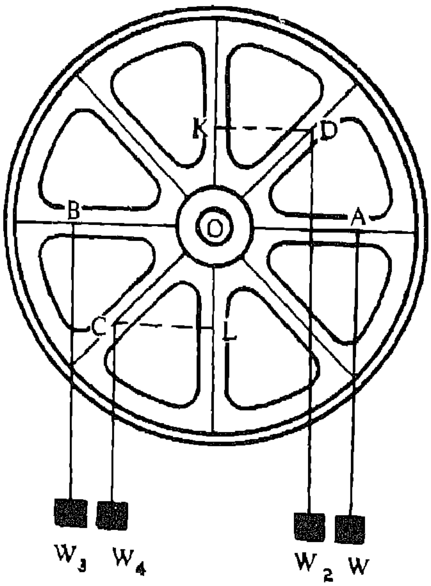


FIG. 8.6

Here O is the point about which the wheel can rotate. Forces W and W_2 are tending to rotate the wheel clockwise and forces W_3 and W_4 tend to rotate it anti-clockwise. But the wheel is at rest, it is in rotational equilibrium.

Now clockwise Moment $= (W \times OA + W_2 \times DK)$, *i.e.*, moment$=(W \times OA + W_2 \times DK)$.

Anti-clockwise Moment $= (W_3 \times OB + W_4 \times CL)$, *i.e.*, moment $= +(W_3 \times OB + W_4 \times CL)$.

So the condition for rotational equilibrium is

$$(W_3 \times OB + W_4 \times CL) - (W \times OA + W_2 \times DK) = 0.$$

If this condition is satisfied, the wheel will have no tendency to rotate. This is what the Principle of Moments states.

It is clear that the condition for achieving translational equilibrium is that the vector sum of all the forces acting on it must be zero.

We must note in this connection that a body which is in translational equilibrium

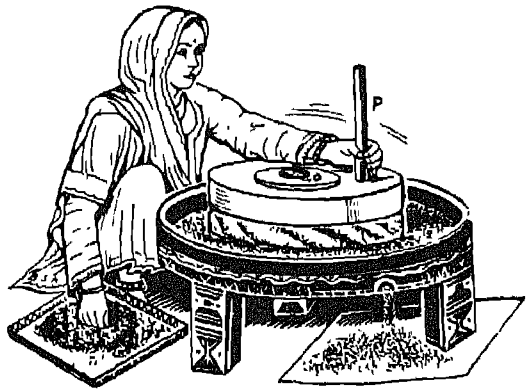


FIG. 8.7

may not necessarily be in rotational equilibrium also. Take the case of a grinding wheel. The stone wheel can rotate about the vertical axle passing through the centre O (Fig. 8.7). When a force is applied at the peg P as shown, an equal force is produced at the axle which acts on the stone wheel in a direction opposite to that of the force applied at P. Hence the resultant force is zero and the wheel has no translational motion, it is in translational equilibrium.

What about the turning effects of these forces? Force at P has a moment about O. If the magnitude of the force is F, the moment will be $F \times PO$ in an anti-clockwise direction. The opposite force at

O has no moment because here the moment arm has length equal to zero. The anti-clockwise moment $F \times PO$ causes the grinding wheel to rotate anti-clockwise. The wheel is not in a rotational equilibrium.

Again take the case of a light scale pivoted at its middle point O (Fig. 8.8). Equal forces P and Q act on the scale at points A and B so that these forces are parallel but oppositely directed It is clear that the vector sum of these forces is zero. Hence the scale will have no translatory motion. It is in translational equilibrium.

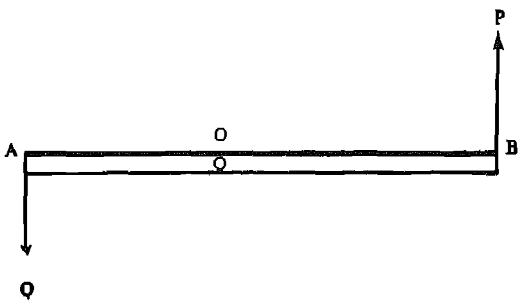


FIG. 8 8.

Now consider the moments. Force P exerts a rotating effect in the anti-clockwise direction : the moment is $OB \times P$ while the force Q exerts an anti-clockwise turning effect and the moment is $OA \times Q$.

Each of the forces P and Q exerts a turning effect in the same direction. They will not cancel each other So the body will experience a rotation in anti-clockwise direction. Hence the body is not in rotational equilibrium.

Thus here too we see that forces acting on the scale produce no linear or translational motion but they do produce rotational effect.

Conditions for complete equilibrium

In order that a body achieves translational as well as rotational equilibrium, two conditions must be satisfied; one for translational equilibrium and the other for rotational equilibrium.

These conditions are :

1. The vector sum of all the forces acting on the body must be zero (to ensure translational equilibrium)

2 The vector sum of moments of forces about any point must be zero (to ensure rotational equilibrium).

These conditions can be written mathematically as $\vec{F}_1 + \vec{F}_2 + \ldots . \vec{F}_n = 0$

which may be written as $\sum \vec{F} = 0$

and $\vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \ldots \ldots \vec{r}_n \times \vec{F}_n = 0$.

which may be written as $\sum \vec{r} \times \vec{F} = 0$.

Here $\sum$ is a letter of the Greek alphabet called sigma and denotes summation over all the forces acting.

The turning effect of a couple

The case of two equal and opposite forces acting on a body such that they are

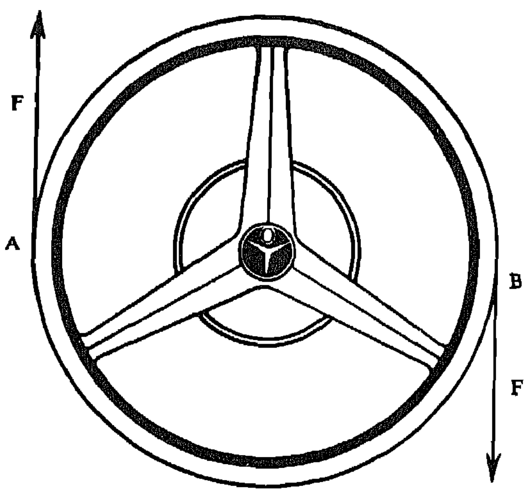


FIG. 8·9. *Steering wheel of a motor car.*

not in the same straight lines deserves our special attention. Such a pair of forces is known as a *couple*

If we wish to tighten the cap of an inkpot, we apply two equal and opposite forces with our thumb and forefinger forming a couple. At the same time we hold the inkpot with the other hand so that only the cap will turn. When a house wife wishes to churn curd, she applies with her hands two equal and opposite forces to the stem of the churner. When we try to turn the steering wheel of a motor car, we apply two equal and opposite forces F and F at the extremities of the diameter of the steering wheel as shown in Fig. 8·9. Although the two forces are equal and opposite, the steering wheel is not in complete equilibrium. They have no resultant, hence the wheel is in translational equilibrium no doubt. But it is not in rotational equilibrium. Both the forces tend to rotate the wheel clockwise. Clearly the sum of their moments $= -(OA\times F+OB\times F) = -F\times(OA+OB) = -F\times AB$.

The product $F\times AB$ gives the moment of the couple. It is equal to the product of either one of the forces and the perpendicular distance between the two forces constituting the couple.

The moment of the couple is independent of the position of the point about which the body is supposed to rotate This will be clear from the following example :

Consider a light rod BD such that two equal and opposite forces act on it at points B and A as shown.

1. Let the rod be pivoted at O. Taking moments about this point we have,

The sum of moments $=F\times BO$ (anti-clockwise) $+F\times AO$ (anti-clockwise) $=F(BO+AO)=F\times AB$ (anti-clockwise).

2 Let the rod be pivoted at E, then the sum of moments $=F\times BE+F\times AE$ both

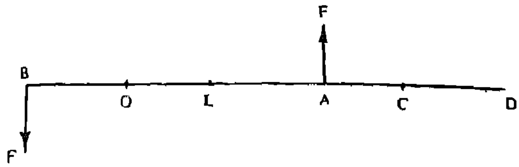


Fig. 8·10.

anti-clockwise $=F(BE+AE)=F\times AB$ (anti-clockwise).

3 Let the rod be pivoted at C, then the sum of moments $=F\times AC$ (clockwise) $+$ $F\times CB$ (anti-clockwise)

$=F\,(CB-AC)=F\times AB$ (anti-clockwise).

Thus in all these cases the moment of the couple is the same; it is equal to the force multiplied by the perpendicular distance between the two forces. It can be easily seen that a couple can neither be replaced by a single force nor be neutralized by a single force. A given couple can only be balanced by another couple having an equal and opposite moment.

We have here considered two equal and opposite parallel forces constituting a couple. *More generally couple is the name given to any system of forces which tend to cause rotation only.* For example, when unscrewing the lid of a jar we apply a couple with our hand, the wind on the vanes of a wind-mill applies a couple which rotates the mill and the head of a wrench on the nut applies a couple which turns the nut on the bolt.

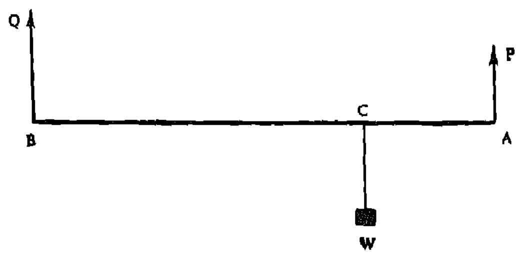


Fig. 8·11.

8.5 Parallel Forces

Suppose two men carry a heavy load on a stick AB and each of them holds one end of the stick (Fig. 8 11).

Obviously, the men exert upward forces P and Q and the load exerts a downward force W.

All of these forces are parallel to one another and are called parallel forces.

We must note here that parallel forces do not act in *the same line* and their points of application are different. They may act in the same or opposite directions. In the first case they are called like parallel forces; in the second case they are called unlike parallel forces.

In the example given above, it is clear that the resultant of the like parallel forces P and Q must be equal in magnitude to W and act at C upwards, *i e*, $W=P+Q$

Hence the resultant of like parallel forces (P and Q) is equal to the algebraic sum of their magnitudes and its direction is the same as that of P or Q.

What will be the point of application of the resultant force ?

To obtain the position of the point C, note that the stick is in rotational equilibrium. Taking moments about the point C, we must have,

$$Q \times AC = P \times BC,$$
$$\frac{AC}{BC} = \frac{P}{Q}.$$

Thus the resultant of the like parallel forces acts at a point which divides internally the distance between them in the inverse ratio of the forces.

8.6 Unlike Parallel Forces

P and Q are two unlike forces (Fig. 8.12). It is clear that if we apply a parallel force W downward anywhere between A and B, the stick will not remain in equilibrium It will rotate anti-clockwise or

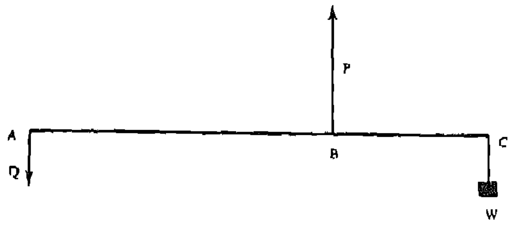


FIG. 8.12

may have a translational motion. If, however, the force W is applied at some point C outside AB in a downward direction then it is possible that the rotational equilibrium may be obtained.

Taking moments about C, we find that for rotational equilibrium

$$P \times BC = Q \times AC,$$
$$\frac{AC}{BC} = \frac{P}{Q}.$$

That is, the point of application of the resultant must divide the line AB externally in the inverse ratio of the forces. Considering the translational equilibrium it is easy to see that the magnitude of the resultant force will be P—Q which will be equal and opposite to W. This resultant will have the same direction as the greater force P.

8.7 Principle of Moments Applied to Levers

Lever is probably the earliest device that our ancestors used for raising heavy loads.

Lever is simply a bar free to turn about some point of the bar. This pivot point is called the *fulcrum* The applied force is called the effort, while the force

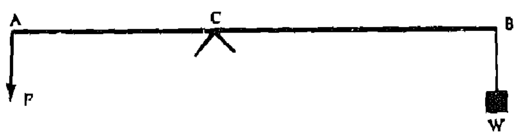


FIG. 8.13

offered by the object to be lifted is called the *load.*

The perpendicular distance of the applied force (the effort) from the fulcrum is called the effort-arm and the perpendicular distance of the resisting force (the load) from the fulcrum is called the resistance-arm.

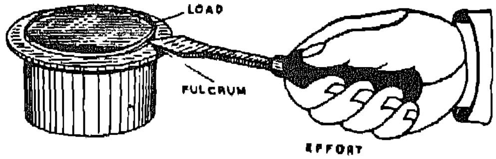


FIG. 8.14(a). *Opening the lid of a can*

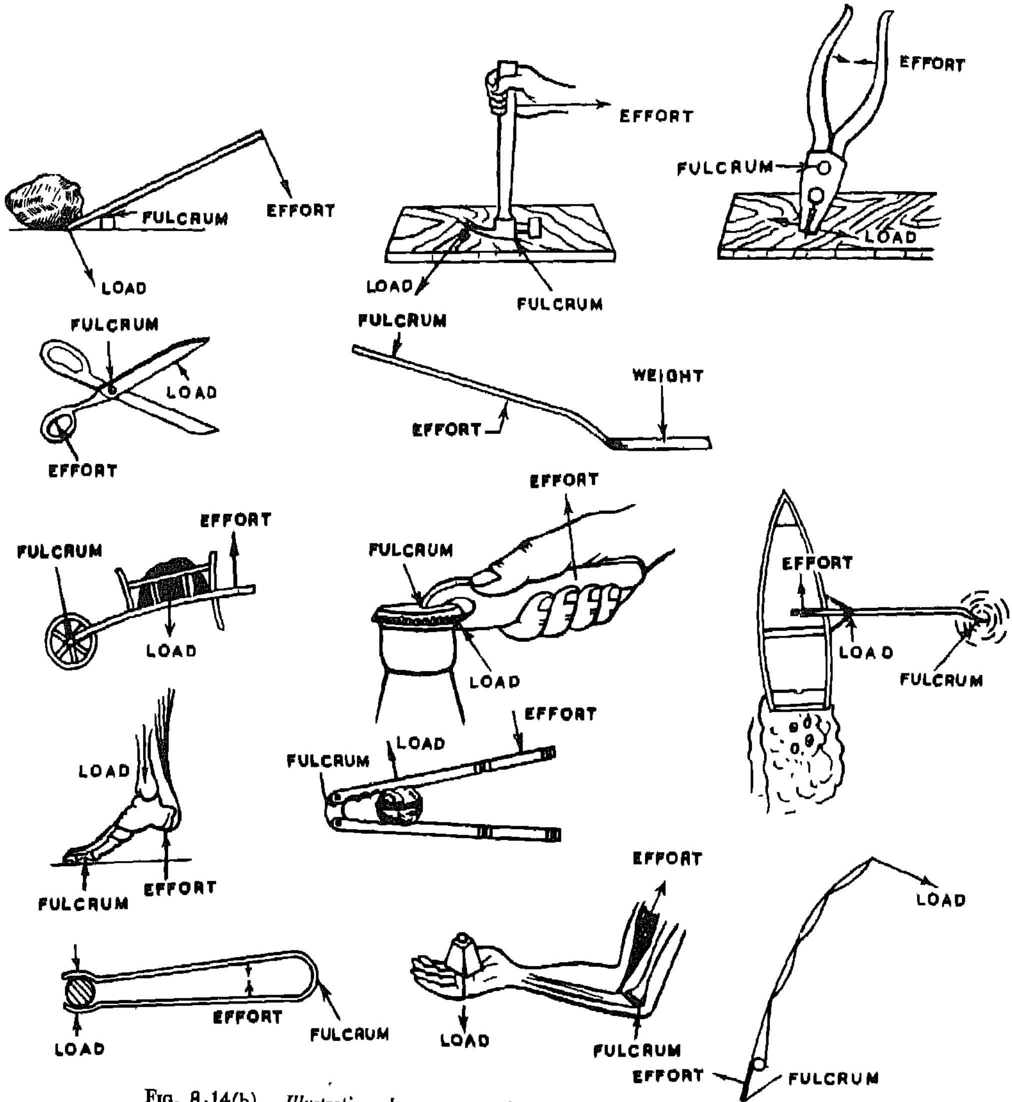


FIG. 8.14(b). *Illustrating devices in which the principle of the lever is involved.*

From the principle of moments we have, effort $\times$ effort-arm $=$ load $\times$ resistance-arm, or

$$\text{effort}=\frac{\text{load}\times\text{resistance-arm}}{\text{effort-arm}}$$

If the effort-arm is longer than the load-arm, then the ratio of load to the effort will be greater than one, so that a small effort can balance a big load.

When we open the lid of a can with the help of a screw-driver as shown in Fig. 8 14(a), the screw-driver is being used here as a lever. The point where the screw-driver rests on the can is the fulcrum and the load (resistance offered by the lid) acts on the lip of the screw-driver while the effort acts downwards on the handle As the effort-arm is much larger than the load-arm, a small effort produces a large force at the point of contact with the lid so that the latter is forced to open.

Another familiar example is a nut cracker (double lever). The hinge or fulcrum is at one end and the effort is applied at the other end. The nut placed between the two experiences a large force sufficient to crack it.

Several more typical examples of levers have been shown in Fig. 8.14(b).

8.8 Centre of Gravity

Have you ever wondered why a porter while carrying a heavy bag on his back has to lean forward ? When you carry a heavy suitcase in your right hand, you have to lean towards the left in order to keep your balance, but when you carry two suitcases, one in each hand, you can walk upright. Why is it not necessary for you to bend your body either to the left or to the right in this case ? Again on hill stations you must have seen the hillmen carrying long heavy beams balanced on the back so that the middle portion of the beam rests on the back Here too the man bends his body forward In fact the heavier the beam, the more has he to lean forward. Why does he do so ?.

In order to find a satisfactory explanation to questions posed above, we have to make ourselves familiar with the conception of what we call the centre of gravity. Let us consider the following .

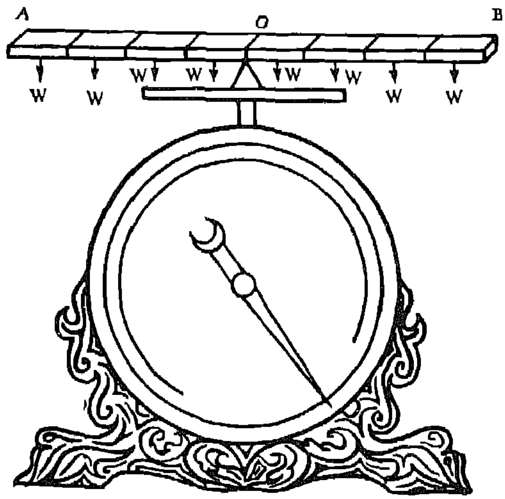


FIG. 8.15 *Illustrating the centre of gravity with weights suspended from the two arms of a beam balance.*

Suppose a platform spring balance is provided with a knife-edge fitted on the platform as shown in Fig. 8 15. A uniform rod AB when placed on the knife-edge balances only when its middle point O is on the knife-edge. As is clear from the diagram, the weight of each element of the scale acts vertically downwards. Each of these will be exerting a turning effect on the rod about the knife-edge. Since the rod is in rotational equilibrium, the clockwise moment produced by the forces on right of the point O must be equal to the anti-clockwise moment produced by forces acting on the left of O. Thus the sum of all gravitational torques about the point O of the rod is zero. This point is called the centre of gravity.

Again consider the translational equilibrium of the rod. Parallel forces of magnitude W each are acting on the elements of the rod downwards and only one force R, the reaction is acting vertically upwards at O. Hence R must be equal to the resultant of all the downward parallel forces. This means that the resultant of weights of all the elements of the rod must pass through this point O. Of course this resultant is equal to the weight of the entire rod. But we have already defined this point O as the centre of gravity of the rod

So that the centre of gravity can also be defined as the point where the entire weight of the body may be supposed to act.

We can verify the truth of this statement by performing the following experiments. Take a metre-scale AB and weigh it with a spring balance Let its weight be 200 gf.* Support the scale on a knife-edge O so that its end A is at a distance of 20 cm from the knife-edge (Fig. 8.16). You will find that a force of 300 gf must be applied at A in order to keep the scale horizontal.

The anti-clockwise moment due to the force applied at A

$$=300\times 20 \text{ gf cm},$$
$$=6000 \text{ gf cm}.$$

This must be balanced by the clockwise moment due to the weight of the scale. We know the weight of the scale is 200 gf. This

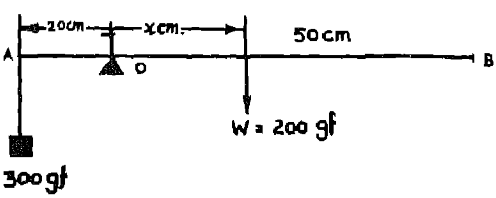


FIG. 8.16.

* A force of 1 gf means the force with which the earth pulls a mass of 1 g towards it Thus a force of 1 gf = 0.98×10^{-2} newton. This force is denoted by the symbol gf

weight should act at some point towards the right of the knife-edge so that it may produce a clockwise moment

Let the distance of the point be x cm from the knife-edge.

The clockwise moment due to the weight of the scale $=200$ gf$\times x$ cm. Since the scale is in rotational equilibrium, the anti-clockwise moment=the clockwise moment.

$$300\times 20=200\ x,$$
$$\text{or,}\ x=30 \text{ cm}.$$

Thus the point where the entire weight of the scale appears to act is at 30 cm to the right of the knife-edge, that is, the centre of gravity is at the *middle* of the scale.

Hence it is proved that under all circumstances the centre of gravity of the body remains at a fixed position in the body. From the above discussion we arrive at two important conclusions

1. If a body is suspended on a horizontal axis which passes through its centre of gravity G, the body will remain balanced in whatever orientation it is placed, for the sum of the gravitational torques about the centre of gravity is always zero A body so suspended will not tip either clockwise or anti-clockwise.

2. When a body is suspended from a point other than the centre of gravity, the body comes to rest in a position such that the centre of gravity is vertically below the point of suspension. Since the entire weight of the body may be taken to act at the centre of gravity, the weight will have a moment about the point of suspension unless the centre of gravity comes directly below the point of suspension when the gravitational moment vanishes.

In Fig. 8.17 (a) we see that the body has an anti-clockwise moment due to its weight; hence it will rotate so as to bring the centre of gravity G directly below the point of suspension O [Fig. 8.17(b)].

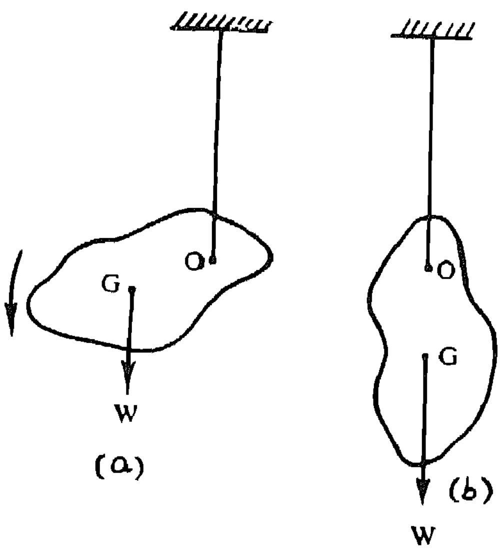


FIG. 8 17 *The centre of gravity of any freely suspended object is directly beneath the point of suspension*

Now the body is in rotational as well as translational equilibrium. Condition for the latter is satisfied because the downward force, the weight, is equal and opposite to the tension in the string and the two are acting in the same straight line.

8.9 Position of the Centre of Gravity

If the body has a simple geometrical shape and its thickness and density are the same throughout, then its centre of gravity will naturally be situated at the geometrical centre of the body.

Thus the centre of gravity of a uniform rod will be situated at the mid-point of its axis. For a rectangular plate the centre of gravity will lie midway on the line joining the points of intersection of the diagonals of the two opposite faces.

If such a body is supported directly below the centre of gravity, the body will balance on the point and will not rotate (Fig. 8.18).

This fact gives us an easy method of determining the centre of gravity. Even in the case of a body of irregular shape we can find its centre of gravity by balancing it on a horizontal knife-edge. The centre of gravity will lie in the vertical plane through the knife-edge By balancing the body in different directions, the centre of gravity can easily be determined.

Or we can suspend the body by means of a thin piece of thread. The body will take up a position of rest such that its centre of gravity is vertically below the line of the thread and also in the lowest possible position. For example, by suspending the plate, usually called lamina, from two different points from which a plumb-line is also suspended, we can obtain the position of the centre of gravity.

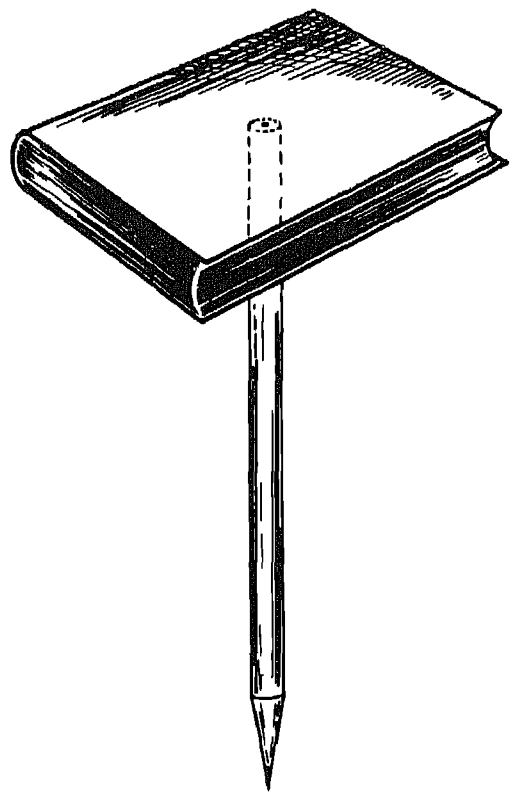


FIG. 8·18. *Balancing a book at its centre of gravity.*

In both the cases the centre of gravity would lie on the plumb-line, so the point of intersection of the plumb-lines in the two cases would give the position of centre of gravity (Fig. 8.19).

It is not necessary that the centre of gravity of a body must lie within the body. For example the centre of gravity of a ring lies at the centre of the ring outside the body of the ring. The centre of gravity of a horse-shoe or that of motor tyre will not lie within the material of the body. Similarly a cup or a tumbler has its centre of gravity lying outside the main body.

The positions of centre of gravity of a few geometrically shaped bodies are given below :

Body	Position of the centre of gravity
1. A uniform rod	At the mid-point of its axis.
2. A circular disc	At the mid-point of the line joining the centres of the two circular faces.
3. A circular ring	At its centre (outside the body).
4. A triangular lamina	At the point of intersection of its medians
5. A cylinder	At the mid-point of its axis.
6. A solid cone	On its axis at distance h/4 from the base where h is the height of the cone.
7 A hollow cone	On its axis at distance h/3 from the base

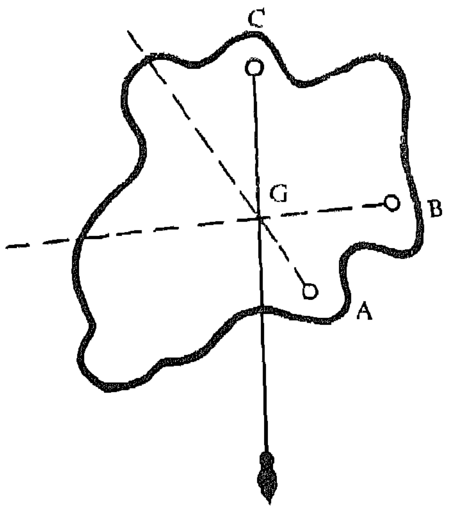


FIG 8·19. *Determining the centre of gravity of a lamina of irregular shape by suspension.*

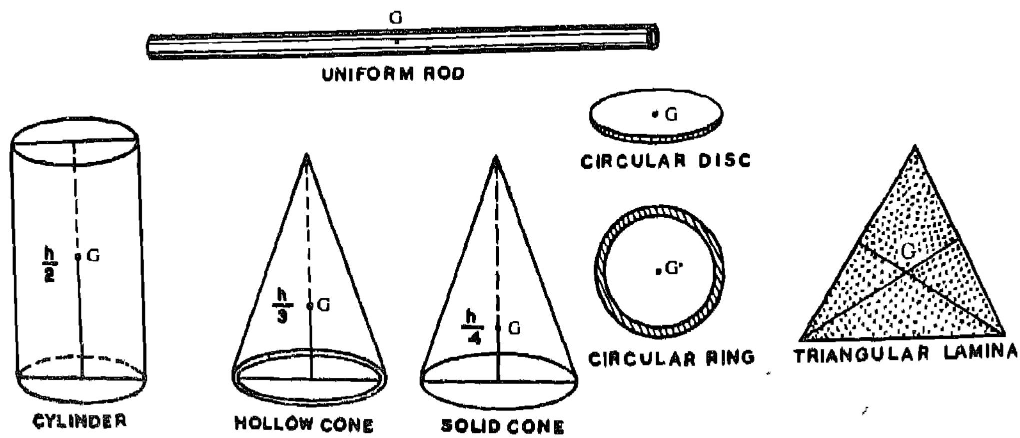


FIG. 8·20. *Illustrating the centre of gravity of regularly shaped objects.*

8.10 Centre of Gravity and Equilibrium

We have already said that a body will be in a state of equilibrium when (i) the resultant force acting on it is zero and (ii) when the resultant couple (torque) acting on it is zero. Let us now consider some typical cases of equilibrium. A book lying on a table (Fig. 8.21), a scale balanced on a knife-edge (Fig 8 22) and a ball placed on a table (Fig. 8 23). No doubt all three are in equilibrium. The resultant force as well as the resultant couple in the above three cases are zero.

Now raise one side of the book a little and then let it go. The book returns to its original position of equilibrium. We then say the book is in stable equilibrium.

A body is said to be in stable equilibrium if when given a slight displacement and then released, it comes back to its original position of equilibrium.

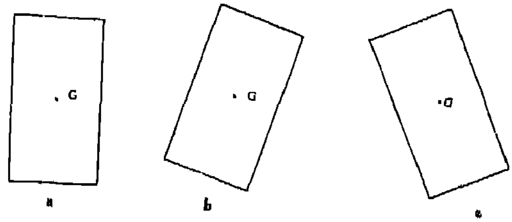


Fig. 8.21.

Next consider the scale balanced horizontally on the knife-edge. Even a slight displacement given to the scale causes it to tip The scale when balanced on the knife-edge was in unstable equilibrium.

A body is said to be in unstable equilibrium if when given a slight displacement and then released, it continues to move further from its original position.

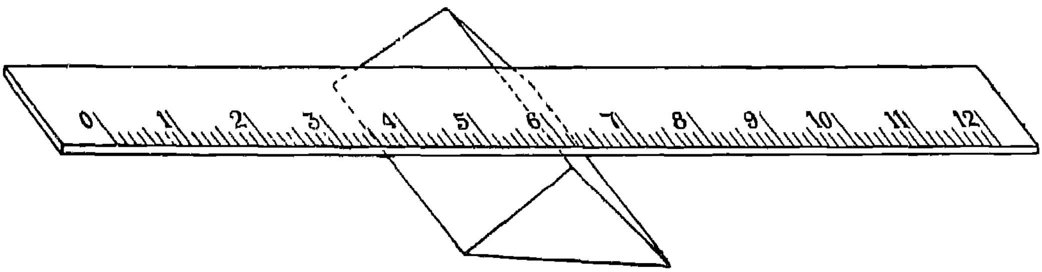


Fig. 8 22

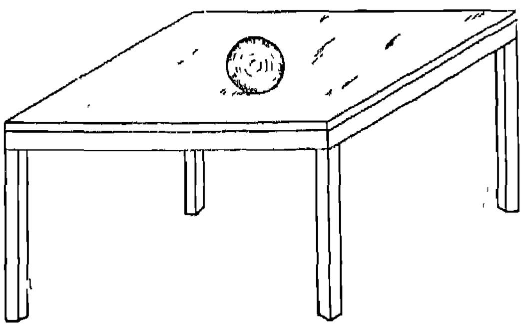


Fig. 8 23.

Lastly take the case of the ball placed on the table.

On receiving a slight displacement the ball rotates to one side and then it stays in the new position. It does not return to the original position nor does it continue to move further. It stays in the position in which it was left. The ball is in neutral equilibrium.

A body is said to be in neutral equilibrium if when given a displacement and then released, it stays in the new position.

Relation between the position of the centre of gravity and the states of equilibrium

Once again consider Fig. 8 21 On raising one end of the book the centre of gravity is raised. In contrast the centre of gravity of the scale balanced on a knife-edge is lowered when it is slightly rotated about the edge. Finally the centre of gravity of the ball resting on a horizontal table is neither raised nor lowered on being rolled.

On analysing similar situations one finds that if a body is in stable equilibrium then any slight displacement raises its centre of gravity. On the other hand when it is in unstable equilibrium the slight displacement lowers the centre of gravity and when it is in neutral equilibrium, the centre of gravity is neither raised nor lowered by the slight displacement.

Even when two bodies are in stable equilibrium, one may be more stable than the other. This may be found out by determining how much the body may be tilted so that when released it would regain its original position. The following examples will illustrate the point under discussion :

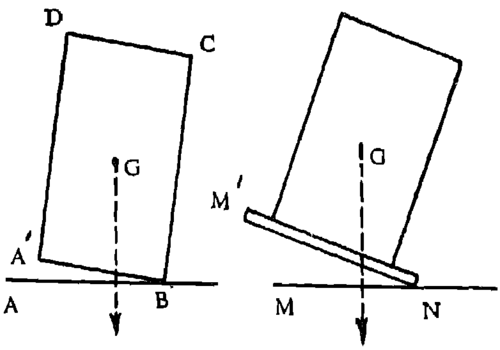


Fig 8 24

Consider an empty box ABCD (Fig. 8.24) standing on its end face. It is in stable equilibrium. It can be tilted through the angle GBC without the risk of its toppling over. Here G is the centre of gravity of the empty box. Now attach a broader base of negligible weight to its face. This time the box can be tilted through a much greater angle without falling over on its side. Hence the greater the width of the base, the greater is the degree of stability.

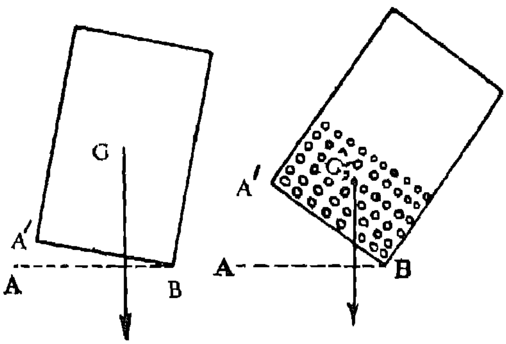


Fig. 8·25

Again consider the same box standing on its end face (Fig 8.25). Place inside the box some bricks filling about half of the box. Now the centre of gravity of the composite body will be situated at G′, a point lower than G. In this case the box can be safely tilted about B through an angle ABA″ and it will not topple It is only when the tilt goes beyond this angle that the box would topple over. We have seen that if the box were empty then it could be tilted only through angle ABA′ without its toppling over. So the stability has been increased by lowering the position of the centre of gravity.

We conclude therefore that the stability can be increased,

(i) By increasing the width of the base.
(ii) By lowering the centre of gravity of the body.

Of course the reverse is also true. That is, the stability is decreased if the base area is reduced or if the centre of gravity is raised.

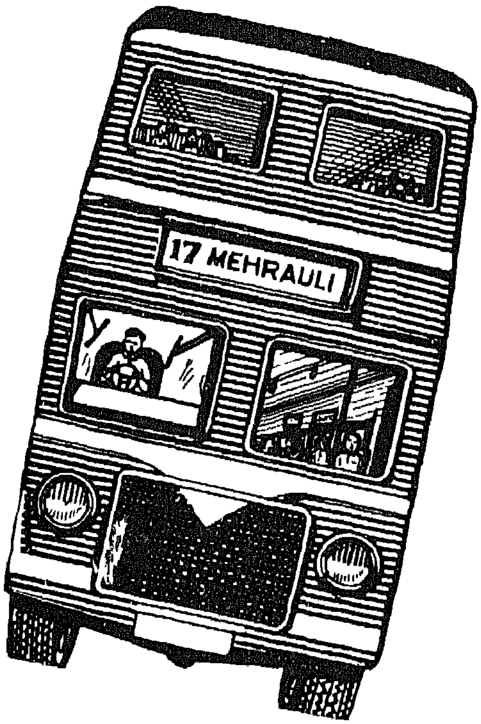


FIG. 8.26. *A double decker bus tilted to one side.*

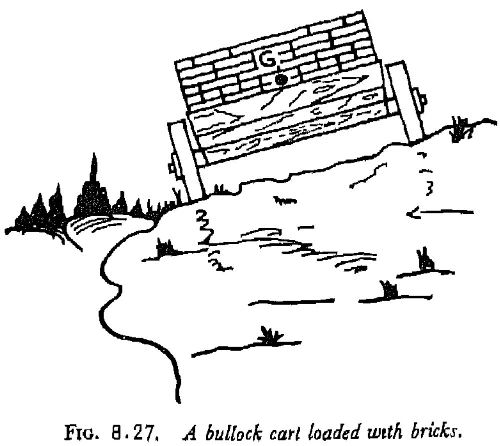


FIG. 8.27. *A bullock cart loaded with bricks.*

A few examples

The following examples help to illustrate the above principles. In a double-decker bus the chasis and the engine being very heavy the centre of gravity of the empty bus remains at a low height. When all the seats are occupied in the lower as well as the upper deck, the centre of gravity is raised (Fig. 8.26). Now the bus becomes less stable. If the loaded bus passes over the surface of a road such that it tilts to one side, there is a danger of its being overturned if the tilt is too much. In our country the buses are so designed that in the loaded condition they will retain the equilibrium even when tilted by 28°.

Similarly a bullock cart when fully loaded with bricks (Fig. 8 27) may not overturn when going on the road with a tilted surface but if the same cart is fully loaded with straw as shown in Fig. 8.28, it may overturn on the same road. The reason is that in the latter case the centre of gravity may be much higher so that the plumb line from the centre of gravity lies outside the space between the two wheels in the tilted position

Self erecting doll—Dolls have been designed so that they always stand erect. The doll has a curved base and its centre of gravity is situated below the centre of curvature of the curved bottom as shown in Fig. 8.29. As a result when the doll is tipped, its centre of gravity is raised and the point of support shifts. Now the centre of gravity is no longer above the point of support but it is in such a position that the force of gravity tends to rotate the doll until the doll resumes the erect position bringing the centre of gravity in the lowest possible position and vertically above the point of support.

Similarly the design of a typical Indian *lota* (Fig. 8 30) has a curved bottom and

its centre of gravity is kept low. When resting on level surface, the centre of gravity is vertically over the point of contact When the *lota* is tilted a little to one side, the point of contact shifts and the centre of gravity is raised slightly so that the weight acting through the centre of gravity applies a moment tending to bring the *lota* back to its original position

Again the *patili* used in Indian kitchen has a typical design which gives it a high degree of stability. The lower half of the vessel is concave such that its centre of curvature lies above the dotted level (Fig. 8.31). But the bottom is made so heavy that the centre of gravity of the whole vessel lies in the line AB. It can be easily seen that the *patili* can be tilted through 90° so that the line AB becomes vertical. Up to this stage the *patili* will not overturn. It is only when the *patili* is tilted beyond this stage that it will overturn; thus the *patili* is very stable.

Similarly the tumbler or bucket is usually so designed that the sides slant outwards as we go up. Such a design increases the capacity of the vessel without raising its centre of gravity unduly. Naturally this goes to increase its stability.

We can cite a number of examples where the stability gets reduced because the centre of gravity has been raised. For instance, a man in the standing posture can be more easily knocked down than one who is sitting In the former case the centre of gravity is higher. Again a man standing on one foot is less stable than one who stands on both feet. In the latter case the base area has been increased so the stability too is greater. In a boat if the passengers abroad stand up, then the centre of gravity gets raised and the boat is liable to be overturned when it tips on either side of the central keel.

FIG. 8.28. *A bullock cart loaded with straw*

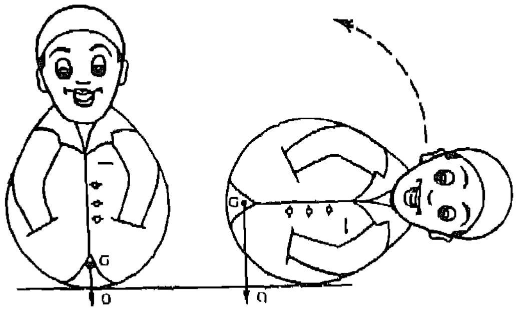


FIG. 8.29. *Self erecting doll.*

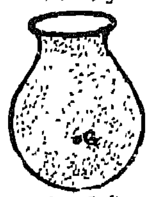


FIG. 8.30. *Indian lota.*

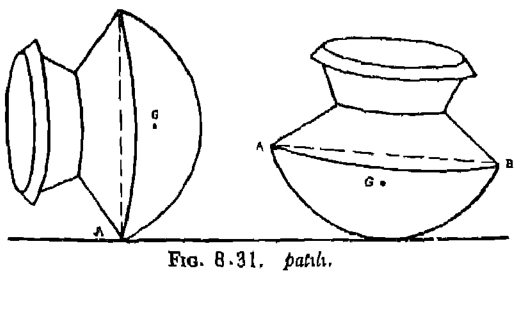


FIG. 8.31. *patili.*

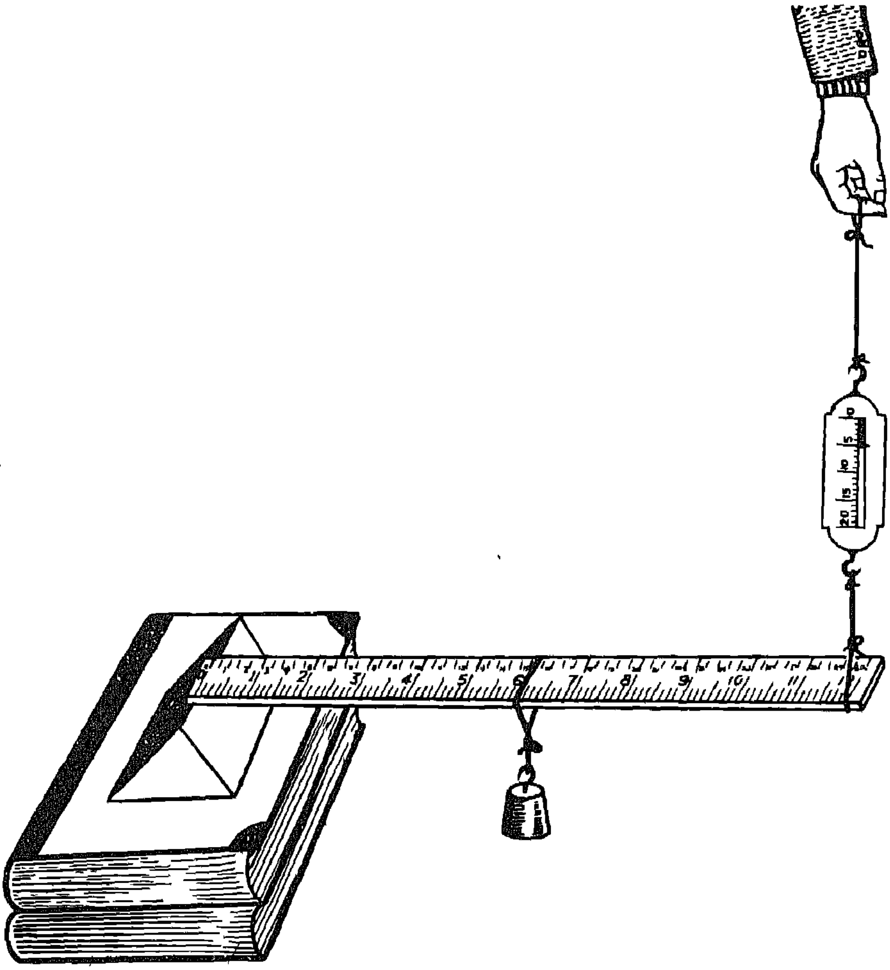
The tight rope walker usually carries a long pole held in a horizontal position. The pole sags a little at both ends. Thus its centre of gravity lies below the middle point of the pole. The result is that the centre of gravity of the rope-walker and the pole combined is at a point lower than the centre of gravity of the rope-walker alone. Thus he is able to increase his stability while walking on the tight rope.

Classroom Activities

1. Take a long heavy plank and fix it like a see-saw. Now sit on a side of this see-saw and ask your younger brother to sit on the other side. How can your younger brother balance you ? Is it possible for your brother to balance you and your friend sitting together on one side of the see-saw ?
2. Place a flat ruler so that one end acts as a fulcrum as shown in the figure on page 104. Hang a 500 g wt from the middle of the ruler. Attach a spring balance by means of a piece of spring to the other end of the ruler and lift. What class of lever do you have ? Move the weight closer to the fulcrum at the 10 cm mark What is the reading when you lift the ruler now ?

Questions

1. A 50 kg boy sits at a distance of 1·4 metres from the fulcrum of a see-saw. How far from the fulcrum must a boy of mass 35 kg sit in order to balance the board ?
2. A 10 kg child and a 12 kg child sit at the opposite ends of a 4 metre see-saw pivoted at its centre. Where must a 8 kg child sit so that the see-saw may be balanced ?
3. A 100 kg bag of wheat is suspended from a 5 metre long uniform pole at a distance of 2 metres from one end. The mass of the pole itself is 5 kg Two men carry the load, one at each end. Calculate in newtons the force experienced by each one of them (the force with which the earth attracts one kg is 9·8 newtons).
4. A metre scale of mass 50 g is suspended by a string through a hole at the 30 cm mark. A mass of 500 g is suspended at the 10 cm mark. How much mass should be suspended at the 90 cm mark to balance the scale ? Calculate the tension in the string.
5. A 10,000 kg elephant is crossing a bridge (single span) 20 metres long. When he is 6 metres from one end, how much of the weight is supported by each pier ?
6. The sides of a right-angled, uniform, triangular lamina are 12 cm, 13 cm and 15 cm. What is the distance of the centre of gravity from the right-angled vertex ?
7. Each of the front wheels of a car are found to support 500 kg and each of the back wheel 400 kg. If the distance between the front and back axles is 3 metres, what is (a) the total mass of the car; and (b) the centre of gravity of the car ?
8. Two forces of 100 gf and 150 gf are applied at the two ends of a metre stick. If the mass of the metre stick is 50 g, what is the maximum weight that can be supported by the stick and what is its location ?



9 What is the minimum force required to lift a uniform iron rod 10 metres long and mass 150 kg?

10. A five metre long teak-wood log weighs 300 kg. If the centre of gravity is at a distance of 2 m from the heavier end, what is the minimum force in newtons required to lift the log from the lighter end ?

11. The fulcrum of a 5 metre long lever is at distance of 1 metre from one end. An effort of 50 kgf is required to lift a load at the other end What is (a) the weight lifted, (b) the mechanical advantage, and (c) the reaction at the fulcrum ?

12. A six metre long uniform lever has a mass of 10 kg. If the fulcrum is at a distance of 1·5 metres from one end and a mass of 100 kg is to be lifted, what is the effort to be applied and the reaction at the fulcrum ?

13. A uniform horizontal drawing board is 4 m long. It is fixed to a beam at one end and rests on a support 1·5 m from this end. A 50 kg man stands at the other

end of the board and another 60 kg man is standing 2 m behind him If the mass of the board is 80 kg, calculate the reactions at the support and the beam.

14. A tapering fishing rod weighing $2 \cdot 5$ kgf has its centre of gravity at $0 \cdot 75$ m from the thicker end. To determine the weight of a fish, the fisherman hangs the fish from the thicker end and can balance the rod by placing his finger under it at $0 \cdot 5$ m from this end. Calculate the weight of the fish

15. When a boat is rowed by oars, the fulcrum at the end of the oars is in water. If the total length of the oars is 5 m and the distance between the effort and the hinge is 1 m, calculate (a) the force exerted by the oars on the boat, and (b) the net force which tends to move the boat if the effort is 100 newtons.

16. A uniform stick can be balanced on a knife-edge 10 cm from one end when a weight of 200 gwt is hung from that end. When the knife-edge is moved 5 cm farther from that end, the weight has to be moved to a point $8 \cdot 75$ cm from the knife-edge to obtain a balance. Find the length of the stick and its weight.

17. A circular hole of radius 2 cm is punched out of a uniform circular plate of radius 4 cm, the centre of the hole being 2 cm from the centre of the plate Calculate the distance of C.G of the remaining portion from the centre of the circular plate.

18. A square hole of side 1 m is cut from a uniform circular sheet of radius 2 m, the sides of the square lying along two diameters of the circle. Calculate the distance of the C.G of the remaining portion from the centre of the circle.

Further Reading

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CHAPTER IX

Measurement of Mass and Density

9.1 Mass and Weight

In a previous chapter we have explained what mass is and we have also described a method of comparing masses. However, in actual practice, that is not the method used for comparing masses. To measure mass we make use of the fact that the forces with which the earth attracts two equal masses are the same. The force with which the earth attracts any given body is called its weight and is measured in newtons. If the mass of a body is m kg, its weight is mg newtons, where g is the acceleration due to gravity. When we lift a book our muscles experience a force which is equal to the force with which the earth attracts the book and this is the weight of the book. The distinction between mass and weight must be understood clearly. The value of 'g' changes slightly from place to place on the surface of the earth. Therefore the weight of a body is different at different places. Hence, if we hang the same mass from a very sensitive spring balance, the readings of the balance will be different at different places. A body which has a weight of one newton at sea-level in the latitude of 45°, weighs only 0·997 newton at the equator but weighs 1·002 newton at the poles.

An example where the weight of the body becomes zero is that of a body in a satellite going round the earth. This reduction in weight to zero will become clear later.

We must also remember that the mass of a body is a scalar quantity but its weight is a vector quantity, the direction of this vector being the direction in which the earth pulls it

To determine the masses of different bodies we use the physical balance in which the masses are measured by making use of the principle of moments

9.2 The Physical Balance

We are familiar with the physical balance shown in Fig. 9 1. It consists of a light but rigid metal beam capable of rotation about a knife-edge. The knife-edge is made of a hard material like agate or steel and rests on a platform of similar material

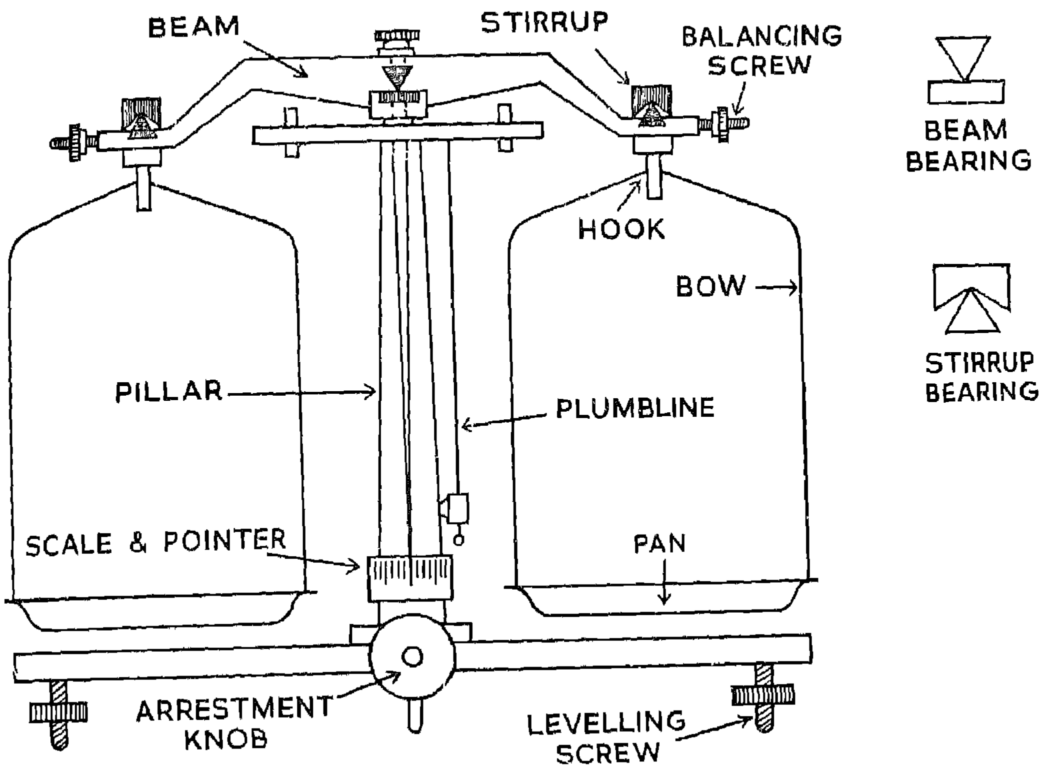

FIG. 9.1. *Physical balance*

When the balance is not in use, the beam rests on another support so that the knife-edge does not rest on the platform to save the knife-edge from wear and tear. When the balance is to be used the agate platform is raised so that now the beam is free to rotate in a vertical plane about a horizontal axis passing through the agate knife-edge. Two pans are suspended from the beam by knife-edges The beam bearing and the stirrup bearings supporting the pans are shown separately in the figure The balance is enclosed in a glass case to protect it from air currents and rapid changes of temperature outside. The position of the beam is indicated by a long metal pointer fixed rigidly to it so that its direction is at right angles to the line joining the two outer knife-edges. The position of the pointer is read on a scale fixed to the base of the pillar. The beam also carries at its two ends two balancing screws which can be moved slightly to make the pointer swing through equal divisions on either side of the central mark of the scale. The centre of gravity of the beam and pan system is directly below the central knife-edge and its position can be raised or lowered by moving the bob on a screw attached to the beam.

To use the balance, the levelling screws fixed to the base of the balance must be adjusted so that the plumb line hangs centrally through its ring. We must then ensure that the stirrups are resting properly on the knife-edges. We now raise the beam. If the pointer is not swinging through equal

divisions about the central mark, we lower the beam and adjust the levelling screws. One should never put weights on the pans or disturb the balance in any way while the beam is in the raised condition. The object to be weighed should be put in the left hand pan and the weights on the right hand pan. The weights should never be touched with the hand which are always greasy and forceps should always be used for handling them.

A good balance must possess the following qualities :

(a) It must be reliable, *i e.*, the beam must be horizontal or swing through equal angles about the horizontal directions when equal masses are put in the two pans. This requires that (i) the distances between the central knife-edge and the two end knife-edges must be exactly equal, (ii) the masses of the two knife-edges must be equal; and (iii) the masses of the pans must also be equal.

(b) The balance must be stable, *i.e.*, it must not suffer any change in shape and must return quickly to the equilibrium position when disturbed slightly.

(c) The balance must be sensitive, *i.e.*, the deflection of the beam from the horizontal position must be large for a small difference in the masses placed on the two pans.

For a simple discussion of the theory of the balance, we will assume that the knife-edges are in the same plane, horizontal and parallel to each other. The sensitivity of the balance then does not depend on the masses put on the pans. If the knife-edges are not in the same plane, the sensitivity of the balance depends on the masses on the pans. But we will not discuss this case.

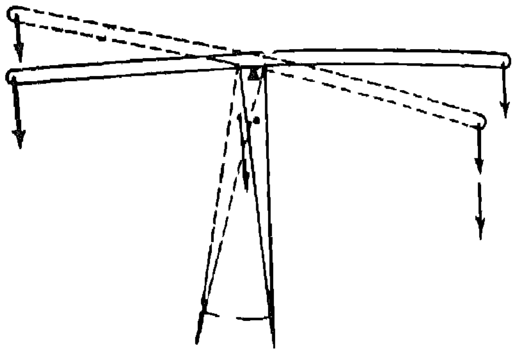

FIG. 9·2.

Let O be the position of the central knife-edge and G the position of the centre of gravity of the beam and its attachments, their mass being equal to M. Let p be the mass of each pan and m the mass on the left hand pan and $(m+\triangle m)$ the mass on the right hand pan. Let l be the length of each arm of the balance and h the distance between O and G If the beam is deflected by an angle θ, then taking movements about the point O, we have $(p+m)\ gl \cos\theta + Mgh \sin\theta = (p+m+\triangle m)\ gl \cos\theta$

or, $\tan\theta = \dfrac{l}{Mh}\triangle m$

Since the angle θ is small we can replace $\tan\theta$ by θ, and we have $\theta = \dfrac{l}{Mh}\triangle m.$

The sensitivity of a balance is, therefore, the greater, the longer and lighter the balance arms are and the nearer the centre of gravity is to the fulcrum, *i.e.*, the central knife-edge. As pointed earlier the centre of gravity can be raised by moving up the bob. But if the centre of gravity is raised to such an extent that it moves above the knife-edge, the balance becomes unstable. Also as the sensitivity is increased by reducing h, the period of oscillation of the balance increases and weighing becomes inconvenient.

If a body is placed on the left hand pan and a standard one kg mass is placed on

the right hand pan and the beam remains horizontal then it means that the force with which the earth attracts the given body is the same as the force with which the earth attracts the standard kilogram. Hence the mass of the body must be one kg Chemical balances are also constructed on this very principle but they are more sensitive and measure masses up to one thousandth of a milligram.

Such a balance will compare masses at all places accurately, even though g changes from place to place, from the equator to the north pole. If we take the balance to the moon or to any planet the force of attraction on both the pans of the balance will be the same provided the masses placed on the two pans are equal. Thus two masses that balance on the earth will balance anywhere else also.

9.3 Very Small and Very Large Masses

To measure extremely small or very huge masses indirect methods are used For instance the mass of the earth or that of the sun has been calculated from the gravitational attraction exerted by these bodies. The mass of the earth and sun are given in the table. You will learn about these methods in the chapter on gravitation Masses of atoms on the other hand are extremely small and have been determined by electromagnetic methods. The mass of the hydrogen atom is $1 \cdot 67 \times 10^{-27}$ kg. You will learn about these methods later in the book

Below you find the masses of some of the more familiar objects

Object	Mass	
Comma of hair; wing of a fly	50 microgram $= 50 \times 10^{-6}$ g	can be measured on a balance
a postage stamp	20 mg	
a pound	$453 \cdot 59$ g $= 0 \cdot 45359$ kg	
a litre of water	10^3 g $= 1$ kg	
metric ton	10^3 kg	
air in an ordinary room	100 kg	
the earth	$5 \cdot 98 \times 10^{24}$ kg	can be measured by indirect means only.
the sun	$1 \cdot 99 \times 10^{30}$ kg	
a red corpuscle of blood	10^{-13} kg	
a molecule of oxygen	$5 \cdot 3 \times 10^{-26}$ kg	
an electron	$9 \cdot 1 \times 10^{-31}$ kg	

9.4 Conservation of Mass

In all chemical and physical changes the total mass of the reacting substances remains constant This principle is called the conservation of mass. For example when a block of ice melts into water, the mass of water formed is the same as that of the original block of ice Similarly in the changes of state, *e.g.*, boiling, the mass of the end products is the same as that of the initial substances. The law of conservation of mass also holds in chemical reactions. One can verify this in the photographer's flash bulb which has the same mass before and after the flash. Other common chemical reactions with which we are familiar, *e.g.*, burning of coal or wood, burning of petrol in a car obey the same law.

In the beginning of the twentieth century Einstein showed that mass and energy are equivalent, *i.e.*, they can be converted into each other In a reaction some energy is either absorbed or given out. This will mean that the mass after the reaction will either increase or decrease. However, the change in mass for a chemical reaction is so negligibly small that it can not be detected by chemical balances. However, in an atom bomb an appreciable fraction of the mass is converted into energy. Similarly in the sun also mass is being converted into energy. It has been calculated that 4×10^9 kg of the solar mass is being converted into energy every second. But the

mass of the sun is so enormous that about a hundred billion years must elapse before its source of energy is exhausted.

For ordinary chemical and physical changes the mass of the substances taking part in these reactions remain practically constant.

9.5 Density

In every day experience we come across light objects and heavy ones. What do we exactly mean when we say that wood is lighter than iron ? A log of wood is certainly much heavier than an iron nail. Actually, when we say that wood is lighter than iron, we mean that a piece of iron is more massive than an equal volume of wood. Clearly then matter is more densely packed in iron than in wood. Therefore iron is said to be denser than wood. This property is given the name density. Density can be conveniently compared by comparing the mass of a unit volume of various substances. Hence density is defined as the mass per unit volume of the substances.

In MKS system density is expressed in kilograms per cubic metre or kilograms per litre*. For example, the density of water is 1,000 kg per cubic metre or one kg per litre at 4°C.

Mathematically, we can write

$$\text{density} = \frac{\text{mass of the body}}{\text{volume of the body}},$$

$$\text{or } d = \frac{m}{v} \text{ (d=density),}$$

(m=mass of the body),

(v =volume of the body)

We have then,

mass of the body=volume of the body × density.

To find the density of a given body

If the body is regular in shape, we can calculate its volume v from the measurement of its sides. Its mass m can *be measured on the equal arm balance. So density* d *can be found by* applying the formula $d = \frac{m}{v}$. For example, if the body has a shape of a cuboid with sides measuring a, b and c metres then its volume will be abc cubic metre If its mass is m kg, then density $= \frac{m}{abc}$ kg/m³.

If the body is a sphere of radius r metre then its volume will be $\frac{4}{3}\pi r^3$ m³ and if mass is m kg then density $= \frac{3m}{4\pi r^3}$ kg/m³. Some formulae for finding volumes of bodies of regular shape are given below :

Body	Volume	Formula
Cuboid	Length × breadth × thickness	$= abc$
Cube	(length of its side)³	$= a^3$
Sphere	$\frac{4}{3}\pi \times$ radius³	$= \frac{4}{3}\pi r^3$
Solid right cone	$\frac{1}{3}\pi \times$ radius² × height.	$= \frac{1}{3}\pi r^2 h$
Cylinder	$\pi \times$ radius² × height.	$= \pi r^2 h$

Here we must keep in mind the fact that in determining density when we divide mass by volume, we not only divide the number expressing mass by the number expressing volume but we also divide the unit of mass by the unit of volume

If 10 cc of alcohol has mass 8 g, then

$$\text{density of alcohol} = \frac{8 \text{ g}}{10 \text{ cc}} = \frac{8 \times 10^{-3} \text{ kg}}{10 \times 10^{-6} \text{ m}^3}$$
$$= 800 \text{ kg/m}^3$$

*According to the recommendation of the twelfth General Conference of Weights and Measures, held at Paris in October 1964, it has now been decided to define the litre as one cubic decimetre and not as the volume of one kilogram of water at the temperature of its maximum density, 4°C, as this volume is 1 000028 cubic decimetre But for all ordinary purposes we may take the density of water as 1 00 kilogram per litre

2 If bodies are irregular in shape, then it is not possible to find the volume of such a body by measuring the length of its sides. In such cases we find the volume of the body by the method of displacement of water.

TABLE OF DENSITIES OF SOME COMMON SUBSTANCES

	kg/m^3
Solids	
Aluminium	$2 \cdot 70 \times 10^3$
Brass	$8 \cdot 5 \times 10^3$
Copper	$8 \cdot 9 \times 10^3$
Cork (approx.)	$0 \cdot 24 \times 10^3$
Diamond	$3 \cdot 53 \times 10^3$
Glass	$(2 \cdot 5 \text{ to } 3 \cdot 6) \times 10^3$
Gold	$19 \cdot 3 \times 10^3$
Human body	$1 \cdot 07 \times 10^3$
Ice	$0 \cdot 92 \times 10^3$
Iron	$(7 \cdot 1 \text{ to } 7 \cdot 8) \times 10^3$
Lead	$11 \cdot 3 \times 10^3$
Nickel	$8 \cdot 6 \times 10^3$
Platinum	$21 \cdot 4 \times 10^3$
Silver	$10 \cdot 5 \times 10^3$
Wood	$(0 \cdot 6 \text{ to } 1 \cdot 1) \times 10^3$
Uranium	$18 \cdot 7 \times 10^3$
Liquids	
Alcohol	$0 \cdot 81 \times 10^3$
Carbon tetrachloride	$1 \cdot 6 \times 10^3$
Glycerine	$1 \cdot 28 \times 10^3$
Kerosene oil	$0 \cdot 82 \times 10^3$
Liquid hydrogen	$0 \cdot 07 \times 10^3$
Mercury	$13 \cdot 60 \times 10^3$
Petrol	$(0 \cdot 68 \text{ to } 0 \cdot 72) \times 10^3$
Water	$1 \cdot 00 \times 10^3$
Gases at 0°C and at Atmospheric pressure (N.T.P.)	
Air	$1 \cdot 293$
Chlorine	$3 \cdot 214$
Helium	$0 \cdot 1785$
Hydrogen	$0 \cdot 08988$
Oxygen	$1 \cdot 429$
Gases in the inter-stellar space	10^{-21} (approx.)

9.6 Relative Density

We often need to know how many times a substance is heavier than some other substance. Naturally while comparing the masses of two bodies we must take equal volumes. For the sake of convenience we compare the mass of the given body with the mass of an equal volume of water This ratio is called Relative Density Hence relative density is the ratio between the mass of a given body and the mass of an equal volume of water

$$\text{Thus Relative Density} = \frac{\text{mass of the given body}}{\text{mass of an equal volume of water}}.$$

Dividing the numerator and the denominator by the same volume, we get Relative Density

$$= \frac{\dfrac{\text{mass of the substance}}{\text{volume of the substance}}}{\dfrac{\text{mass of same volume of water}}{\text{same volume of water}}}$$

$$\text{or, Relative Density} = \frac{\text{Density of the substance}}{\text{Density of water}}$$

$D_r = \dfrac{D}{D_w}$, where D_r = relative density of the substance,
D = density of the substance,
D_w = density of water.

So that Relative Density can also be defined as the ratio between the density of the substance and the density of water.

As an example let us take copper. Its density is $8 \cdot 9$ g per cc or 8,900 kg per m^3 and that of water is 1 g per cc or 1,000 kg per m^3.

$$\text{Relative Density of copper} = \frac{8 \cdot 9 \text{ g per cc}}{1 \text{ g per cc}},$$
$$= \frac{8{,}900 \text{ kg/m}^3}{1{,}000 \text{ kg/m}^3},$$
$$= 8 \cdot 9.$$

Since relative density is a ratio between two masses of the same volume, it comes out to be a number alone. No units are to be attached to it.

Questions

1. Express the density of iron in g per cc.
2. How will you determine the density of an irregular piece of metal ?
3. Describe an experiment to determine the density of cork.
4. A rectangular block of wood weighs 150 g. Its dimensions are 8 cm $\times$ 5 cm $\times$ 5 cm. What is its density ?
5. Give a method for determining the relative density of lead shots.
6. A tank is 2 metres long, 1·5 metres wide and 1 metre deep. How many kilograms of water will it hold ?
7. You have to manufacture a 1,000 g brass weight. How many cubic centimetres of brass will be required ? Given density of brass = 8·5 g per cc.
8. An ice-box measures 30 cm $\times$ 20 cm $\times$ 15 cm. How many grams of ice will it hold ? The density of ice is 0·9 g per cc.
9. Two liquids A and B are mixed and no chemical action takes place. Their volumes and densities are as below :

Liquid	A	B
Volume	10 cc	8 cc
Density	0·6 g/cc	0·9 g/cc

If there is no change in their volumes, find the relative density of the mixture.

10. Explain why the relative density is numerically equal to the density of the body expressed in CGS system. Is it true in MKS system also ?
11. What volume of water must be added to 100 cc of alcohol of density 0·75 g/cc so that the resulting mixture has a density 0·85 g/cc ? Assume that there is no change in volume when the two liquids are mixed.
12. Calculate the mass of air at NTP inside a room measuring 6 metre $\times$ 4 metre $\times$ 3 metre. Given the density of air at NTP is 1·29 g/litre.
13. 3·0 g of salt is dissolved in 2 litres of water Find the density of the solution assuming that the solution has a volume 2 litres.
14. A density bottle can hold 25 g of water. How many grams of spirit of density 0·8 g/cc will it hold ?
15. What is the mass of a solid sphere of iron of diameter 5 cm? The density of iron is 7·5 g/cc.
16. A thread of mercury inside a circular capillary tube is 15 cm long and its mass is found to be 11·2 g. If the density of mercury is 13·6 g/cc, find the diameter of the capillary.
17. A piece of wire 52 cm long weighs 0·322 g. If the density of the metal be 7·7 g/cc, find the diameter of the given piece of wire
18. Describe a method for determining the relative density of sand.

19. In an experiment for determining the relative density of sand the following readings were obtained;

    Mass of sand $= 8 \cdot 2$ g.

    Mass of density bottle full of water $= 37 \cdot 58$ g.

    Mass of bottle containing the sand and filled with water $= 42 \cdot 5$ g.

    Calculate the relative density of sand.

20. A ball made of gold weighs 675 g and its diameter is 5 cm. The density of gold is 19 3 g per cc. Is the ball solid or hollow? If hollow, find the volume of the empty space inside it.

21. A hollow sphere of copper has its wall 0 3 cm thick If its other diameter is 5 cm, find its mass (relative density of copper is $8 \cdot 9$ g cc)

22. A column of mercury stands up to a height of 76 cm inside a vertical glass tube of cross sectional area 1 sq cm. Find the mass of the column of mercury in grams (relative density of mercury $= 13 \cdot 6$) Calculate the force exerted by the column of mercury at the bottom of the tube.

23. In an experiment for determining the relative density of lead-shots with the help of density bottle, the following readings were obtained,

    Weight of empty bottle $= 11 \cdot 2$ g

    Weight of bottle containing some lead-shots $= 31 \cdot 4$ g.

    Weight of bottle containing lead-shots and water to fill it $= 41 \cdot 6$ g.

    Weight of bottle completely filled with water alone $= 23 \cdot 1$ g.

    Find the relative density of lead-shots

24. A density bottle weighs $10 \cdot 2$ g when empty. When some salt is put inside, the bottle weighs $16 \cdot 4$ g. When the remaining space inside the bottle is filled with turpentine the bottle weighs $35 \cdot 4$ g When the bottle is filled completely with turpentine alone it weighs $32 \cdot 0$ g. When the bottle is completely filled with water, it weighs $35 \cdot 3$ g. Find the relative density of turpentine and salt.

25. How much water should be added to one litre of salt solution of density $1 \cdot 2$ g per cc so that the resulting mixture may have its density equal to $1 \cdot 1$ g per cc ?

26. Assuming that the earth is a sphere of radius 6,400 km, calculate its average density. (Mass of the earth is $6 \cdot 0 \times 10^{24}$ kg). If the average density of the surface rocks is $2 \cdot 7$ kg/litre, what conclusion do you draw about the density of the interior of the earth ?

27. The maximum mass a man can lift at the equator is $50 \cdot 00$ kg. What is the maximum mass he can lift at the pole ?(Assume g at the equator is $9 \cdot 77$ m/sec^2 and g at the pole is $9 \cdot 82$ m/sec^2).

28. A honey-bee is sitting at the bottom of a closed glass case balanced on one pan of a physical balance. If the bee is now supporting itself in the air contained in the case, how will it affect the equilibrium of the balance? Justify your answer.

29. On burning 13 g of coal, we get 1 g of ash and 44 g of carbon dioxide. Explain how the law of conservation of mass is obeyed in this reaction.

Further Reading

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CHAPTER X

Circular Motion

10.1 Introduction

Suppose we tie one end of a string to a piece of stone and firmly hold the other end in the hand. Now we can swing the ball in a horizontal circle with our hand as the centre. Here the stone moves all the time along a circle whose radius is equal to the length of the string. This is an example of *Circular Motion.*

Whenever a body moves along a circle, it is said to revolve about the centre of the circle. If it travels on the circumference with a constant speed, then the circular motion is said to be uniform. The axis about which the body 'revolves' is called the *axis of revolution*

There is another type of circular motion called *rotation.* In this the axis about which the body spins lies somewhere in the body itself, as for example in the case of a rotating flywheel attached to an engine. We first consider revolution.

10.2 Angular Velocity

Since the body is moving along the circumference of a circle, the speed of the body may be expressed either in terms of the linear distance traversed per unit time along the circular path or in terms of the angle through which the radius vector joining the body to the centre turns in unit time.

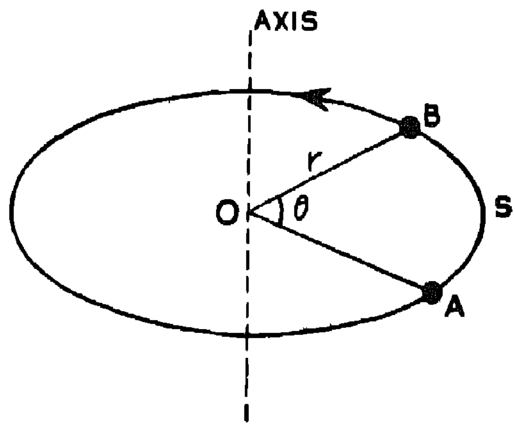

FIG. 10.1.

In Figure 10.1 the stone moves from A to B on the circumference in time t sec.

The linear speed at any point along the path$=\dfrac{\text{length of the arc AB in cm}}{t}$,

$$V=\frac{S}{t}\text{ cm/sec.}$$

On the other hand, the angular speed

$$\omega = \frac{\angle AOB}{t} = \theta/t.$$

If the $\angle \theta$ is expressed in radians then

$$\theta = \frac{\text{length of the arc AB}}{\text{radius}} = \frac{S}{r},$$

$$S = \theta r,$$

or, $$\frac{S}{t} = \frac{\theta r}{t},$$

or, $$V = \omega r.$$

In uniform circular motion, the magnitude of the velocity of the body remains constant but the direction keeps on changing. Hence the body describes the circular path with a varying velocity. Thus we arrive at the conclusion : linear speed = angular speed × radius, *i. e*, the linear velocity at any point is equal to the product of the angular velocity and the radius provided the angle is measured in radians. If the time taken to describe one complete revolution by the body is T, its angular velocity is

$$\omega = \frac{2\pi}{T} = 2\pi n,$$

where n is the number of revolutions described in one second.

Hence, $$V = \frac{2\pi r}{T} = 2\pi nr = \omega r.$$

It should be noted that at every point the direction of V is perpendicular to that of r, *i.e*, it is always in the direction of the tangent.

Example : A piece of stone is whirled at 100 revolutions per minute in a horizontal circle at the end of a string 60 cm long. Find the angular as well as the linear speeds.

In one complete revolution the angle described $= 2\pi$ radians.

In 100 revolutions the angle described

$= 100 \times 2\pi$ radians.

Time elapsed = 60 sec,

angular speed $\omega = \frac{100 \times 2\pi}{60}$ radians/sec,

or, $\omega = 10\ 47$ radians/sec

Now, linear speed = angular speed × radius,

$$V = \omega r,$$

$$V = \frac{60 \times 100 \times 2\pi}{60} \text{ cm/sec},$$

$$= 628 \text{ cm/sec}.$$

10.3 Acceleration of the Body Moving in a Circle

We know from Newton's first law that if no force is acting on a moving body, it will continue to move with constant speed along a straight line. But if a body is moving along a curved path there must be a force acting on it. Circular motion is a special case of curved motion.

In the case of the whirling stone tied at the end of a string, we know from experience that we have to exert a pull on the stone along the string in order to keep the stone whirling in the circular path.

This shows that a body moving in a circular path with constant speed possesses acceleration. We will now derive an expression for the acceleration of a small body (Fig. 10.2) moving at constant speed along the circumference of a horizontal circle with centre O. When the body is at A, its velocity is along the tangent AC and when it is at the point B, its velocity is along the tangent BD. Since the speed is the same throughout, the magnitudes of these instantaneous velocities at A and B are equal but their

The Relation $V = \omega r$ holds good not only as regards the magnitude of the quantities involved but also when we write it in the vector notation, *i.e.*, $\vec{V} = \vec{\omega} \times \vec{r}$ provided we take ω to be a vector in the axial direction, its positive direction being given by the right-handed screw convention. It can be shown that $\vec{\omega}$ is a vector.

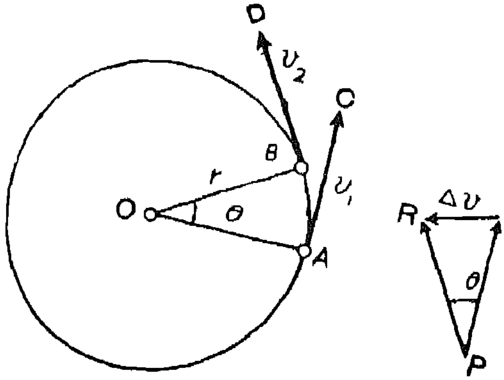

Fig 10·2.

directions are different Thus as the body moves along the circle uniformly, its velocity changes continuously though the speed remains constant. A change of velocity means that the body is experiencing an acceleration. Hence a force must be acting on the body to produce this acceleration and its direction must be the same as the direction of the acceleration. Let the instantaneous velocities of the body shown at A and B be $\vec{v}_1$ and $\vec{v}_2$ respectively In the diagram, PQ represents $\vec{v}_1$ and PR represents $\vec{v}_2$. Hence QR represents the change in velocity $\vec{\Delta v} = \vec{a}\Delta t$ that has occurred while the body moved from A to B because $\vec{\Delta v}$ must be added to $\vec{v}_1$ to produce $\vec{v}_2$.

Now the triangle OAB is similar to the triangle PQR because $\angle\theta$ is the same in both of them. Hence the corresponding sides must be proportional to each other.

$$\frac{QR}{PQ} = \frac{AB}{OA}$$

$$\text{or, } \frac{\Delta v}{v_1} = \frac{AB}{OA}$$

$$\text{or, change in velocity} = v_1 \times \frac{AB}{OA}$$

This change of velocity takes place in time taken by the body to move from A to B. Therefore

$t = \frac{\text{arc AB}}{V}$, where V is the speed of the body,

$$\text{or, acceleration} = (v_1 \times \frac{AB}{OA})/t,$$

$$= v_1 \times \frac{AB}{OA} \times \frac{V}{\text{arc}AB}.$$

For a small angle θ, the arc AB is nearly equal to the chord AB, and of course the magnitude of $v_1 = V$ Hence,

$$a = V \times \frac{AB}{OA} \times \frac{V}{AB}$$

$= \frac{V^2}{r}$, where r is the radius and V the speed along the circumference.

Let us now determine the direction of this acceleration. As the angle θ is made smaller, the point B approaches A and the chord AB becomes more and more equal to the arc AB, so that the direction of acceleration given by QR becomes more and more nearly perpendicular to PQ or to v_1. Now the direction perpendicular to v_1 is along the radius AO The acceleration is thus along the radius and is directed towards the centre O.

Thus we conclude that a body having a uniform circular motion experiences an acceleration directed towards the centre of its path The magnitude of the acceleration is $\frac{V^2}{r}$, where V is the constant speed of the body and r the radius of the circular path.

This acceleration can also be expressed in terms the angular speed ω. Substituting ωr for V in the above expression we have,

$$\text{the acceleration} = \frac{V^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}.$$

Since this acceleration is directed towards the centre we may call it a centrally directed acceleration. It is also called *centripetal acceleration.*

10.4 Centrally Directed Force

Since the body having a uniform circular motion experiences an acceleration $\frac{V^2}{r}$ in the direction of the centre, it must be acted upon by a force directed towards the centre Its magnitude must be equal to the product of the mass of the body and its acceleration

The centrally directed force $F=\frac{mV^2}{r}$ where m is the mass of the body.

This centre seeking force is called 'centripetal force'.

It is this centripetal force $\frac{mV^2}{r}$ which is responsible for continually changing the direction of the velocity of the particle so that it can move along the circle. In the case of the stone whirling along a circle, this central force is applied to the body through the medium of the string attached to it. Suppose while the stone is at A, the string breaks. At once the central force disappears. At this point the stone has an instantaneous velocity of magnitude V along the tangent to the circle at A. So in the absence of the centrally directed force, the stone will continue to move with the same velocity along the straight line given by the tangent at A (see Fig. 7·3, chapter VII).

This explains why in the absence of the centrally directed force the body flies off along the tangent to the circle. This motion of the body is in accordance with the Newton's first law of motion.

A centrally directed force is essential for maintaining the uniform circular motion of a particle.

This force acts continuously on the particle and is directed towards the centre of the circle.

This force is directly proportional to the mass of the particle and to the square of its uniform speed and is inversely proportional to the radius of the circle

The uniform circular motion is an interesting example of the case where the force responsible for maintaining the motion is directed at right angles to the direction of motion all along the path. This force has no component along the direction of the velocity of the body and this is why the magnitude of the velocity remains unchanged throughout.

10.5 Experiments to Illustrate Motion in a Circle

Obtain a disc, with a smooth surface, mounted in such a way that it can be rotated about a vertical axis passing through its centre O. By means of a hook, attach one end of a string to the axis, and the other end to a spring balance. Place the spring balance along the radius of the disc and attach a piece of metal (mass 100 g) to the other end of the spring balance (Fig. 10.3).

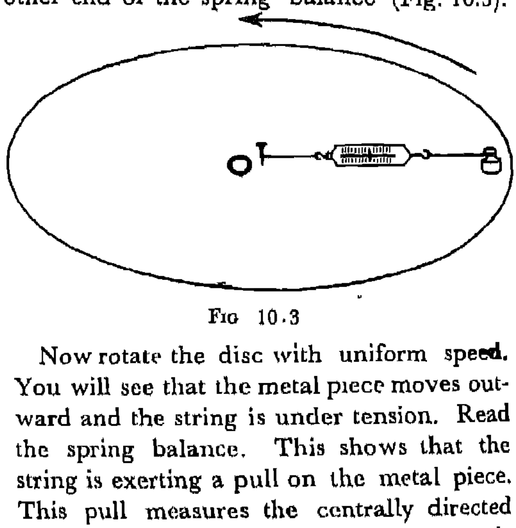

Fig 10.3

Now rotate the disc with uniform speed. You will see that the metal piece moves outward and the string is under tension. Read the spring balance. This shows that the string is exerting a pull on the metal piece. This pull measures the centrally directed force acting on the metal piece. Not only the string, but the metal piece also exerts an equal but opposite pull on the string. These two forces thus balance each other

By varying the mass and keeping the radius of the circular path as well as the

speed of rotation constant, we can show that the pull exerted by the spring balance (the centrally directed pull) is proportional to the mass of the metal piece. That is, if the mass of the metal piece is doubled, the pull recorded on the spring balance is also doubled.

Using the same length of the string and the same metal piece, rotate the disc with different speeds The mass will describe circles of the same radius each time but the speeds will be different. Take the readings of the spring balance each time and record the linear speed of motion of the metal piece.

The pull recorded by the spring balance will be found to vary as the square of the speed of the metal piece

Finally aking the same metal piece use different lengths of the string and adjust the rotation of the disc such that the speed of the metal along its circular path remains the same in all cases. Here you will find that the centrally directed force is inversely proportional to the radius of the circular path. That is, if the radius is halved, the centripetal force gets doubled, and if the radius is reduced to one-third of its previous value, then the centrally directed pull is increased three-fold.

Combining these empirical results, we get the same formula as we derived earlier, namely centrally directed force$=\frac{mV^2}{r}$.

10.6 Some Examples

Whenever there is motion in a circular path, a force must come into play to maintain the body moving along the circle This centrally directed force may be due to elasticity of the materials holding the body or it may be an electrical force or a gravitational one. In fact, wherever you come across a body moving with a uniform circular speed, you should always try to find out the origin of the centrally directed force responsible for maintaining the circular motion of the body.

When the wheels of a cycle rotate rapidly the mud attached to the tyre requires a centrally directed force to enable it to pursue the circular path. Generally the adhesive force between the mud and the tyre of the wheel is very small so that the mud breaks off and flies along the tangent to the rim of the wheel. Similarly, sparks from a grinding wheel too fly off along the tangent to the wheel.

A fly-wheel is a heavy metallic wheel rotating about an axis. When it rotates at a high speed, the central force necessary to keep the rim in circular motion is applied through the parts of the wheel joining the rim to the central part.

This force varies as the square of the speed of rotation While the wheel is rotating, the whole fly-wheel is under a great tension. (Fig. 10.4). If the angular speed becomes

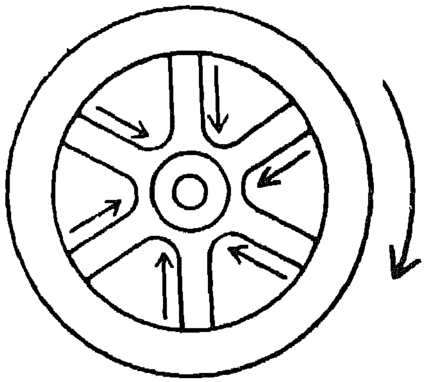

FIG. 10.4.

too high the cohesive force holding the molecules of the wheel together may no longer be greater than the necessary centripetal force, so that the wheel may disintegrate and its parts fly off along the tangent to the circles along which the parts were moving. It should be noted that the central force is greatest near the axis of the wheel for each inner ring has to supply the force needed to maintain the motion of all

the rings situated farther out. This is why the stout spokes connecting the hub of the wheel to its rim are thickest near the hub.

The fly-wheels are usually made of cast iron or cast steel. Since the breaking stresses for these materials are known, we can specify the diameter if the maximum speed of the fly-wheel is known For cast iron, the fly-wheel rim speed is approximately 1,500 to 1,800 metres per minute. With fly-wheels made of steel, higher speeds can be permitted.

10.7 A Stone Whirling in a Vertical Circle

When a piece of stone is whirled in a vertical circle by applying a tension through a string tied to it, it does not fall down even when it is at the highest point of the circle provided its speed is high enough.

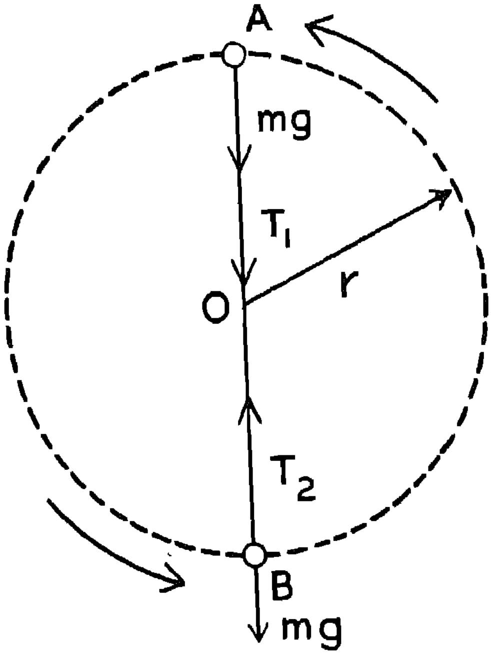

Fig 10·5. *A stone whirling in a vertical circle.*

Why does it not fall down in spite of the attraction downwards due to gravity ?

We know that a central force applied through the tension of the string must be directed to the centre O, in order to make the stone move along the vertical circle, (Fig. 10.5). If V is the speed and r the radius of the circle, then the force towards O must be mV^2/r where m is mass of the stone.

In position A, the stone is being attracted towards the earth in the direction of O by a force mg. So that the tension on the string and force mg must supply the central force $\frac{mV_1^2}{r}$.

$$T_1 + mg = \frac{mV_1^2}{r},$$

$T_1 = \frac{mV_1^2}{r} - mg$, where V_1, is the speed at position A So that, so long as the central force $\frac{mV_1^2}{r}$ is greater than or equal to mg, the stone will stay in the circular path at A, and the string will not get slack in the position OA. Otherwise the string will get slack and the stone will not reach the highest position of the circle.

This means that the speed V_1 must be greater than a certain critical speed if the stone is to stay in the circular path at A. This critical value is given by the equation

$$\frac{mV_1^2}{r} = mg \text{ or, } \frac{V_1^2}{r} = g \text{ or, } V_1 = \sqrt{rg}$$

We note that at the bottom B of the path, the tension of the string must supply the centripetal force mV_2^2/r as well as the force to balance the weight of the stone.

Tension at B, $T_2 = \frac{mV_2^2}{r} + mg$, where V_2 is the speed at B. We have chosen the speeds V_1 and V_2 because they are not the same. We shall find out the relation between them in the next chapter using energy considerations.

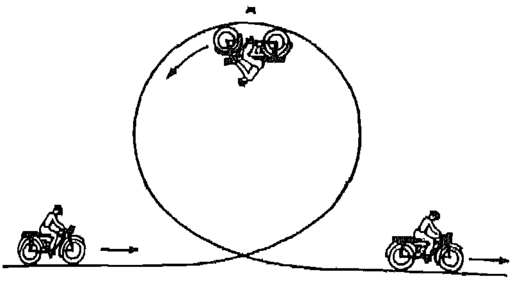

FIG. 10 6. *Acrobat rounding along a circular track*

In a circus show the acrobat often rides a motor-cycle and makes a loop along a circular track in a vertical plane (Fig. 10.6). How is he able to accomplish this feat ?

In this case too his speed should be large enough so that the central force $\frac{mV^2}{r}$ required to maintain the motor-cycle in the circular path is greater or just equal to the force with which gravity attracts the motor-cycle and the acrobat

So long as $\frac{mV^2}{r}$ is greater than the weight of the motor-cycle and acrobat (mg), an additional force acting downwards will be required to maintain the circular motion at A. This additional force $\frac{mV^2}{r} - mg$, must come in the form of the reaction at the track even at the highest position.

This means that the motor-cycle will stay in the track as long as $\frac{mV^2}{r}$ is greater than or equal to mg.

Such a behaviour explains why a whirling bucket of water describes a vertical circle without spilling any water. Here too the speed must be above a certain critical value given by the relation $V > \sqrt{rg}$, if the experiment is to succeed.

10.8 Centrifuge

It is a device in which a vessel is rotated at a high speed. It is used for separating fluids of different densities, for example, cream from milk.

The principle will be clear from the following :

When a vessel containing a liquid is whirled round with uniform speed, the side of the vessel applies a reaction inwards which maintains the circular motion of the liquid. The side of the vessel presses on the layer of liquid in contact with it, that layer in turn exerts a pressure on the next layer and so on. Clearly in each layer the pressure all over the layer must be the same, otherwise the liquid will not stay in that layer.

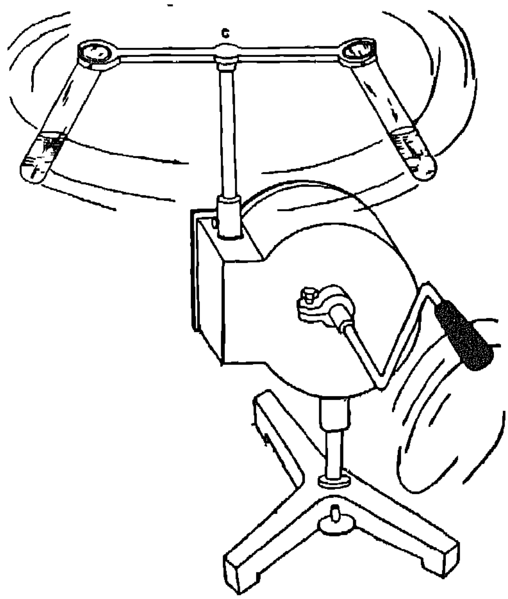

FIG. 10·7.

Suppose a liquid of uniform density ρ is placed in a test tube, and the test tube is whirled in a horizontal circle about the point C as the centre (Fig. 10.7).

Consider a small element of volume x of the liquid in the layer which is at distance r from the centre C. The central force exerted upon it by the surrounding liquid must be just sufficient to maintain this circular

motion, that is, it must be equal to $\frac{x\rho v^2}{r}$ where v is linear velocity in that layer. Now let a particle of volume x and density σ replace the element of the liquid we have considered in that layer, then a force $\frac{x\sigma v^2}{r}$ will be required to maintain it in that layer

If σ is greater than ρ, that is, if the particle is denser than the liquid, then the actual central force in that layer will fall short of what is required to hold the particle in that layer by $\frac{xv^2(\sigma-\rho)}{r}$; therefore, the particle drifts towards the closed end of the tube, just as a car skids when the friction of the road fails to provide a sufficiently large central force for negotiating a curve on the road at a given speed.

It follows from the above discussion that particles denser than the liquid will drift outward and those which are less dense than the liquid will drift inwards.

For instance, in the cream separator when milk is churned rapidly, cream being lighter occupies a position near the centre of the vessel and can be taken out. In the same manner blood corpuscles are separated from blood fluids when the whole blood is rotated in a centrifuge with high speed. Germs and viruses are also separated from fluids in this manner. Very high speed centrifuges called ultra-centrifuges have been developed which can be rotated at 1,300,000 revolutions per minute. With high speed centrifuges the heavier molecules (oxygen) in air can be separated from the lighter ones such as nitrogen.

10.9 The Centrifugal Governor

The Governor is a device for controlling the speed of an engine. It works on the principle of a centrifuge. It consists of two equal masses A and B attached to arms hinged on a vertical spindle which rotates at a speed proportional to the speed of the engine. The spindle controls a valve which regulates the supply of steam or fuel to the engine (Fig 10.8) As the speed of the engine increases, the circular motion of the masses A and B also increases, so the masses move farther away from the axis. Consequently the spindle is depressed enabling the valve V to decrease the supply of steam or fuel, thus reducing the speed of the engine.

The reason why the masses A and B separate out farther from the axis will be clear from the following

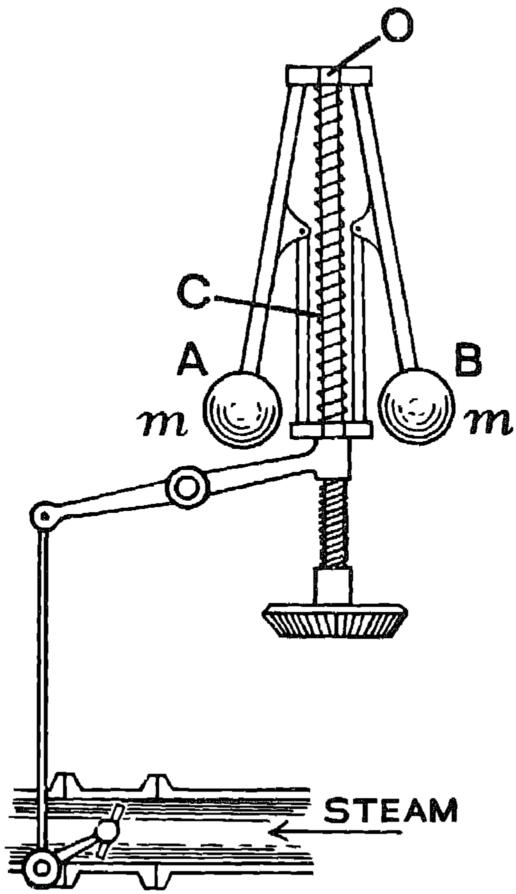

FIG 10 8 *Governor of an engine.*

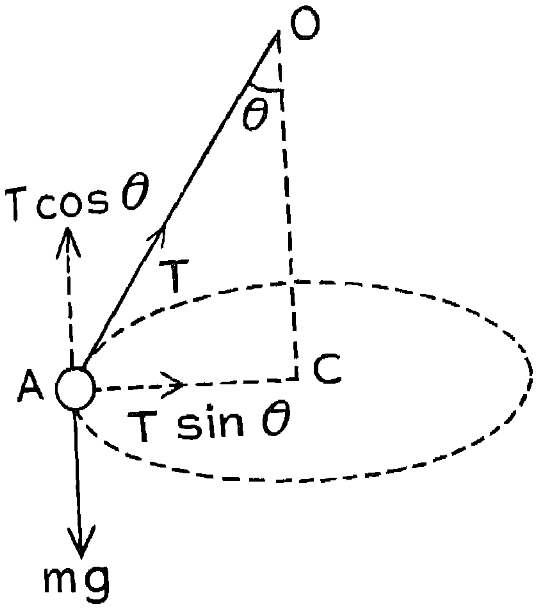

FIG. 10.9

In order to keep the ball A moving in a horizontal circle about OC, a central force must act along AC. This force is necessarily supplied by the combined action of the tension T along the arm AO and the weight mg acting vertically downwards (Fig. 10.9).

Now mg has no component along AC, while the resolved part of T along AC is T sin θ.

$$T \sin\theta = \text{central force} = \frac{mV^2}{r}, \text{ where}$$

$$r = AC;$$

also $T \cos\theta = mg$.

$$\text{So } \tan\theta = \frac{V^2}{rg},$$

$$\text{or, } \frac{r}{OC} = \frac{V^2}{rg},$$

$$\text{or, } OC = \frac{r^2g}{V^2} = \frac{g}{V^2/r^2} = \frac{g}{\omega^2},$$

where ω is the angular velocity of A in the circle So that the distance OC decreases as the angular velocity of the ball round the axis increases. That is, the balls A and B are thrown up farther away from the vertical axis as the speed of rotation increases. However, the arm OA (Fig. 10.8) will never become horizontal because this will require the angular speed ω to be infinite.

10.10 Why is the Earth an Oblate Spheroid ?

The above example explains why a circular loop of wire, when rotated rapidly about its vertical diameter, acquires the shape of an eclipse. The parts near the equatorial line are thrown out farther from the axis of rotation.

Similarly the spinning of the earth about its imaginary axis through its poles has caused equatorial regions to bulge out. This has resulted in a flattening of the earth at its poles.

10.11 Banking of Roadways on Curves

Whenever a car negotiates a curve at a high speed, it requires a central force to enable it to do so. If the centrally directed force is missing or is insufficient, the car will not follow the circular track; rather it will tend to fly off along the tangent to the track. In such a case we say the car tends to skid. If the central force is insufficient the car may even tip towards the outside of the curve.

Ordinarily this inward force on level road is supplied by the friction between, the road surface and the wheel If the curve is sharp and the speed of the car is high, the frictional force on the road may not be sufficiently large to maintain the circular motion. On this account, therefore, in modern times this inward force is obtained by tilting the surface of the roadways on the outside suitably. Under the circumstances, the perpendicular reaction from the surface of the road and the weight of the car acting downwards combine to give a resultant force

in the horizontal direction which is just the required central force.

Consider the diagram (Fig. 10.10) where the surface on the outside is raised making an angle θ to the horizontal; here A is the centre of gravity of the car.

To find the resultant of the reaction N and weight mg, construct the triangle ABC such that AC is parallel to N and proportional to its magnitude and from C draw CB parallel to the force mg and proportional to its magnitude; then AB is the resultant in the horizontal direction.

From the geometry of the figure,

in $\triangle ABC$, $\angle ACB = \theta$

$$\therefore \tan\theta = \frac{AB}{BC}.$$

The central force $AB = BC \times \tan\theta$.

If the mass of the car is m, its speed V and the radius of curvature of the bend of the road is r, then the central force required to maintain the motion of the car along the curve $= \frac{mV^2}{r}$.

AB must be equal to $\frac{mV^2}{r}$ and we know that $BC = mg$,

$$\frac{mV^2}{r} = mg\tan\theta,$$

$$\text{or,}\quad \frac{V^2}{r} = g\tan\theta,$$

$$\text{or,}\quad \frac{V^2}{rg} = \tan\theta.$$

If the above equation is satisfied then the car will negotiate the turn without skidding.

The tilt of the surface of the road should be at an angle θ such that $\tan\theta = \frac{V^2}{rg}$. Thus for a given curve (r fixed) the angle of tilt depends upon the speed of the car so that the road can be ideally tilted for only one speed. Therefore, on roadways the banking is graduated, getting

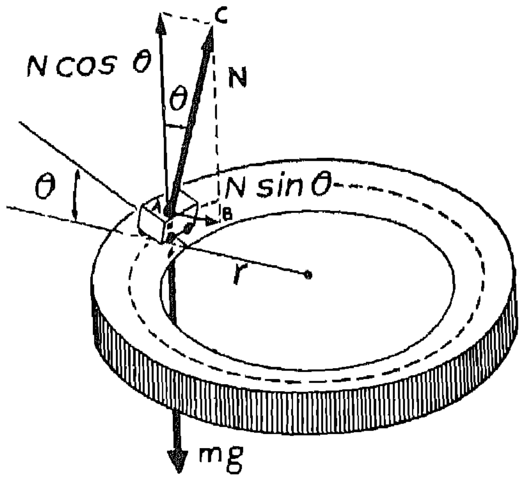

Fig 10.10.

steeper towards the outside edge. Cars moving at higher speeds keep towards the outer edge and those with smaller speeds keep towards the inner side.

In the case of railway trains too, on every curve the tracks are banked and the tilt is chosen for the average speed. At higher speeds there is a side thrust on the outer rail outward and for smaller speeds there is a sideways thrust on the inner rail towards the inside.

10.12 Artificial Satellites

The moon revolves about the earth in an approximately circular orbit. A body that revolves round a planet is called a satellite. So the moon is a satellite of the earth.

The centrally directed force necessary to keep the moon in uniform motion along its circular orbit is provided by the gravitational attraction between the earth and the moon.

If m is the mass of the moon, V its uniform speed and R the distance between the centres of the earth and the moon, then the required central force $= \frac{mV^2}{R}$.

Now the gravitational force between the two bodies (see chapter XX) $= \dfrac{GMm}{R^2}$ where M=mass of the earth and G=gravitational constant.

$$\frac{GMm}{R^2} = \frac{mV^2}{R} \text{ or } \frac{GM}{R^2} = \frac{V^2}{R} \text{ or } V = \sqrt{\frac{GM}{R}}.$$

Hence the speed of a satellite can be easily calculated if we know its distance from the centre of the earth. This speed is inversely proportional to the square root of the distance R from the centre of the earth. We also note that this orbital speed of the satellite is quite independent of the mass of the satellite It depends only on the distance of the satellite from the earth.

Example · Given the distance of the moon from the earth as $3 \cdot 84 \times 10^8$ m and mass of the the earth as 6×10^{24} kg, find the speed of the moon in its orbit round the earth (Assume that the moon moves in a circular orbit round the earth and $G=6 \cdot 67 \times 10^{-11}$ Nm^2 kg^{-2}).

Applying the relation $\sqrt{\dfrac{GM}{R}} = V$

we have,

$$V=\sqrt{\frac{6 \cdot 67 \times 10^{-11} \times 6 \times 10^{24}}{3 \cdot 84 \times 10^8} \cdot \frac{\text{newton m}^2}{\text{kg}^2} \cdot \frac{\text{kg}}{\text{m}}}$$

$$= 1021 \left(\frac{(\text{newton . m})}{\text{kg}} \right)^{\frac{1}{2}} = 1021 \text{ m/sec.}$$

We can check this result easily for we know that the moon takes $27 \cdot 3$ days to complete one revolution,

$$V = \frac{2\pi R}{T} = \frac{2\pi \times 3 \cdot 84 \times 10^8}{27 \cdot 3 \times 24 \times 3600} \text{ m/sec.}$$

$=1022$ m/sec.

So the agreement is quite good.

Suppose we have a body in the sky moving parallel to the surface of the earth (in a circular path) If its velocity is not high enough, then the central force required to hold it in the circular path will be less than the gravitational attraction of the earth and the body will fall to the ground. If the speed of the body is increased to a value such that the central force required to maintain it in the given circular orbit is just equal to the gravitational attraction of the earth on the body, then the body will continue moving in the given circular orbit. Such a body forms an artificial moon or an artificial satellite of the earth.

For example, if we have to establish an artificial satellite at a height of a 500 km above the surface of the earth, then we can beforehand calculate the orbital velocity required to keep the satellite moving in that orbit. Next the satellite is placed in the nose cone of a rocket which is powerful enough to take the satellite to the required height of 500 km and to give it a horizontal speed equal to the calculated speed. At this instant the satellite separates from the rocket and from now onwards it continues to move in the given orbit.

The launching of an artificial satellite is shown in Figure 10.11.

In 1957 the Russians succeeded in establishing the first artificial satellite of the earth. It was given the name Sputnik I. It took about 96 minutes to complete one revolution of the earth. Its speed was about 8 km per second and the maximum height above the earth was 900 km and the minimum about 230 km.

It should be remembered that the orbit of a satellite of the earth must lie in a plane containing the centre of the earth. The central force here originates from the centre of the earth, hence this very point is necessarily the centre of the circle along which the satellite moves with the uniform speed.

Since we know the speed V of the earth satellite at a given distance R from the centre of the earth, we can easily calculate its period, that is the time it takes to complete one revolution.

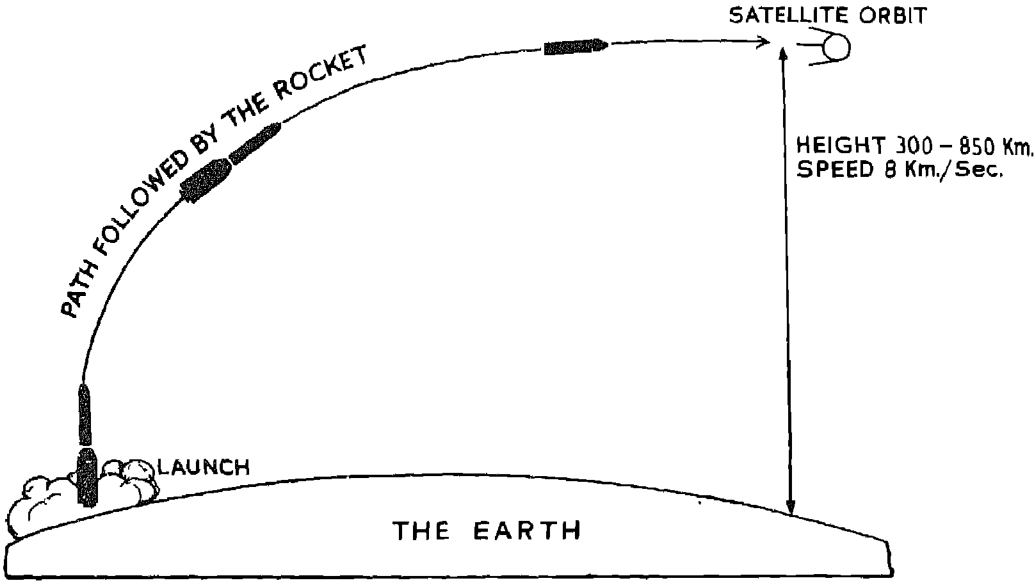

FIG. 10 11. *An artificial Satellite going into orbit*

Now $V=\sqrt{\frac{GM}{R}}$,

the period $T=\frac{\text{length of the circular orbit}}{\text{speed}}$,

or, $T=\frac{2\pi R}{V}=2\pi R\div\sqrt{\frac{GM}{R}}$,

$$=\frac{2\pi R\sqrt{R}}{\sqrt{GM}}=\frac{2\pi R^{3/2}}{\sqrt{GM}}.$$

By squaring both sides we have

$$T^2=\frac{(4\pi^2)R^3}{(GM)}.$$

Since the quantity inside the bracket is a constant, it follows that the square of the period is proportional to the cube of the radius of the circular orbit. In fact, this was discovered by Kepler in the 17th century with regard to the motion of the planets round the sun.

Example · Find the period of revolution of an artificial satellite of the earth at a height of 224 km above the surface of the earth. (Given radius of earth 6,400 km)

Here the distance of the satellite from the centre of the earth = the radius of the earth + 224 km,

$=(6400+224)$ km $=6624$ km $=6624\times10^3$m.

We have,

$$T^2=\frac{4\pi^2}{GM}R^3$$

$$=\frac{4\pi^2\times(6624\times10^3)^3}{6\cdot67\times10^{-11}\times6\times10^{24}}\frac{m^3}{kg}\left(\frac{\text{newton m}^2}{kg^2}\right)^{-1},$$

$$T=\sqrt{\frac{4\pi^2\ (6624)^3\times10^9}{6\cdot67\times10^{-11}\times6\times10^{24}}}\left(\frac{\text{kg m}}{\text{newton}}\right)^{1/2},$$

$=5350$ sec $=89\ 2$ minutes.

The following Table gives the approximate periods of satellites moving at different heights above the surface of the earth. (It is assumed that the orbits are circular).

Altitudes (km) from the surface of the earth	Period in hours
Very near the surface	1·41
260	1·50
560	1·59
1720	2·01
35,680 (22,300 miles)	24·00
3,80,800 (2,38,000 miles)	665·00

Example : What should be the distance of a satellite from the surface of the earth so that its period of revolution is exactly 24 hours ?

Applying the relation $T^2=\frac{4\pi^2R^3}{GM}$

we have ,

$$R^3=\frac{GMT^2}{4\pi^2}$$

$$\text{or, } R^3=\frac{(24\times3600)^2\times6\cdot67\times10^{-11}\times6\times10^{24}}{4\pi^2}\cdot\frac{\text{newton m}^2}{\text{kg}^2}\cdot\text{kg sec}^2 \text{ or,}$$

$$R=\left(\frac{(24\times3600)^2\times6\cdot67\times10^{-11}\times6\times10^{24}}{4\pi^2}\right)^{1/3}\text{m},$$

$=4,20,80,000$ m$=42,080$ km.

Now the radius of the earth is 6,400 km.

The height of the satellite$=(42080-6400)$km

$=35,680$ km $=22,300$ miles.

This means that a satellite revolving about the earth at a height of 35,680 km will have a period exactly equal to the period of rotation of the earth, so that if the satellite is moving from west to east along an orbit lying in the plane of the equator, it would appear to be standing still with regard to any point on the earth Since such a satellite has the same period as the period of rotation of the earth it has been given the name 'Syncom'. American scientists have succeeded in establishing satellites which move in a nearly 24 hours orbits.

10.13 The Electron inside an Atom

In the atomic model proposed by Bohr in 1913 for the hydrogen atom, the atom consists of a nucleus with a positive electric charge about which an electron (negatively charged particle) whirls in a circular orbit.

Here the central force, which maintains the electron, revolving in the circular orbit originates from the electrical attraction exerted by the nucleus on the electron. This is a mutual force of attraction between the electron and the proton. Thus the atom is a sort of solar system on a much reduced scale where the gravitational forces are replaced by electrical forces (Fig 10 12).

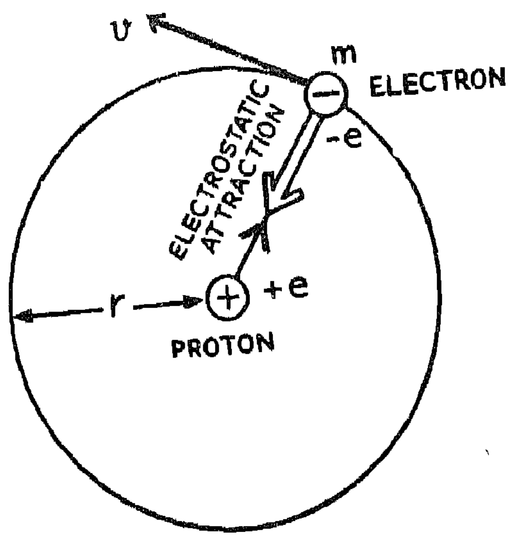

FIG. 10 12 *Bohr's atomic model.*

Classroom Activities

Pass a cord through a tube bent at right angle in the middle. Attach a small stone to one end of the cord and a stone ten to fifteen times as heavy to the other end. Hold the tube in one hand and pull a few centimetres (say 60 cm) of the cord up through the tube. Hold the heavy stone in your other hand. Now start swinging the small stone around your head in a circle faster and faster and release the heavy stone Repeat the experiment several times, increasing the speed with which the smaller stone swings each time. What happens to the heavy stone ? Slow down the speed of the smaller stone. What happens to the heavier stone now ?

Questions

1 Why does a cyclist lean inwards while negotiating a curve ?

2. Is angular velocity a vector quantity ? If so, how is its direction indicated ? The earth rotates once in 24 hours Calculate its angular speed in radians per second.

3 When a heavy piece of metal on the end of a string is revolved rapidly enough, the string may break. Why does the string break ?

4 How is it possible for a cyclist in a cirus-show to loop a vertical loop ? Discuss the possibility of his cycle losing contact with the circular track.

5. In a circus-show a motor cyclist is able to ride against the circular walls of a 'Death-well' which are almost vertical. How is he able to accomplish this feat ? Can he ride against the wall of the circular track if it were perfectly vertical ? Give reasons for your answer.

6. Assuming the earth to be a perfect sphere of radius 6400 km and rotating about an axis passing through the poles, calculate the difference in the readings of spring balance at the equator and the poles when a mass of 1 kg is hung from it.

7. If a body in uniform circular motion reverses its direction of motion, how will the direction of the centrally directed force be affected ?

8 A pendulums is oscillating about its mean position. Is the tension in the string constant in all positions of the swinging bob ? (Hint : consider the central and the extreme positions).

9 A heavy bob is suspended by means of a string which is just strong enough to support the weight of the bob. What will happen to the string when the bob is set swinging ? Give reasons for your answer.

10. Does an artificial satellite, orbiting round the earth, carry an engine to enable it to keep on moving ? How is the satellite able to maintain its circular motion ?

11. A piece of heavy metal tied to a 50 cm length of string is whirled in a circle in a vertical plane. What should be the minimum number of revolutions made by the metal piece per second so that the string may not become slack at the top of the circle ?

12. A string with a breaking strength of 5000 newtons, has a stone of mass 500 g attached to one end of the stone. It is whirled in a horizontal circle on a smooth table. The other end of the string is fixed at the centre of the circular path.

Find the maximum number of revolutions which the stone could make without breaking the string.

13. A circular track of radius 100 metres is to be used for a car speeding at 80 km per hour. What should be the banking angle in order that no skidding takes place ? (The track is supposed to be perfectly smooth.)
14. What should be the period of rotation of the earth so that a body placed at the equator may have no weight ? Take the diameter of the earth to be 12800 km.
15. A wheel of radius 40 km is rotating about its axis with a speed of 5 turns per sec. Find (a) the angular speed in radians per sec, (b) the linear speed of a point on the rim of the wheel.
16. A gramophone record of diameter 25 cm has a tiny weight of $0 \cdot 10$ g placed on the edge. The record rotates 2 time per sec. What is the central acceleration on the weight ? What is its linear speed ? What should be the minimum coefficient of friction that will keep the weight from slipping ?
17. A piece of stone of mass 10 g is attached to a string 20 cm long. The stone is whirled in a vertical plane by means of this string. If the stone is revolving 5 times per second, find the tension in the string when (a) the stone is at the highest position, (b) the stone is in the lowest position, (c) the string is horizontal.
18. A car of mass 200 kg is negotiating a curve at 20 m/sec. If the radius of curvature of the track is 200 metres, find the magnitude and direction of the central force experienced by the car.
19. The governor of an engine has its arms 20 cm long; while in rotation the arms stretch out so as to make an angle of 45° with the vertical. Find the number of rotation the shaft of the governor is making per minute.
20. How fast must a plane fly in a loop of radius 1 00 km if the pilot experiences no force from either the seat or safety belt when he is at the top of the loop ? In such circumstances the pilot is often said to be "weight-less". (PSSC)
21. A electron (mass—$0 \cdot 90 \times 10^{-30}$ kg) under the action of a magnetic force moves in a circle of $2 \cdot 0$ cm radius at a speed of $3 \cdot 0 \times 10^{6}$ m/sec. At what speed should a proton (mass—$1 \cdot 6 \times 10^{-27}$ kg) move in a circle of the same radius if it is acted upon by the same force ?

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CHAPTER XI

Work, Power and Energy

11.1 Introduction

Whenever we apply mental or physical effort, we say in everyday life, we are doing work. But in Physics the word 'work' has a special meaning. Work is said to be done only when we apply a physical force and as a result of this, the point of application of the force moves

When we raise a bucket of water we perform work. Raising the bucket means applying a force to lift it from the ground and then moving the bucket up. Similarly, when you climb up a tree, you do work. When the bullock is ploughing the field, it is doing work. If you keep a mass of one kilogram in your hand, you are applying a force to support it. But if you keep your palm stationary, you are not moving the point of application of the force and hence you are not doing any work. You will do work only when you raise your palm.

When we lift a mass of one kilogram against gravity through one metre we do a certain amount of work. If we now lift a mass of two kilograms through one metre, we say that the work done against gravity now is twice the work done in the previous case. Similarly, if we lift a mass of one kilogram through two metres, we again say that the work done is twice the work done to lift it through one metre. Taking these two factors together we define work as follows :

Work=magnitude of force $\times$ distance moved in the direction of the force.

or, $W=\vec{F}\cdot\vec{S}$, where W=the work done,
F=the force applied,
S=the distance moved in the direction of the force

In the above relation you will notice that we have taken the distance in the

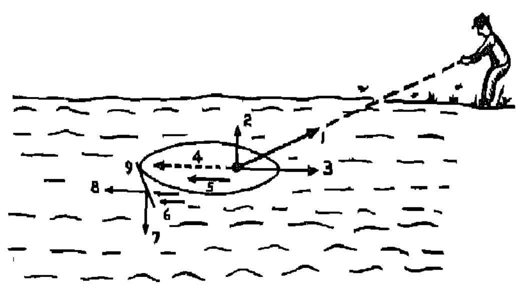

FIG. 11.1 *Pulling a boat.*

direction of the force The idea will be clear from the example given below :

If in the previous example of a weight in the palm of your hand you move your palm horizontally, say from the point A to the point B, you have not done any work as you have not moved the palm along the direction of the force (vertical) at all. If you now move the palm in the vertical direction from the point B to the point C, the work done is equal to force $\times$ BC. If you had moved the palm in the inclined direction directly from A to C the work done would still be equal to force $\times$ the vertical distance BC. Or the work done=Force $\times$ AC $\cos\theta$, where θ is the angle between the direction of the force and the direction of motion.

Thus if the body moves in a direction other than that of the force causing motion, then we must take the projection of the distance moved on the line of action of the force.

For example, if a stone of mass m falls vertically downwards through a distance h, then work done by gravity is mgh. Here distance moved, h, is in the line of action of the force due to gravity. On the other hand if a train moves on the horizontal track, the force of gravity does not do work on the train, because motion here is in a direction perpendicular to the force of gravity In such a case no work gets done because the force and the direction of motion are at right angles to each other. Take another example. When a stone is whirled with a uniform speed in a horizontal circle, we have to apply on the stone a force all the time directed towards the centre of the circle That is, the force is acting in a direction perpendicular to the line of motion. Hence no work is done in this case also.

Unit of work

The unit of work is taken as the work done when a unit force causes the point of application to move through unit distance in the direction of the force.

Therefore in MKS system one unit work is done when a force of one newton moves its point of application through one metre This unit is termed 'newton-metre' or 'joule'.

1 joule=1 newton $\times$ 1 metre.

Let us calculate the work done in raising a tumbler of water through, say, 20 cm. Suppose the mass of the tumbler together with water is 500 g

Work done in lifting the tumbler through a vertical distance of 20 cm=

$$\left(\frac{500}{1000} \times 9 \cdot 8\right) \text{newton} \times \frac{20}{100}\text{m},$$
$$=0 \cdot 98 \text{ joule},$$
$$=1 \text{ joule approximately}.$$

So that everytime we lift a glass of water from the table to have a drink, we perform approximately one joule of work.

Again while weighing on a beam balance, if you transfer a 100 g weight from the box of weights, you raise it through a height of say 1 cm. Here work is

$$\frac{1}{10} \times 9 \cdot 8 \times \frac{1}{100} = 9 \cdot 8 \times 10^{-3} \text{ joule}$$

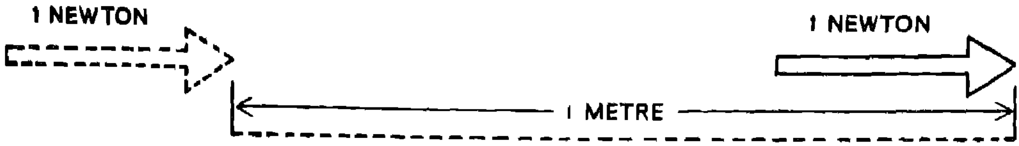

Fig. 11·2.

which is approximately equal to $\frac{1}{100}$ joule.

This means that one joule is approximately equal to one hundred times the work done in lifting a 100 gram weight.

Let us further calculate the work done by an athlete when he jumps up to a height of one metre Suppose the athlete weighs 60 kg. In jumping he has to raise the centre of gravity of his body through a height of one metre

Now, force required to lift his body $=60\times9\cdot8$ newton

Work done in jumping up one metre $=60\times9\cdot8\times1$ joules$=588$ joules.

11.2 Power

Power in everyday life may mean a variety of things. In Physics, however, the word 'Power' has a precise meaning. It means the time rate of doing work, that is, the work done per unit time. A machine is said to have more power if it can do more work per unit time.

$$\text{Power}=\frac{\text{Work}}{\text{Time}}=\text{time ratio of doing work.}$$

Power thus does not depend only upon work but also upon the time taken in doing the work. Take the case of a man climbing a staircase. If he goes up slowly, then on reaching the top he does not feel exhausted. On the other hand if he climbs the same staircase rapidly, he feels exhausted. In both the cases he did the same amount of work but in the latter case power developed by him was greater for the rate of doing the work was greater. Similarly, the rate of doing work is higher in the case of a pair of bullocks drawing water from a well than the rate of work done by a human being in drawing water from the same well because the bullocks lift more water than a human being in a given time.

Unit of Power

From the definition of power given above we can easily derive the unit of power. In MKS system unit power will be the power when the rate of doing work is one joule per second. This unit of power is called 'watt'.

1 watt$=$1 joule per second.

For practical purposes the unit 'watt' proves to be rather small, so that a larger unit equal to 1,000 watts is often used It is called a kilowatt.

1 kilowatt$=$1000 watts$=$1000 joules/sec

In industry power is also measured in terms of another unit called horse-power One horse-power is equal to 746 watts.

A powerful automobile engine may develop from 60 to 300 horse-power. Modern steam locomotives possess 1,000 to 4,000 horse-power An aeroplane may develop 2,000 to 6,000 horse-power while a rocket plane could develop about 10,000 horse-power A good sportsman can develop for a very short time as much as 1 horse-power As an example if an athlete runs up a flight of stairs, we can easily determine the horse-power developed by him.

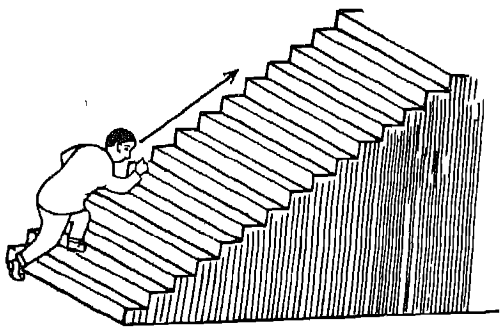

FIG 11 3. *An athlete running up a flight of stairs*

Suppose he weighs 50 kg and the flight of stairs has a vertical height of 6 metres. If he takes 4 seconds to climb the entire flight of stairs, then

Work done in 4 sec $=50\times9\cdot8\times6$ joules

Rate of doing work$=\dfrac{50\times9\cdot8\times6}{4}$ joules/sec.

Power $=\dfrac{50\times9\cdot8\times6}{4}$ joules/sec,

$=\dfrac{735}{746}=1$ horse-power (approx.).

So the athlete develops 1 horse-power.

But no human being can maintain this power for a long time. Over a period of time a man can develop only about one seventh of a horse-power. A fast sprinter may develop a maximum of 2 horse-power but no more.

Determine your own horse-power by climbing a flight of stairs briskly.

11.3 Energy

From experience we know that a fast moving ball on striking the cricket stumps pushes them backwards or can even uproot them. Similarly, if a heavy stone is allowed to fall from a height so that it strikes a stake standing in a field, it drives the stake further into the ground. In both these cases the objects in motion do work. If a body has the ability to do work, it is said to possess energy. If the ability to do work is on account of motion of the body, then this energy is called 'kinetic energy'. A moving motor car, a bullet shot from a gun, strong wind and running water, all possess kinetic energy.

We know however that many bodies are capable of doing work even when they are not in motion. Consider a brick lying on the floor. If we want to lift the brick we must apply a force upwards exactly equal to its weight. Suppose we lift it to a height of say one metre above the floor In doing so the point of application of the upward force had to move up through a distance one metre, so that work has been done upon the brick. What has happened to this work? Actually this work is stored in the brick. If this brick is allowed to fall to the floor, it will do the same amount of work as was done on it. That is the brick in the raised position possesses a potentiality to do work—it possesses energy and this energy is called *potential energy*. Since the brick has acquired this energy because work has been done on it against gravity, we say that the brick possesses *gravitational potential* energy Hence, the water behind the Bhakra Dam too possesses gravitational potential energy due to its high position.

Again consider a coiled spring having one of its ends attached to a rigid wall while the other end is free. If we compress the spring by applying a force at the free end, we do work against the elastic forces developed in the spring. Obviously, the work done upon the spring has been stored in it because if the spring is released, it will do work. Thus the compressed spring in the strained condition possesses the potentiality of doing work—it possesses *elastic potential* energy (Fig. 11.4). When we wind a watch, the spring in it is tightened. It then possesses potential energy (elastic) which is used in keeping the watch running.

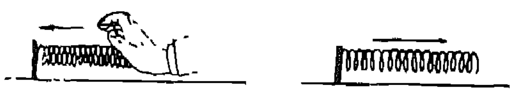

Fig. 11.4.

Similarly, a gas compressed in a cylinder has the potentiality of doing mechanical work, so that the compressed gas too possesses potential energy. Again a bent bow possesses potential energy when its string is pulled taut, because in bending the bow we do work upon the bow.

Further we see that potential energy in chemical form is stored in coal or gasoline, so that under suitable conditions the coal

or gasoline is able to do mechanical work by virtue of its potential energy. Lastly potential energy in yet another form is stored inside the nucleus of atoms like uranium which manifests itself in the form of a powerful explosion.

Thus the capacity of doing work possessed by a body on account of its position or state of strain is called potential energy.

11.4 Measurement of Kinetic Energy

Since energy is the ability to do work, it is obvious that energy can be measured in work units. Therefore we may use the joule to measure either kinetic or potential energy.

To compute the kinetic energy of a body of mass m having a velocity v we can argue that the body has the capacity to do a certain amount of work because the same amount of work has been done on it to impart the velocity v.

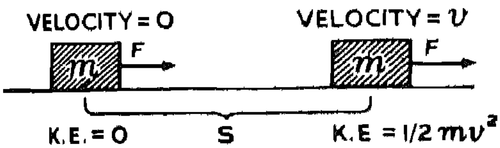

FIG. 11.5.

Suppose a constant force F has been applied to the body initially at rest so as to give it an acceleration a and that the force acts through a distance s such that the body acquires a velocity v (fig. 11.5).

Work done on the body $=$ F. s. (i)

From Newton's Second Law of motion,

$$F = ma, \qquad \text{(ii)}$$

and from equation of motion

$$2a.\ s = v^2 - 0 = v^2$$

$$\text{or, } s = \frac{v^2}{2a}. \qquad \text{(iii)}$$

Substituting in (i) the values of F and s obtained from (ii) & (iii), work done on the body

$$= m\ a \times \frac{v^2}{2a},$$

$$= \tfrac{1}{2}\ mv^2.$$

Kinetic energy $= \frac{1}{2}\ mv^2$.

We may note here that the kinetic energy of a body is proportional to the square of its velocity. So that if the speed of a motor car is doubled, then the work that must be done by its brakes to stop the car will be quadrupled. Now work is given by the product of force and distance. If we assume that the force applied by brakes remain constant then we have $\frac{1}{2}\ mv^2 = F.s$ and

$$v^2 \propto s.$$

That is, the stopping distance for the car is proportional to the square of its speed. Actual demonstration verifies that this result is nearly correct. For example, a car speeding at 15 km/hour has a stopping distance of about 1·5 m, while one travelling at 30 km per hour has a stopping distance of 6·0 m, so that, doubling the speed increases the stopping distance four times.

Table 11·1· kinetic energy of a cyclist at different speeds. The total mass of the cyclist and the machine is taken to be 100 kg.

Speed of the cyclist km/hr	Kinetic energy in joules	Stopping distance
10	38·6	0·64 m
15	87·0	
20	154·3	
25	241·1	
30	347·2	
35	475·7	

In table 11.1 we give the kinetic energy of a cyclist who with his machine has a total mass of 100 kg.

Calculate the stopping distance on applying the brakes for the other speeds in the Table.

If a particle of mass m is moving in a circle with a uniform speed v, the magnitude of its angular velocity of revolution ω is given by $\omega = \frac{v^2}{r}$. Hence, the kinetic energy of the particle is given by K.I.

$$= \tfrac{1}{2} m v^2$$
$$= \tfrac{1}{2} m r^2 \omega^2$$
$$= \tfrac{1}{2} I \omega^2,$$

where I is known as the rotational inertia or the moment of inertia of the particle with respect to the axis of revolution.

11.5 Measurement of Potential Energy

Clearly the gravitational potential energy acquired by a body must be equal to the work done against gravity in raising the body to its present position.

If the body of mass m is raised through a a height h, then work done $=$ mgh.

$\therefore$ Gravitational potential energy of the body $=$ mgh.

Similarly, the potential energy of a spring which has been compressed is equal to the work done against elastic forces.

Of course, the gravitational potential energy is computed with reference to some arbitrary level. This reference level from which the height of the position of the object is to be measured may be the floor level of the laboratory, the surface of the table or the sea-level.

In determining the potential energy due to height we must bear in mind that the value of the potential energy will be the same whether we carry the body straight upwards, or through a stair case or through a zigzag path. The potential energy gained will always be equal to the weight of the body multiplied by the vertical height through which the body has been raised.

11.6. The Relationship Between Potential and Kinetic Energies

Suppose a stone is raised to a height h above the ground level. If the mass of the stone is m, then here the potential energy of the stone is mgh (fig. 11·6). The stone is at rest there, so its kinetic energy is nil (Position A). Now the stone is allowed to fall. At the instant it reaches the ground, its height is reduced to zero Hence the stone possesses no potential energy (Position C), but it possesses a speed v here, so its kinetic energy is $\frac{1}{2}$ mv².

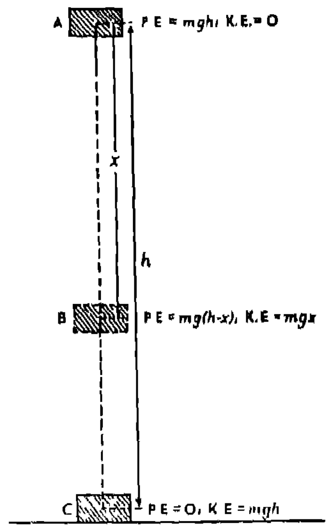

FIG. 11·6.

Let us find the value of speed v. The stone falls through a distance h under gravity from the position of rest.

According to the equation of motion

$$2a.s = v^2 - u^2.$$

We have in this case $2\,gh = v^2 - 0$,

$$\text{or, } v = \sqrt{2\,g\,h}.$$

At the ground level, the kinetic energy

$$= \tfrac{1}{2}mv^2 = \tfrac{1}{2}m \times 2\,gh = mgh.$$

Thus by the time the stone reaches the ground its entire potential energy has been converted into kinetic energy.

What about its potential and kinetic energies at any instant during the fall?

Consider the stone at point B when it has fallen through a distance x.

Here the velocity $= \sqrt{2\,gx}$.

Kinetic energy at $B = \frac{1}{2}m \times 2\,gx = mgx$

Again, the potential energy at $B = mg\,(h-x)$

Potential energy + kinetic energy $= mg\,(h-x) + mgx = mgh$. Thus we see that

1. At the highest point the entire energy of the body is potential.
2. At the lowest point the whole of the potential energy of the body gets converted to kinetic energy.
3. At any intermediate point, part of the original potential energy is converted into kinetic energy.
4. At every point of the path the sums of the potential and kinetic energies remain constant.

Hence, potential energy can be wholly or partially converted into kinetic energy. The reverse is also true as will be clear from the following example.

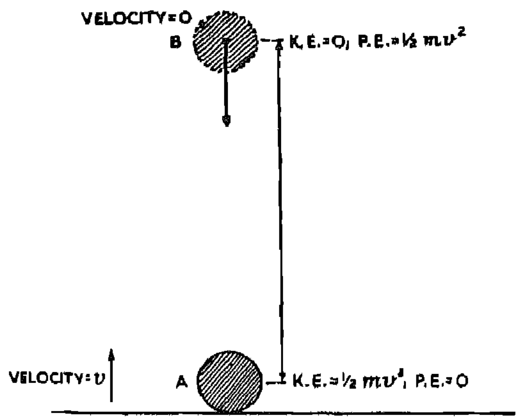

Fig 11.7.

A ball of mass m is projected vertically upwards with a velocity v. It will reach a point B at a certain height h when its velocity becomes zero and then it will begin falling down

On the ground at A, the energy of the ball is wholly kinetic

$$\left.\begin{array}{l}\text{So its kinetic energy} = \frac{1}{2}\,mv^2\\ \text{and its potential energy} = 0\end{array}\right\}\text{ at A.}$$

At B, the highest point, its velocity is zero

$$\left.\begin{array}{l}\text{So its kinetic energy} = 0\\ \text{and its potential energy} = mgh\end{array}\right\}\text{ at B.}$$

What is the value of h in terms of v?

The body is moving upwards, so the acceleration due to gravity is negative for this case.

$$0^2 - v^2 = -2\,gh,$$
$$\text{or, } v^2 = 2\,gh,$$
$$\text{or, } h = \frac{v^2}{2g}.$$

Potential energy at $B = mgh$

$$= mg\,\frac{v^2}{2g} = \tfrac{1}{2}mv^2.$$

Thus, we see that the whole of the kinetic energy of the ball appears as its potential energy at the point B. Similarly it can be shown that at any intermediate point along the path AB, the sum of the kinetic and potential energies remains equal to the original kinetic energy, $\frac{1}{2}\,mv^2$.

In the case of a simple pendulum too, the energy of the bob in the extreme position B is wholly potential (Fig 11·8) By the time the bob reaches the middle position A, the whole of its potential energy gets converted into kinetic energy. And this kinetic energy is again changed wholly into potential energy when the bob reaches the extreme position C on the other side. At any intermediate point, the bob possesses both kinetic as well as potential energy and the sum always remains equal to the potential energy at B or the kinetic energy at A.

We, therefore, conclude that potential and kinetic energies are interchangeable. This principle holds good whether the potential energy is due to change in position

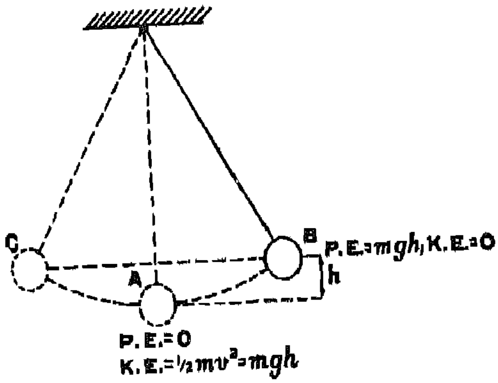

Fig. 11·8.

against gravity or due to strain against elastic forces etc.

Consider a thin strip of steel with one end fixed in a vice. Pull the end A to one side and then release it. The strip will vibrate as shown in Fig 11·9. Now at the position B, energy of the strip is wholly potential because the strip in this position is at rest. In the position A, the whole of the elastic potential energy appears as its kinetic energy. At C again it changes wholly into potential energy.

11.7 Law of Conservation of Energy

In all these examples we have seen that energy can be transformed from potential to kinetic and back again to potential. In these transformations the total amount of energy remains unchanged, no part of the energy is destroyed. Experiments show that energy can neither be created nor can be destroyed. You can throw a stone and thereby give it kinetic energy, but you do not create energy in this process. All you do is to transfer it from your body to the stone

This fact, that energy can never be

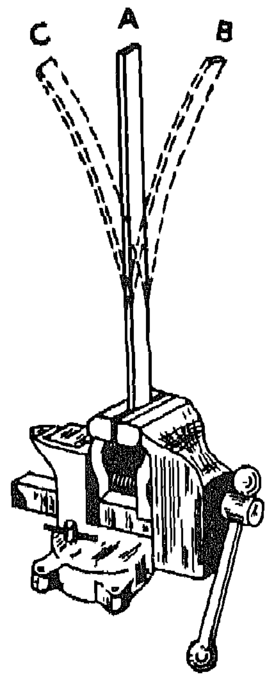

Fig. 11·9.

created or destroyed but only transformed from one form into another, was first stated by a German scientist Robert Mayer. It is called the Law of Conservation of Energy. This is a powerful law.

Questions

1. (a) The moon revolves round the earth in an almost circular orbit Does the earth exert a force on the Moon ? Does the earth perform work on the moon ? Give reasons for your answers.
   (b) A pendulum is oscillating to and fro. Does the tension in the string of the pendulum do any work ? Why does a pendulum not continue to swing indefinitely after it has been once set in motion ?
   (c) A hammer strikes a nail driving it into a wall Does the hammer do any work on the nail ? Does the nail do work on the hammer ?
2. Explain the terms ' energy ' and ' work ' and distinguish between the two.
3. How much work must be done to raise a body of mass 10 kg to a height of 35 metres ?
4. The hammer of a pile driver weighs 500 kg. If it is to be lifted 2 metres in 1 5 seconds, what should be the horse power of the engine?
5. A 400 horse-power engine of an aeroplane maintains its speed at 400 km per hour. Find the force of friction due to air at this speed.
6. A two kg stone is dropped from a height of 10 metres above the ground. Find the potential energy and the kinetic energy of the stone when it reaches a point 4 metres above the ground.
7. A ball having a mass of one kg is projected vertically upward with a speed of 50 m/sec. Compute from energy considerations the maximum height that the ball will attain.
8. A man weighing 800 newtons climbs a staircase 4·5 m high. Calculate the amount of work done and also determine the horse-power if he takes 4 sec to climb.
9. Determine the power of an engine in kilowatts which can raise the cage of an elevator of mass 3000 kg to a height of 2·5 m in 5 sec.
10. A man whose mass is 60 kg climbs a staircase 10 metres high in 20 sec. Find (a) work done (b) increase in his potential energy and (c) power developed.
11. Compare the kinetic energies of the objects A and B, identical in every respect except one.
    Assume that the single difference is :
    (a) A has twice the velocity of B.
    (b) A moves north, B south.
    (c) A moves in a circle, B in a straight line.
    (d) A is a projectile falling vertically downward; B is a projectile moving vertically upward at the same speed.
    (e) A consists of two separate pieces attached by a light string, each equal in mass to the mass of B (PSSC).
12. A metre scale is pivoted at the upper end and its mass is 250 g. The scale is hanging vertically downward. It is now displaced through 45° from the vertical. Find its potential energy.
13. On a body of mass 1 kg a force of 1 newton acts towards the east and another force of 2 newtons acts on it towards the North. Find the kinetic energy of the body after 5 seconds. Also calculate the energy when each force acts on the body

separately and show that their sum is equal to the kinetic energy acquired by the body, when both forces are acting simultaneously. Repeat the calculation when the forces are in the same direction Discuss the difference in the two cases.

14. A motor car weighing 1 tonne climbs to the top of a hill 100 m high in 1 minute. Neglecting friction, determine the horse-power developed by the car.
15. A crane can lift a load of 1 tonne through a height of 10 metres in 4 seconds. Determine the horse-power of the engine which operates the crane.
16 An engine can develop 20 kilowatt of power. How much time will it take to lift a mass of 100 kg to a height of 50 metres ?
17. A piece of stone attached to a string is moving in a vertical circle Show from energy considerations that its velocities at the highest point and the lowest point of the circle will be different.

Further Reading

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CHAPTER XII

Machines

12.1 Introduction

Even in pre-historic times man devised contrivances to help him in his work Such devices are termed machines. In modern times we use machines of all sorts in our everyday life. A cycle, a motor car, a lathe, an electric fan and a screw all are examples of machines.

If we examine various types of machines, we shall find that they generally perform one or more of the following tasks :

1. To apply force at one point and to produce its effect at some other point. For example, when we apply force at one point of the lever, load attached at another point can be raised

2. To magnify force. By applying a small force, a much greater force can be produced at some other point of the machine as in the case of levers.

3. To change the direction of the force. A string passing over a fixed pulley is employed for changing the direction of force. The pulley attached to the pillars on a well is used for drawing a bucket of water from the well. By applying a downward pull on the free end of the rope, we produce an upward pull on the bucket.

4. To multiply speed. By moving certain part of the machine at a small speed, a much larger speed may be produced at some other point of the machine as in the case of a bicycle.

5. Finally to transform energy from one form into another In a motor car the chemical energy of petrol is changed into mechanical energy. In an electric fan the electrical energy supplied to it through the wires is converted into mechanical energy which causes the blades of the fan to revolve. In a steam engine the chemical energy of coal is changed into motion of the train. In a nuclear power station nuclear energy is converted into heat energy which is further converted into other forms. It should be emphasized that it has not been possible to devise a machine which could create energy out of nothing. Had it been possible to do so, it would have violated the law of conservation of energy. A machine can only transform one form of energy into another.

In this chapter we shall be dealing only with machines which employ mechanical energy and do work against mechanical force The usefulness of such a machine consists chiefly in the fact that it enables us to perform some work conveniently by changing the magnitude of a force, its direction or the point of application of the force

12.2 Simple Machines

These are machines, simple in design so that energy is supplied to them by a single applied force and the machines do work against a single resisting force. There are six simple machines, namely the lever, the pulley, the wheel and axle, the inclined plane, the screw and the wedge.

12.3 The Lever

We have learned in chapter VIII that a lever is a rigid bar straight or bent which is pivoted at a fixed point F on the bar, called the fulcrum (Fig. 12·1). The lever is mainly used in overcoming a large resisting force called the 'load' by employing a smaller applied force called the 'effort'.

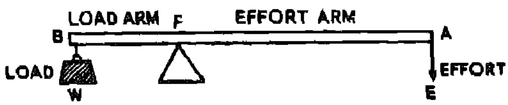

FIG. 12·1.

With such a lever we are able to raise a heavy load by means of a smaller effort. This advantage of being able to balance a large force with a smaller one is called the Mechanical Advantage and is defined as the ratio of the load to the effort.

Mechanical Advantage=load/effort.

We may note here that since the mechanical advantage is a ratio, it is just a number and no units are involved in expressing it.

In raising a load by means of a lever, usually the speed with which the point of application of effort moves is different from the speed of the point on which the load acts. That is, *in a given time* the point of application of effort and the point where the load acts move through different distances. The ratio of the distance moved by the effort to the corresponding distance moved by the load is called the velocity ratio

$$\text{Velocity ratio} = \frac{\text{distance moved by the effort per sec}}{\text{distance moved by the load per sec}}$$

By applying the principle of moments (§ 8·7), we find that

Mechanical advantage of a lever

$$= \frac{\text{effort arm}}{\text{load arm}},$$

provided there is no friction at the fulcrum.

Let us find the velocity ratio of a lever

As shown in the figure, during the time the effort moves from A to A′, the load moves from B to B′.

$$\text{Velocity ratio} = \frac{\text{arc AA}'}{\text{arc BB}'}$$

Opposite angles being equal,

$$\angle \text{AFA}' = \angle \text{BFB}' = \theta.$$

Now arc $AA' = AF \times \theta$ and arc $BB' = BF \times \theta$

$$\therefore \quad \frac{\text{arc AA}'}{\text{arc BB}'} = \frac{AF \times \theta}{BF \times \theta} = \frac{AF}{BF} = \frac{\text{effort arm}}{\text{load arm}}$$

$$\text{or V R.} = \frac{\text{effort arm}}{\text{load arm}}.$$

We should note here that for a given lever the velocity ratio is always fixed, being equal to the ratio of the effort arm and the load arm.

We have seen that in the ideal case, that is, when there is no friction and the rod is weightless, the mechanical advantage is equal to the ratio between the effort arm and the load arm. So that in the ideal case the mechanical advantage is equal to the

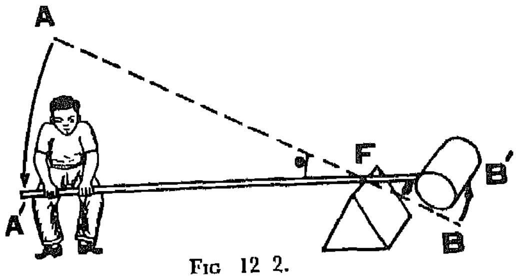

FIG 12 2.

velocity ratio; otherwise it is less than the velocity ratio In actual practice the real mechanical advantage, which is equal to the ratio of the actual output force to the input force, is less than the velocity ratio

Suppose in the ideal case the mechanical advantage is 2. Then it means that a force of one newton can raise a load of two newtons. But here the velocity ratio is also 2, so that the effort has to move a distance of 2 metres in order to make the load move 1 metre. Clearly then what we gain in force, we lose in speed.

Efficiency : When a force is applied to a lever or to any other simple machine, the point of application has to be moved so that the load may be raised If there is no friction, work put into the machine must be equal to the work done by the machine on the load. In actual practice, a part of the energy supplied to the machine is wasted in overcoming friction of moving parts, so useful work done by the machine is always less than the energy supplied to it.

From the law of conservation of energy we have,

energy input = energy output + energy wasted.

The ratio of the output energy to the input energy is called efficiency. Usually this ratio is expressed as a percentage.

$$\boxed{\text{Efficiency} = \frac{\text{output energy}}{\text{input energy}} \times 100\%}$$

Thus, in practice, no machine can ever acquire an efficiency of 100%

The efficiency of a machine will be high if most of the energy supplied to it is spent by the machine in doing useful work against the load and only a small part is expanded in overcoming friction etc.

$$\text{Efficiency} = \frac{\text{actual output force} \times \text{arc BB}'}{\text{input force} \times \text{arc AA}'}$$

$$= \text{actual mechanical advantage} \times \frac{\text{arc BB}'}{\text{arc AA}'},$$

or, actual mechanical advantage

$$= \text{efficiency} \times \frac{\text{arc AA}'}{\text{arc BB}'} = \text{efficiency} \times \text{velocity ratio.}$$

12.4 The Pulley

It is a wheel that turns freely about an axle passing through its centre. If the wheel is fixed to a block such that the wheel does not move as a whole but can turn

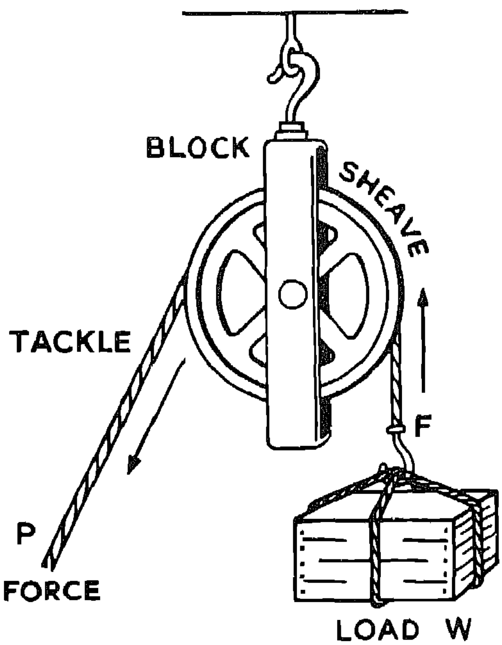

FIG 12.3 *The fixed pulley.*

about the axle freely, then it is called a fixed pulley.

A fixed pulley is always used to change the direction of the effort applied to it. In Fig. 12.3 a string passes over the fixed pulley as shown. The load W is attached to one end of the string and the effort P is applied to the other end. This illustrates one advantage of the pulley, *viz.*, it can be used to change the direction of the applied force. Here the downward force P produces a tension in the string which pulls the load W upwards This tension is the same throughout the length of the string (if the pulley is perfectly smooth). Thus the direction of the force here has been changed

For a frictionless pulley, P=W.

$$\boxed{\text{M.A.} = \frac{W}{P} = 1}$$

If the point of application of P moves x cm down, then the load W also moves the same distance up

The movable pulley: In this case the pulley is suspended on a continuous string passing round it so that it can move up or down as a whole and it can also rotate at the same time

If we suspend a frictionless pulley of negligible weight over a continuous string such that the ends of the string pointing upwards, are attached to two spring balances as in Fig. 12.4, we shall see that each spring balance records the tension in the string as $\frac{W}{2}$ where W is the load suspended from the pulley.

$$P = \frac{W}{2}, \qquad \text{(Fig. 12.4)}$$

or, $$\frac{W}{P} = 2.$$

Thus the ideal mechanical advantage for such a pulley is 2. The actual mechanical advantage is always less than 2 on account of the weight of the pulley and friction and it increases with the load.

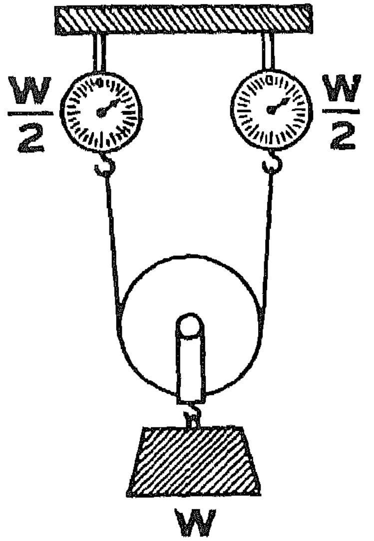

FIG. 12 4. *The movable pulley.*

Pulley systems: We shall consider here the most common system of pulleys known as the 'Block and tackle' system There are two blocks of pulleys, one being fixed and the other moveable. Either both blocks have the same number of pulleys (Fig. 12.5 a) or the lower one contains one less than the upper one (Fig. 12.5b). In practice the pulley wheels on each block are side by side as shown in Fig. 12.14 instead of one below the other, as this enables the load to be raised much higher before the pulleys in the upper and lower blocks touch each other.

The upper block is fixed to a rigid support. If the number of pulleys in both the blocks is the same, then one end of the string is fixed to the lower end of the upper block. On the other hand, if the number of pulleys on the upper block is one more than the number in the lower block, then the string is tied to the top of the lower block. This continuous string passes round all the pulleys so that the force end points

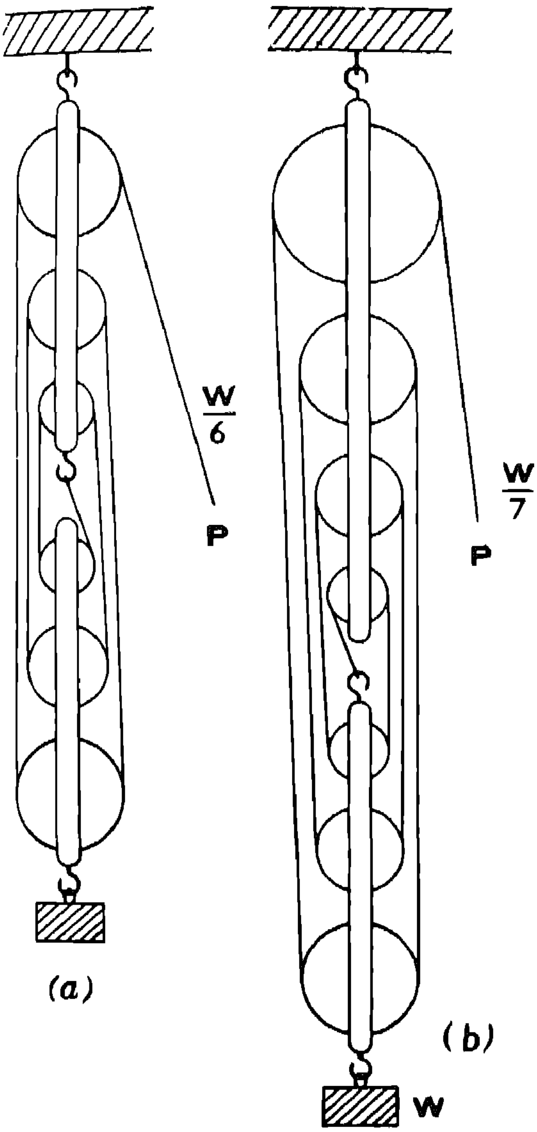

FIG. 12.5 *The pulley system.*

down wards. The effort is applied at this force end.

On examining the figure it will be observed that the number of strands supporting the movable block is equal to the total number of pulleys in both the blocks. One can see that the last strand on which the effort acts does not support the movable block.

Neglecting friction, the tension in each strand is equal to P, the effort.
Thus, the load W=nP, where n is the number of pulleys.

$$M.A. = \frac{W}{P} = n.$$

In deriving the above relation we have supposed for the sake of simplicity, that the pulleys and blocks are weightless. Actually the velocity ratio is n and the mechanical advantage is less than n.

The Weston differential pulley : This is a combination of pulleys such that the mechanical advantage may be increased considerably without increasing the number of pulleys. Due to the high mechanical

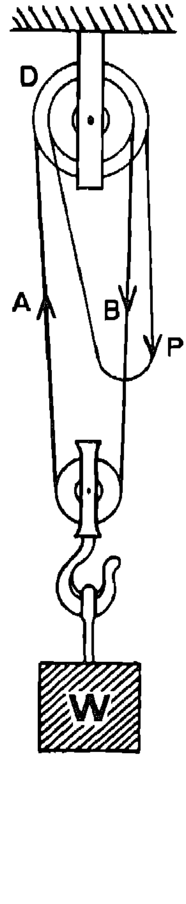

FIG. 12.6 *A differential pulley is used to raise heavy weights*

advantage, the differential pulley is often used when very heavy loads are to be raised.

In this device the upper block consists of two wheels fixed together side by side on the same axle, so that they rotate as one unit. One of them is slightly smaller in diameter than the other. An endless chain passes round both pulleys as shown in Fig 12.6. The links of the chain engage in notches cut on the rim of the pulley wheels so that they cannot slip. The lower block consists of a moveable pulley to which the load is attached and the pulley itself is supported on one of the loops of the chain.

Neglecting friction we see that each of the strands A and B supports half the load.

Effort P is applied on the part of the chain passing through D.

As the effort P pulls the chain downwards, part A of the chain moves up and part B moves down. To calculate the velocity ratio, suppose that the effort pulls the chain so that the large wheel makes one complete rotation. Then, distance moved by the effort$=2\pi R$ and part A of the chain too will move up a distance equal to $2\pi R$. Now during the same time the smaller wheel also makes one complete rotation so that part B of the chain moves down a distance equal to $2\pi r$.

Thus, the moveable pulley carrying the load will move a distance $\frac{1}{2}(2\pi R-2\pi r)$ upwards, that is, the distance moved by the load$=\pi(R-r)$.

Velocity ratio=

$$\frac{\text{distance moved by the effort}}{\text{distance moved by the load}}=\frac{2\pi R}{\pi(R-r)}=\frac{2R}{R-r}.$$

To obtain a large velocity ratio the difference between the radii of the two pulley wheels in the upper block is kept very small.

As the friction in Weston pulley is large, the efficiency is small, say about 50 per cent only.

In such a case,

mechanical advantage=efficiency × velocity ratio$=\frac{1}{2}\times\frac{2R}{R-r}=\frac{R}{R-r}.$

Hence, in spite of a great deal of friction, the mechanical advantage is pretty large, for R-r is much smaller than R.

12.5 The Wheel and Axle

The device consists of a wheel rigidly attached to an axle so that both can rotate together. One end of a string is attached to a point on the rim of the wheel and then it passes round the rim several times. The effort is applied at the free end of the string. Another string is wound round the axle in the opposite sense and the load is attached to the free end of this string.

By applying the force of pull on the string attached to the wheel, it is rotated so that the string attached to it unwinds itself with the result that the string on the axle winds itself, thus raising the load upward (Fig. 12.7).

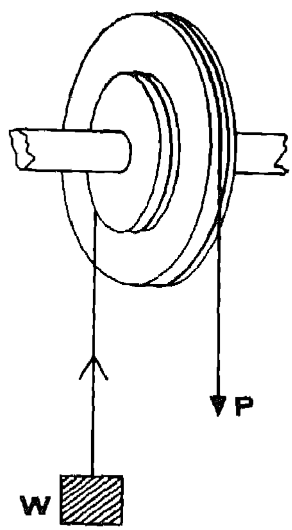

FIG. 12·7. *The wheel and axle.*

It can be shown that velocity ratio

$$= \frac{2\pi R}{2\pi r} = \frac{R}{r} = \frac{\text{radius of the wheel}}{\text{radius of the axle}}.$$

In this machine too due to friction etc. the efficiency is less than one, so that the mechanical advantage never reaches the ideal value $\frac{R}{r}$.

12.6 The Inclined Plane

Every now and then we use the inclined plane device for transporting a heavy load to a higher position without actually lifting it vertically.

For example, if a person wishes to raise a block of ice of weight W from the ground to a platform at a height h, he places a wooden plank so that one end of the plank rests on the ground and the other on the edge of the platform. This plank forms an inclined plane. Let the length of the plank be l. He applies a force P so that the block of ice moves up the plane, and the man has been able to raise the block through a vertical height h. Here the effort acts along the plane in the direction of the greatest slope (Fig. 12.8).

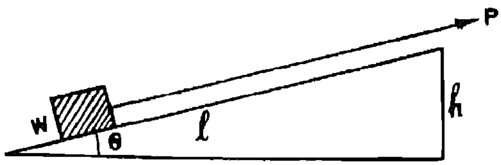

FIG. 12.8.

Neglecting friction and equating the work done by the effort P with that done on the load W, we get, $Wh = Pl$;
hence ideal mechanical advantage

$$= \frac{W}{P} = \frac{l}{h} = \frac{l}{l \sin\theta} = \frac{1}{\sin\theta}.$$

By making the inclination of the plane to the horizontal very small, $\sin\theta$ can be decreased considerably so that the mechanical advantage $\frac{1}{\sin\theta}$ is high. In this case, a small force (effort) can balance a big load on the inclined plane. This is why roads leading to the top of a hill go on winding so that the slope (angle of inclination) may be kept small. On such a road only a small effort is required to pull a heavy cart up the slope. It is thought that the huge stone blocks used in the Egyptian pyramids were hauled from ground level to the required height with the help of inclined planes. Cranes were not known in those days. You can easily show that:

$$V.R. = \frac{l}{h}.$$

Efficiency = 1, in absence of friction

The Wedge. As shown in Fig 12.9 the wedge may be considered a double inclined plane in which the two inclined planes are put base to base. Here the effort is applied not along the inclined plane but along the common base

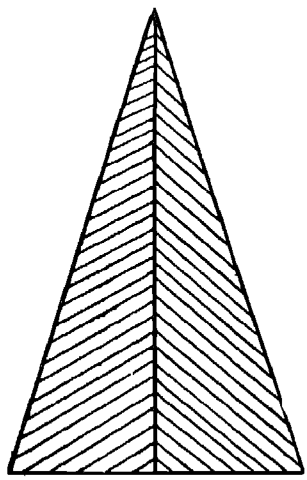

FIG 12.9. *A wedge is commonly two inclined planes placed base to base*

In order to raise a heavy load, we place the wedge under it and then apply a force

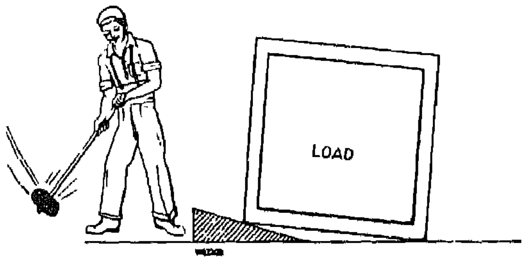

Fig. 12.10. *Heavy loads may be raised by using a wedge.*

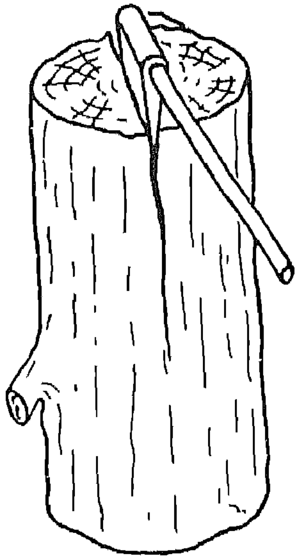

Fig. 12.11.

on the wedge in the direction of the length of the base (Fig. 12.10). Due to a large amount of friction it is not easy to calculate the mechanical advantage of a given wedge. However it can be said that the mechanical advantage is proportional to the ratio of the length of the base to the thickness of the wedge.

All cutting instruments like the axe, the chisel, the carpenter's plane and the blade act as wedges.

12.7 The Screw

In addition to the wheel, the screw is the most wonderful invention made by the ancient people.

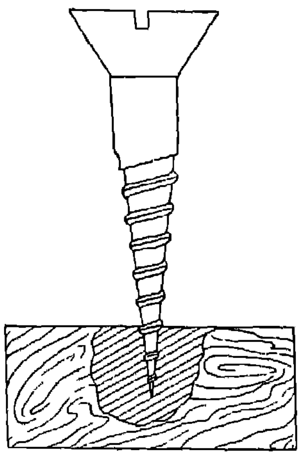

Fig. 12.12 *The screw.*

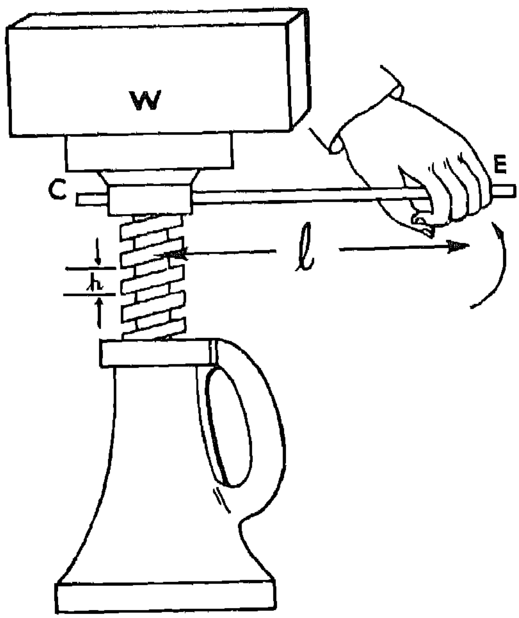

Fig. 12.13. *A screw jack is a combinations of an inclined plane and a wheel and axle*

The screw is really an inclined plane curved to form a spiral about a cylinder. The teeth of the screw form this spiral round the cylinderical axis.

When the tip of the screw is inside a material like wood, and the screw is given one complete rotation, then the screw advances a distance equal to the pitch of the screw in a direction along its axis. In doing so it does work against the resistance.

Generally the effort is applied at the end of a handle or a spanner of which the other end is attached rigidly to the head of the screw. Suppose the handle has length l, then the effort P in giving the screw one complete turn moves a distance $2\pi l$. Meanwhile the screw has moved forward a distance h against the force or resistance.

$$\text{Velocity Ratio} = \frac{2\pi l}{h}$$

Usually the pitch of the screw is very small as compared to the length of the handle attached to the screw head. So the velocity ratio is very high.

Due to the large friction, the mechanical advantage of the screw is always much less than the velocity ratio.

The screw jack : The principle of the screw finds an interesting application in the screw jack. It is usually employed for raising one side of an automobile for changing wheels.

One can easily see that the velocity ratio is $\frac{2\pi l}{h}$, where h is the pitch of the screw. Due to the large friction, here too the actual mechanical advantage is much less than the velocity ratio. A screw jack when used to support a heavy load will usually stay set without running back. This is possible only because of a lot of friction present in the machines; so its efficiency must be less than 50 per cent

12.8 Compound Machines

So far we have considered only simple machines. We shall now consider the situation in which mechanical advantage is increased by combining two or more machines

For instance, a heavy load may be pulled up an inclined plane by means of a pulley system arranged as in Fig 12 14. Here the load W is being pulled up by the force P.

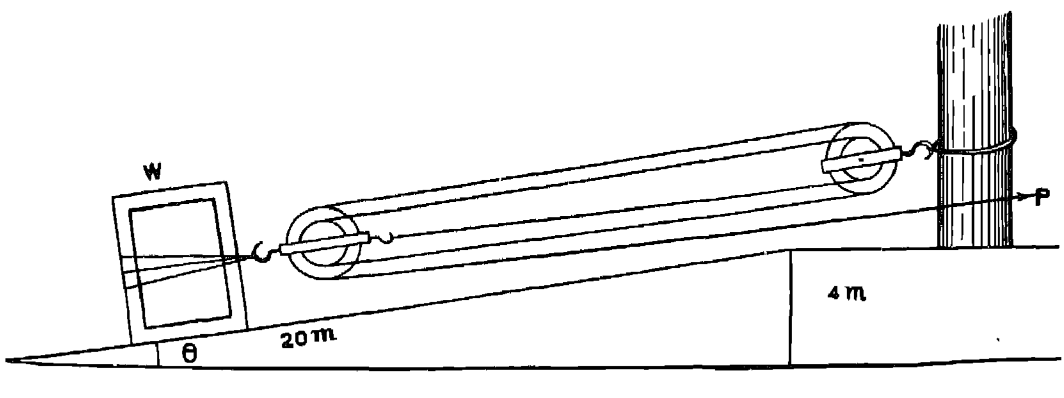

FIG 12·14.

The ideal mechanical advantage of the inclined plane $= \frac{1}{\sin\theta} = \frac{1}{4/20} = 5.$

Hence, a force $\frac{W}{5}$ must be applied to pull the load up the inclined plane directly. Since the mechanical advantage (ideal) for the system of pulleys shown above is 4,

$$P = \frac{W}{5\times4} = \frac{W}{20}.$$

Mechanical advantage for this combination is 20.

In all compound machines, the combined mechanical advantage is the product of the separate mechanical advantages of its component simple machines.

12.9 Gear-wheel

Gear-wheels are commonly used to control speed or to change the direction of force. The device consists of two or more toothed-head wheels of different diameters so arranged that the teeth of one mesh with the teeth of the other.

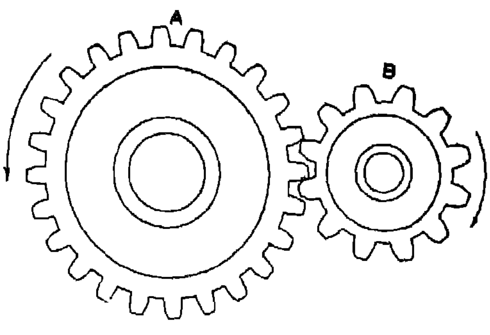

FIG. 12.15. *Gear-wheel.*

To understand the principle involved, consider two wheels, one of which has a diameter twice as big as the diameter of the other. Suppose the larger wheel has 24 teeth (cogs) on its circumference. Then the smaller one will have 12 cogs. Now, if the larger wheel is given one complete rotation, then the smaller wheel will complete two rotations during the same time. Thus the speed of rotation has been doubled. On the other hand, if the effort acts on the smaller wheel, then the speed of rotation at the larger wheel will be halved.

To see how the direction of force can be changed, let us suppose that the effort causes the wheel A to rotate counter-clockwise, then the wheel B will rotate clockwise. So here an anti-clockwise force has produced a clockwise force.

We can easily calculate the velocity ratio of two gear wheels whose cogs engage with each other. Let the larger wheel have n_2 cogs and the smaller one n_1 cogs. For the sake of simplicity, let us suppose that their axles to which they are rigidly attached are of the same diameter. We apply the effort at the force end of a string wound on the axle of the larger wheel so that when the effort pulls downward the load is pulled upward.

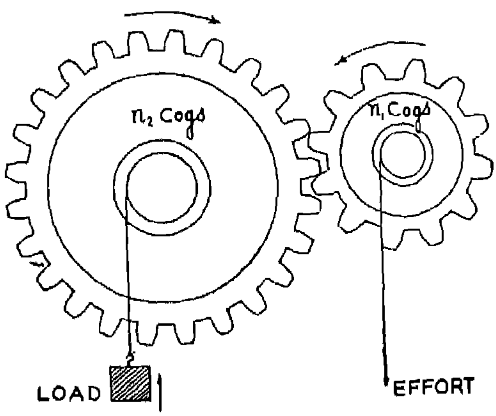

FIG 12·16

When the smaller wheel rotates once, the larger wheel completes less than one rotation. In fact, it will complete only n_1/n_2 of a rotation.

Hence, velocity ratio =

$$\frac{\text{distance moved by the force}}{\text{distance moved by the load}}$$

$$=\frac{2\pi r}{2\pi r \times n_1/n_2}=\frac{n_2}{n_1},$$ where r = the radius of their axles.

Velocity ratio =

$$\frac{\text{number of teeth on the larger wheel}}{\text{number of teeth on the smaller wheel}}$$

If friction is negligible, the mechanical advantage = velocity ratio = $\frac{n_2}{n_1}$.

Therefore, to have a high mechanical advantage the load should be attached to the gear wheel having a larger number of teeth as compared to the number of teeth on the wheel to which the effort is applied.

If we apply the effort to the larger wheel, we gain speed but loose in mechanical advantage. In this case while the larger wheel makes one revolution, the smaller one will make many revolutions. The mechanism of a bicycle affords a good example of the gear drive to gain speed (Fig. 12.17). Suppose the pedal wheel has 28 teeth and the free-wheel attached to the back wheel has 7 teeth. The force is applied to the crank attached to the pedal wheel. Clearly then the back wheel makes 4 revolutions while the pedal wheel makes one revolution. If the back wheel has a radius equal to 45 cm, then the distance covered in 4 revolutions is $2 \times 45 \times 3{\cdot}14 \times 4 = 1130{\cdot}4$ cm. Therefore just one revolution of the pedal wheel moves the bicycle through a distance of 1130·4 cm.

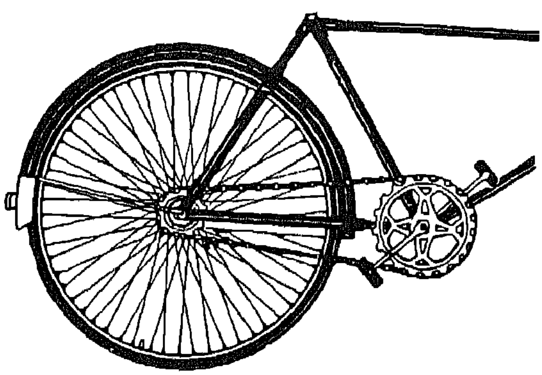

Fig. 12·17.

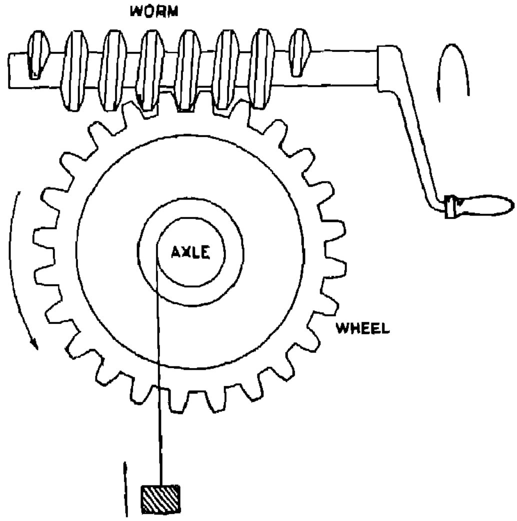

Fig. 12·18 *A Worm-gear.*

12.10 The Worm-gear

It consists of a screw like worm on a shaft which meshes with a cog wheel. One revolution of the crank attached to the work shaft causes the cog wheel to turn through a distance equal to the spacing between two successive cogs. Therefore if the cog wheel has 30 teeth on it, the worm shaft will have to rotate 30 times in order to rotate the cog wheel once. If the effort is applied to the worm shaft, then the velocity ratio will be high; consequently the mechanical advantage too will be very high.

The device is commonly employed for driving the rear axle of a motor truck and also to reduce the speed in speed counters.

Classroom Activities

Use some books to support one end of a metre long board about 30 cm above the table. Fasten a brick on to the roller cart for the resistance and find the total weight. Attach a spring balance to the roller cart and slowly pull it up the inclined plane. What is the reading on the spring balance ? Was your effort multiplied ? Lower the height of the board and repeat the experiment. What is the reading on the balance now ? Would you rather use a high or low inclined plane to lift a heavy object ? Was the work done in both cases the same ?

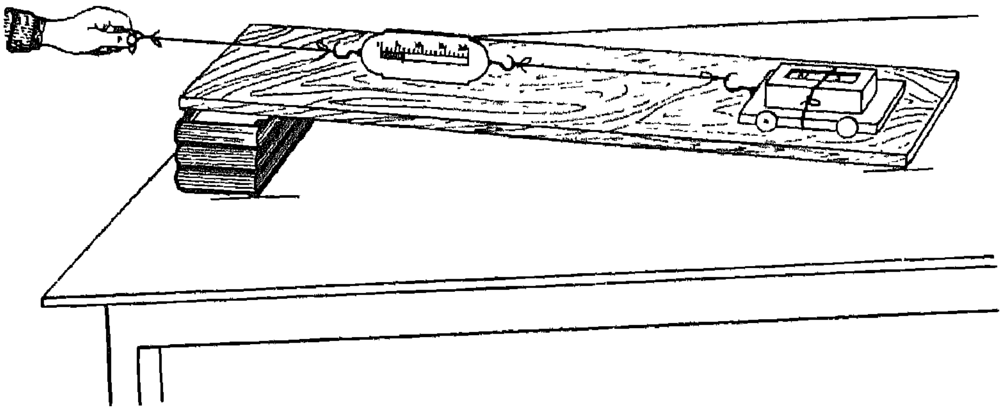

FIG. 12.19.

Questions

1. What are the chief functions of a simple machine ? Give examples illustrating your answers.
2. What is meant when we say that machines save labour ? Do they reduce the amount of work to be done ?
3. A man is able to raise a load of 200 kg through a distance of 6 metres when he uses a machine with a velocity ratio of 6. If the efficiency of the machine is 70 per cent, find the work done by the man and the force he uses.
4. Explain the working of a screw-jack. Assuming there is no friction, calculate the mechanical advantage of a screw jack with pitch 3 mm and handle 40 cm long. What force will be needed to raise a truck weighing 2000 kg with the help of this machine ?
5. In lifting a load through 1 metre with a system of pulleys, the effort moves 5 metres. If the load is 300 kg and the effort 50 kgf find (a) the actual mechanical advantage (b) the input (c) the output and (d) the efficiency.
6. How will you weigh correctly with the help of a faulty balance ?
7. A physical balance has its arms unequal in length, but the beam remains horizontal when the pans are empty. It was found that a given piece of metal when placed on the left hand pan weighs 16 5 g and it weighs 17 g when placed on the right hand pan Find its correct mass.

8. A mechanic uses a block and tackle to hoist himself to the top of a building. Each block consists of a single pulley. If the mechanic weighs 60 kg, find the effort exerted by him on the free end of the rope. What force is exerted by the support of the fixed pulley ?
9. Two pulleys have been installed as shown in Fig. 12.19. Find the effort needed to pull up a load of 100 kg. Neglect friction and the weight of pulleys.

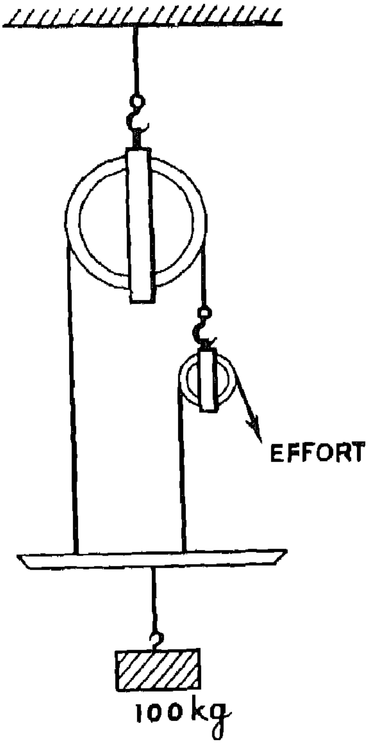

FIG. 12 20

10. A load of 50 newtons is lifted by means of a screw jack of pitch 3 mm. The length of its lever is 18 cm. If the effort needed is 4 newtons, find the efficiency of the machine.
11. A block and tackle has 4 pulleys in each block. What force will be needed to lift a load of 200 kg if the efficiency of the machine is 70 per cent ?
12. Why does a road wind up a steep hill instead of going directly along the line of greatest slope ?
13. Does the use of a machine increase one's power ? Discuss with the help of diagrams.
14. A man weighing 80 kg sits on a chair suspended from a moveable pulley and hoists himself up by a rope passing over a fixed pulley. What force must he exert ? (Neglect the weight of the chair and suppose there is no friction).

15. Suppose the upper pulleys in a Weston differential pulley system have radii 30 cm and 25 cm respectively, what will be the velocity ratio? Also determine its mechanical advantage if the efficiency is 35 per cent.
16. A string passes round a fixed pulley attached to a high ceiling. A monkey stands on a stirrup fastened to one end of the string and holding the other end of the string in his hand, he pulls at it in order to hoist himself up. If the monkey weighs 30 kg, find the force he must exert in order to go up.

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CHAPTER XIII

Friction

13.1 Introduction

Suppose a heavy box is lying on the floor and you push it in a horizontal direction. You may find that the box does not move. Clearly some force is opposing the force of your push. It could not obviously be the weight of the box because it is acting downward and as such it should have no effect on the horizontal motion. The opposing force here must be equal to your force and it must be acting in a direction opposite to that of your force. This explains why the box remains at rest.

If you increase the force of your push, you may ultimately be able to make the box move. In this case, then, the force you apply is greater than the force which was opposing the motion of the box. The force opposing the motion is called the *force of friction.*

When a surface slides over another, the force of friction invariably comes into play to oppose the motion even when no force is acting to maintain the motion. For example, roll a marble ball on a smooth floor; the ball will go quite a long way before it stops. Here its speed decreases continuously on account of friction. Again when a locomotive gives a push to a wagon, the latter moves on the smooth rails over an appreciable distance before it is brought to rest on account of the friction which opposes the motion.

Experience shows that the smoother the surfaces are the less is the friction between them. This explains why skating is possible on ice but not on a concrete floor. Have you ever stepped on a banana skin ? The underside of the skin is so smooth that negligible friction comes into play with the result that you suddenly get a high velocity and you slip badly.

There is friction between the axle and the wheel of the bullock cart. To reduce this friction the cartman puts grease or oil on the axle on which the wheel rotates. In fact frictional forces are developed between sliding parts of any machinery and lubrication is necessary to reduce this friction. The moving parts in delicate machinery of a watch too require proper type of lubrication so that they keep moving without undue wear and tear.

But friction may also be put to use. We are able to walk on account of the friction between the sole of our shoes and the ground. A knot in the rope stays on account of friction. A nut on a bolt continues to stay in the tightened position on account of friction. If there is no friction between the wheels of a railway carriage and the rail, the wheels would slip and the train would not move.

Thus we see that friction plays a very important role in our life. We shall, therefore, study friction in some detail.

Friction opposes motion when one solid slides over another. There is also a resistance to motion when a solid moves in a fluid. We shall also see later that a force of resistance comes into play even when a layer of liquid or any fluid slides over a layer of another fluid.

We will now perform certain experiments to study the friction when a solid slides over another.

Experiment

A block of wood is placed on a horizontal table and a piece of string is attached to it which passes over a pulley as shown in Fig. 13·1. A pan is suspended from the free end of the string. Care should be taken so that the portion of the string above the table remains perfectly horizontal.

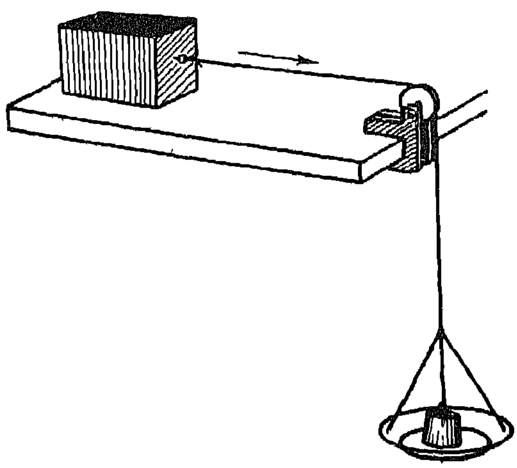

Fig. 13·1.

Place a few weights on the pan. You may find that the block does not move. A force equal to the weight of the loaded pan is pulling the block to the right, but the block does not move. This means that the force of friction is just equal and opposite to the tension in the string. Increase the weights on the pan in small steps, but the block does not move. Clearly then the force of friction can increase its magnitude so that it is equal and opposite to the force tending to move the block. That is, the force of friction is *self-adjusting*.

Continue adding weights to the pan till the block just begins to slide. It means that there is a limit to the increase in the magnitude of the force of friction. In this case, the limiting value of friction is equal to the force which just causes the block to slide.

13.2 Limiting Friction Depends upon the Mass of the Body Sliding

Place an identical block over the first block (Fig. 13.2). You will find that the force necessary to just make the composite block slide is double the force required in the first case. Thus the limiting friction is doubled if the mass of the solid is doubled. In general you will find that the limiting friction increases in proportion to the mass of the block.

As the upward reaction on the block is exactly equal to the weight of the block acting downward (Newton's Third Law of Motion), we may say that the limiting friction F is proportional to the normal reaction R

$$F = \mu R.$$

This constant of proportionately, μ is called co-efficient of friction. It is defined

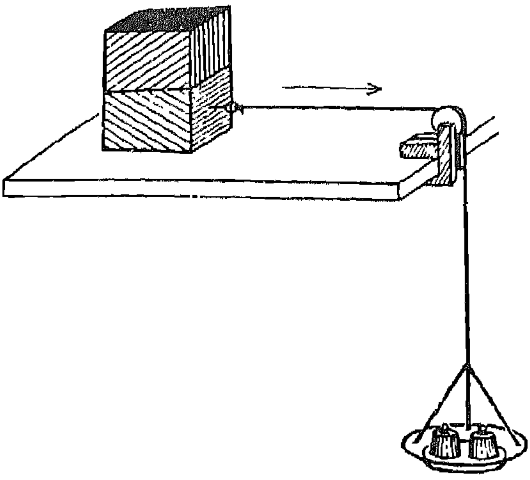

FIG. 13 2.

as the ratio of the limiting friction to the normal reaction.

13.3 Limiting Friction does not Depend on Area of Contact

In Fig. 13.2 the two blocks are placed one over the other. So that the normal reaction is equal to the weights of both the blocks and the area of contact with the table is equal to the area of the base of one block. In Fig. 13·3 the same two blocks are connected one behind the other as shown. Here again the total normal reaction is equal to the sum of weights of the two blocks, but the area of contact is doubled this time. In this case too you will find that the force required to make the blocks just slide is the same as in the case of figure 13 2.

This shows that the limiting friction is independent of the area of contact.

13.4 Static Friction is Greater than Kinetic Friction

In the above experiments, if we apply a force which is just a bit less than the value of the limiting friction and then we give a slight push to the block in the direction of

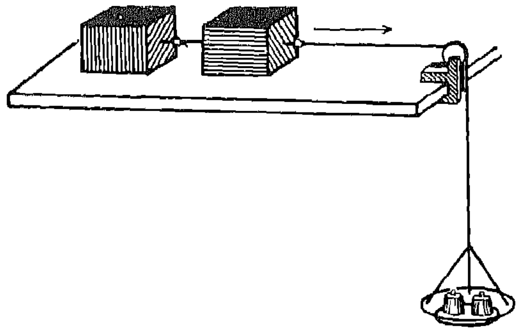

FIG 13.3

the force, then the block will continue to move on. This proves that the sliding friction, also called kinetic friction, is a bit less than the limiting friction

The limiting friction which must be overcome before the body just starts sliding from the position of rest is called Static (limiting) Friction. The co-efficient of sliding (kinetic) friction μ_k is defined as the ratio of the sliding friction F to the normal reaction R.

$$\mu_k = \frac{F}{R}$$

Obviously the coefficient of kinetic friction is less than the coefficient of static friction. This explains why it requires a greater force to start a body sliding over a given surface than the force required to keep it sliding when the body is already in motion.

13.5 Friction Depends upon the Nature of Surfaces in Contact

Experiments show that the coefficient of friction for glass surfaces is much less than that for wooden surfaces. In fact, the force of friction differs with the nature of surfaces in contact, the harder the material and the smoother the surfaces, the less will be the frictional forces.

It has also been observed that friction between two surfaces of the same material

is greater than the friction between two surfaces of different materials This is why in many machines, the axle and the bearing in which the axle rotates are made of different metals. For example the axle may be made of steel while the bearing may be made of bronze.

13.6 Force of Friction Always Acts in the Direction Opposite to the Direction in which the Body Tends to Slide

Consider the following experiments.

(i) In Fig. 13 4, a small force P is applied to the block tending to move it towards the right. But the block fails to move. Thus the force of friction F is opposite to P and is exactly equal to it. As we increase the pull P, it is able to overcome the frictional force acting to the left and the block begins to slide towards the right. Hence, the force of friction is directed in a direction opposite to that in which the body is sliding

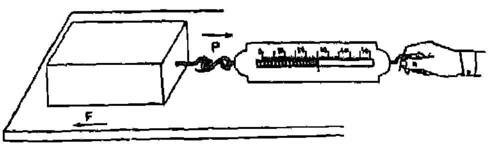

FIG. 13 4

(ii) In Fig. 13.5, the small force P tends to move the block to the left, but the block fails to move; hence, the force of friction F in this case is equal to P but it is directed towards the right That is, the force of friction again acts in a direction opposite to the one in which the block tends to move.

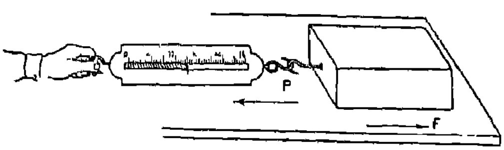

FIG. 13.5.

(iii) Suppose the block of weight W is kept on an inclined rough surface AB as shown in Fig. 13.6. Here the block is attracted vertically downward with a force W; its resolved part W sin θ is responsible for pulling it down along the line of greatest slope on

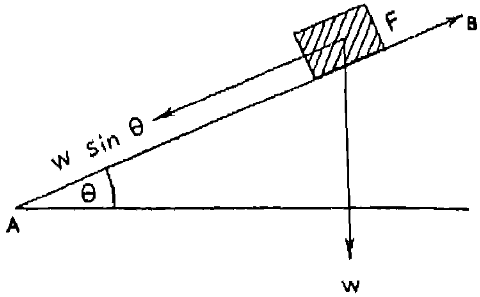

FIG. 13.6

the inclined plane. If the angle of inclination θ is small, the block will not move. Here the force of friction is directed upwards along the line of greatest slope. That is, the force of friction acts in a direction opposite to the direction in which the body tends to move.

Lastly, consider the case (Fig 13 7) in which a force P is applied along the direction A to B so that the body is just on the point of moving upwards. In this case the force of friction F will act downward. Hence the sum of the force of friction and the resolved part of W along the plane AB is equal to the force applied.

$$P=F+W \sin \theta$$

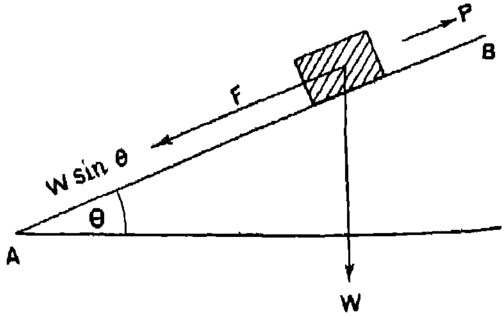

FIG. 13.7.

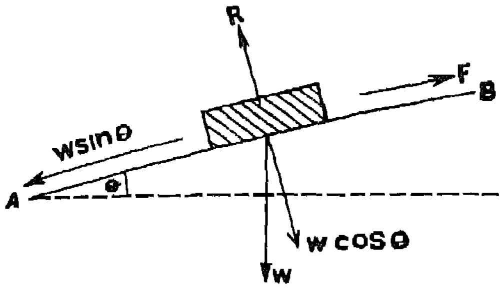

FIG 13.8

If, however, the force P_1 is such that the block is just on the point of moving downward, then the friction F will act upward so as to oppose the motion. We shall then have :

$P_1+F=W \sin \theta$.

We are now in a position to sum up the laws of friction as below :

1. Frictional force comes into play whenever one surface slides or tends to slide over another and its direction is parallel to the surfaces and opposite to that in which the surface tends to slide.

2. When the surfaces are at rest, the frictional force may take any value from zero upto a maximum known as the limiting friction

3. This limiting friction depends upon the nature of surfaces in contact and is independent of the area of contact.

4. The ratio of the limiting friction to the normal reaction between the two surfaces is called the coefficient of Static Friction. This coefficient depends upon the nature of surfaces in contact and the condition of surfaces (*i.e.*, roughness, wetness, etc.) but is independent of the normal reaction.

5. When sliding occurs, the force of friction called kinetic or sliding friction is less than the limiting friction.

The French physicist Coulomb enunciated the law that the sliding frictional force is independent of the sliding speed. But this is not quite correct as is shown by the following Table which gives the values of the coefficient of kinetic friction for unlubricated steel on steel, at various speeds.

TABLE–I

Speed, m/sec	0·0001	0 001	0 01	0 1	1	10	100
Coefficient of Kinetic friction, μ_k	0 53	0 48	0 39	0 31	0 23	0·19	0 18

It is seen that μ_k falls with increasing speed. However, in actual problems such a large range of the variation of the speed is not encountered and the kinetic coefficient of friction may be taken to be independent of the relative speed. Why friction is so high at low speeds and reduces at higher speeds has not been explained satisfactorily.

The following Table gives coefficients of static friction and those of kinetic friction in air for unlubricated surfaces of different materials.

TABLE–2

Material	μ_s	μ_k
Steel on steel	0·15	0·09
Steel on ice	0 03	0·01
Leather on wood	0·5	0 4
Wood on wood	0·25 to 0·5	0·15 to 0 3
Rubber tyre on dry concrete road	1·0	0 7
Rubber tyre on wet concrete road	0·7	0·5

Angle of friction

Suppose a block is resting on a plane whose inclination can be changed. Now increase the inclination by increasing $\angle\ \theta$

between the plane and the horizontal till the body just slides down the plane (Fig. 13.8). Clearly then, the force pulling the block down is equal to the limiting static friction when the block is just on the point of slipping

Limiting friction $F=W \sin \theta$.

The normal reaction $R=W \cos \theta$

$$\mu_s = \frac{F}{R} = \frac{W \sin \theta}{W \cos \theta} = \tan \theta.$$

13.7 Rolling Friction

We know from experience that it requires a lot of force to drag a heavy box along the ground, but a much smaller force is required to do so if we place rollers under the box. This clearly shows that the rolling friction is much less than the sliding friction.

The builders of pyramids in Egypt did not have cranes to move heavy blocks of stone. It is believed that they placed rollers underneath the huge blocks of stone and thus men were able to slide the stone on these rollers up an inclined plane by applying a moderate force only.

This knowledge has been made use of in attaching wheels to vehicles instead of runners as in sledges. The rolling friction of the wheels is far less than the sliding friction of the runners. Instead of sliding on the ground, the wheels pivot about the point of contact with the ground.

Fig. 13.9 explains how the rolling friction acts As the wheel presses the ground, the surface of the latter is deformed as shown in Fig 13.9 a A small hill at C is formed continually and the wheel has to ride over it. Thus the rolling friction is caused by the deformation produced where the wheel presses against the surface on which it rolls.

The deformation produced is less if the rim of the wheel as well as the road surface is hard (Fig. 13 9 b). So in this case the rolling friction too is reduced. Similarly, a cycle moving on a concrete road with its tyres inflated hard experiences a much smaller rolling friction

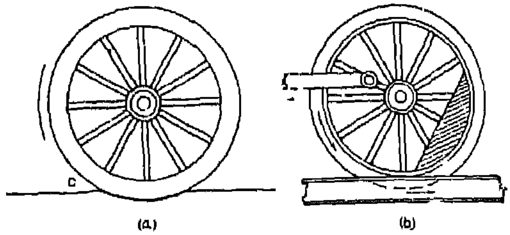

FIG 13.9 *Illustrating rolling friction between soft and hard surface.*
(a) *A bullock cart wheel rolling on the ground*
(b) *A railway engine wheel rolling on the rail.*

It can be shown that the rolling friction is inversely proportional to the diameter of the wheel. For the same weight the larger wheel will depress the road surface less than a smaller wheel. The rolling friction can be further reduced if the rim of the wheel is made wider. In this case the pressure downward gets reduced and consequently there is now less deformation.

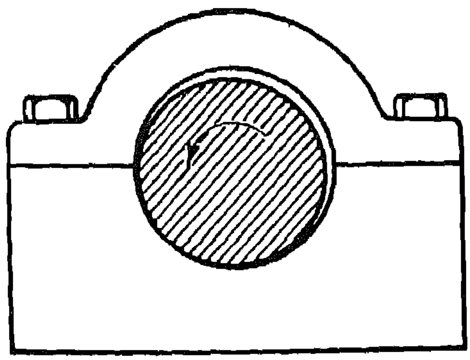

FIG. 13.10. *Sleeve (bush) type bearing.*

In any machinery the shaft rotates inside a bearing In a bush type bearing the surface of the shaft slides on the inside surface of the bearing. Here the sliding friction which opposes the motion may be

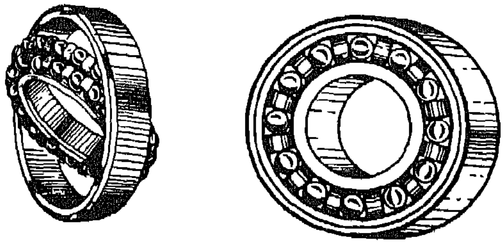

FIG. 13·11. *Ball bearing*

considerable (Fig. 13 10). To avoid this, we make use of ball bearings (Fig. 13.11). In this case as the shaft rotates, the balls too roll round—we have here only rolling friction which is necessarily very small. When does a wheel roll and when does it slide ?

When a bullock pulls at a cart, three forces begin to act on the wheel as shown in Fig 13.12. There is the pull P of the bullock which is applied on the wheel through the shaft and the axle. The wheel has a tendency to slip in the forward direction and this is opposed by the frictional force F, due to the ground In addition there is a frictional force F_1 between the axle and the hub which opposes the rolling of the

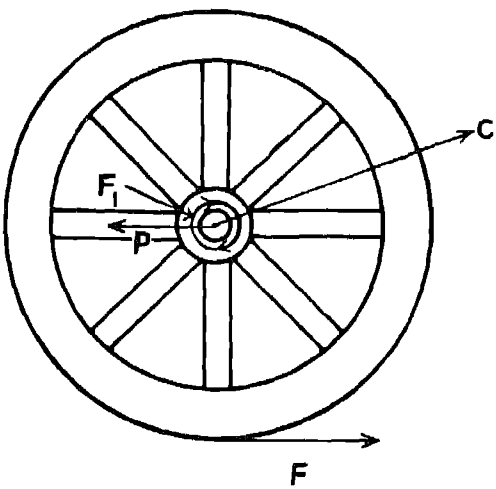

FIG. 13·12. *Forces acting on a bullock cart wheel*

cart. F_1 has a clockwise moment about the centre C and F has an anti-clockwise moment. If the moment of F is greater than that due to F_1 the wheel rolls; otherwise it slips

When a driver has to stop his vehicle he pushes the brakes very hard, thus increasing the frictional forces to the rotation of the wheel The rotation of the wheel is thus decreased very suddenly by increasing a force akin to F_1. But the speed of the car can not be reduced equally suddenly. The wheels now slide on the road and the frictional force akin to F between the wheels and the road leaves a mark on the road and the wheels also are damaged

Sometimes when a railway engine tries to start a stationary train, the wheels begin to spin and the train does not move To give speed to the train, certain force is necessary on account of the inertia of the train. The maximum force that the engine can exert is μ_s R where R is the weight of the engine. If the required inertial force is greater than this, the wheels slip and begin to spin and the train does not start. The frictional coefficient between the wheels and the rails is increased by applying sand and the train is able to start.

13.8 What Causes Friction

Every surface is more or less rough and small projections or irregularities are present in the surface. As one surface presses over the other, the small projections of one surface get interlocked with those of the other surface thus hindering the relative motion between them. The wearing of the surfaces takes place due to this interlocking of surfaces as they slide over each other.

Some physicists think that forces of mutual attraction between molecules of the two surfaces are responsible for offering the resistance to sliding motion. Others think

that electrical forces which hold molecules and atoms together are partly responsible for producing friction.

Small irregularities in the surface produce friction.

Recently experiments were performed with sliding metal surfaces which show that when a metal surface presses upon another

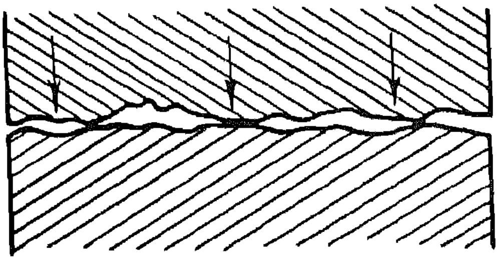

FIG 13.13. *Irregularities on surfaces obstruct sliding motion.*

metal surface and sliding is brought about, the normal pressure between them causes a sort of welding of the two materials at contact spots. Now sliding involves breaking these welding forces which means that resistance has to be overcome. This resistance gives rise to the force of friction.

In fact, even today we do not completely understand the nature of friction Research work is going on in this field.

13.9 How to Reduce Friction

In many cases the presence of friction is a positive hindrance.

Friction between the moving parts of a machinery causes not only a waste of energy but the heat produced by the friotion may also cause damage to the machinery. In fact, wherever one surface slides over another, a part of the force applied is spent in overcoming the friction acting on the surfaces in motion.

Hence it becomes necessary to reduce the friction as far as possible. Several methods are available :

1 LUBRICATION : Oil or grease is put between the two surfaces which slide on each other. The lubricating oil forms a thin layer between the two surfaces. Thus in the case of a shaft rotating inside a sleeve bearing, the thin layer of oil keeps the two surfaces separated, so that, the axle rides on a thin film of oil and thust he frictional forces are reduced considerably.

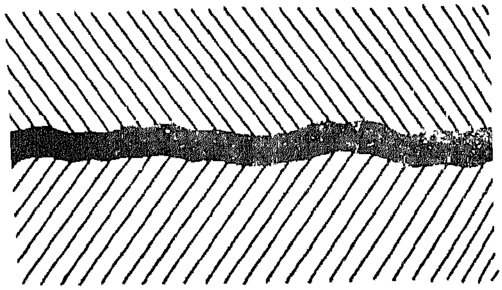

FIG. 13.14.

2. USE OF BALL BEARINGS . If instead of sleeve bearing, we use ball bearing, then the axle does not slide on the inner walls of the bearing. But it rolls over the balls and the balls themselves roll in a groove made in the bearing. Further, the ball bearing is lubricated so that the friction is reduced all the more.

3 USE OF ANTI-FRICTION MATERIALS : We know that the co-efficient of sliding friction for steel on steel is more than the co-efficient of friction for steel over bronze, so that the inside of the bearing is lined with some alloy so that, the steel axle when rotating inside such a bearing experience less friction.

13.10 Friction as a Friend

Friction is not always harmful For instance, while cycling if we apply brakes, the brake shoes rub against the rim of the wheel of the cycle and the friction between the two causes the revolving wheel to stop. In order that the brakes work efficiently, it

is essential that the surface of the inside of the rim should be in a dry condition, and the brake shoes should also be rough, so that they may cause a lot of friction At times the wheels of a car lose grips on slippery ground due to lack of friction. To increase friction either straw is spread on the ground or coir-rope is wound on the wheel so that the wheels may not slip any more. Sometimes sand is strewn on a slippery road to avoid the slipping of wheels.

Similarly more friction is a necessary requirement of the tractor. Hence, the tractor chain is built with large size links and the tyre attached to the vehicle is also heavily cut on the outside.

Even walking on the ground will not be possible if there is no friction between the sole of your shoes and the ground. On account of friction, your foot is able to apply a horizontal force backwards on the ground, and by Newton's Third Law an equal and opposite force acts on your feet so that you move forward. This explains why it is so difficult to walk on slippery ground. It is on account of friction that a nail driven inside wood or wall stays fixed there. Screws, nuts, and bolts get fixed on account of friction.

We had stated in chapter XI that energy of a system is conserved; for example, if a body is moving with a uniform velocity on a frictionless table it will continue to do so. In this case the kinetic energy (equal to $\frac{1}{2}$ mv^2) plus the potential energy remains constant. However, if the table has friction, as is always the case in actual practice, the body will come to a stop after travelling a certain distance. In the rest position, the kinetic energy is zero and there is no gain in potential energy. However, there is no loss in energy. The energy has been converted into another form called thermal energy (in fact this is an example of the conservation law of energy, in a more general form about which we will learn more later).

A familiar example would be when you apply brakes to a moving bicycle to bring it to rest, the rubbing of the brakes along the rim causes the rim to become warm.

Classroom Activities

1. (a) Take a block of wood (say 5 cm × 8 cm × 15 cm). Drive a nail part way into the centre of one end of the block and bend the nail over so that it can be used as a hook. Hook a spring balance on the block and pull it at a steady speed across sheets of sand paper of varying degrees of roughness. Also pull it across a smooth table top. Note the reading on the balance in each instance.
   (b) Support the block on two round pencils placed about 5 cm apart underneath the front end. Pull the block over the pencils and note the reading on the balance.
   (c) Compare the amount of force it takes to pull the block along the sand paper, smooth table and over the pencils.
2. Rub two pieces of sand paper together. Repeat after applying some grease or soap between them. Explain your findings.

Questions

1. A horizontal 100 kg force is required to pull a 500 kg crate at uniform velocity on a level floor. What is the coefficient of friction between the crate and the floor?
2. In sliding a 2,000 kg box on a horizontal floor, the minimum force needed is 50 kg. Find the coefficient of friction.
3. A body is placed on an inclined plane. Discuss the circumstances when the force of friction on the body is directed down the plane. Under what circumstances will the force of friction act up the plane ?
4. The coefficient of friction between ice and the runner on a sled is 0·1. What horizontal force is necessary to pull the sled which weighs 100 kg ?
5. Give reasons for the following:
   (a) One should take short steps rather than long steps when walking on slippery ground.
   (b) A cycle with well inflated tyres moves more easily on the road than one with loosely inflated tyres.
   (c) If the wheels of a car are found to slip on a muddy road, generally coir ropes are wound around the wheels.
   (d) Roller bearings are used in a machinery where a shaft has to rotate on its bearings.
6. Which of the following statements are true ?
   The coefficient of friction between two surfaces is independent of
   (a) The area of contact.
   (b) The lubricant between the two surfaces.
   (c) The smoothness of the surfaces.
   (d) Temperature.
7. If you had to take a jeep on a loose and sandy soil, then which would you prefer and why ? (i) The tyres to be well inflated or (ii) the tyres to be slightly deflated.
8. A slab of ice weighing 810 kgf is to be pushed on a horizontal wooden floor. If the coefficient of friction between ice and wood is 0·1, find the least force required.
9. A block of wood weighing 50 newtons is placed on a horizontal table. A force of 20 newtons is required to make it slide with uniform velocity What is the coefficient of friction ?
10. " Force of friction is independent of the area of contact " If this statement is correct, then why is a broad driving belt preferred to one which is rather narrow ?
11. A block of wood placed on an inclined plane just begins to slide of its own accord when the angle of inclination to the horizon becomes 30°. What is the coefficient of friction ?
12. A plane of wood is inclined at 20° to the horizon. A block of wood weighing 30 kgf is to be slided up the plane at a uniform velocity. If the coefficient of friction is 0·2, find the force required.
13. Distinguish between sliding friction and static friction. Can you give reasons to explain why the sliding friction is less than the limiting static friction ?
14. A block of wood weighing 20 kgf is placed on a horizontal floor. The coefficient of friction between the box and the floor is 0 3. The rope attached to the block

makes an angle of 45^0 to the horizontal. Find the least force which must be applied to the string so that the block may just slide

15 A man standing on a polished floor finds that he is unable to push a heavy box when wearing shoes with leather soles, but he is able to do so when he puts on shoes with rubber soles, Explain the reason.

16 Suppose we take two identical rollers placed adjacent and parallel to each other. If they are rotating in opposite directions with the same speed and a wooden board is placed on the top of then so that the centre of gravity is located just half way between the two rollers, then no motion of the plank takes place Explain the reasons.

If the plank is displaced to the right of the above equilibrium position, it would execute a to and fro motion about the position of equilibrium. Explain why it does so ?

17. You are standing on a very slippery ground. How will you move forward ?

Further Reading

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SECTION II

PROPERTIES OF MATTER

CHAPTER XIV

Fluids at Rest

14.1 Pressure

If we press the top of a table with the palm of our hand we say we are applying force on the table. We do not observe any effect of our pressing the table. Suppose we take 1 kgf, and place it on the table, we still do not observe any visible effect. Let us now take a nail and hold it with its head resting on the table. Then holding the nail vertically support the 1 kgf on it. Even now we do not observe any effect. Now invert the nail so that the sharp end is resting on the table. Holding it in this position, place the 1 kgf on the head. Now we can clearly see a dent on the table. In all these cases we have been applying a certain amount of force on the table. We have also seen that the visible effect is produced for the same force (1 kgf) only when the sharp end of the nail is resting on the table.

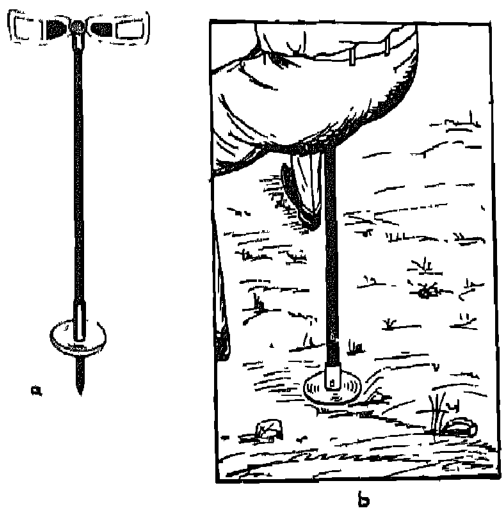

FIG. 14.1. *(a) A shooting stick, (b) the shooting stick in use. The person's weight causes the stick to become embedded in the ground up to the metal plate.*

Most of us are also familiar with the shooting stick used by umpires in cricket matches. This is shown in Fig. 14.1 (a, b) When a person sits on the stick, the sharp end pierces into the ground until the metal disc stops further sinking. Usually tractors are provided with rear wheels of large area to avoid the wheels getting stuck in the soft ground.

From the above examples, it should be clear that the effect produced depends both on the force as well as the area of the surface on which it is acting. The effect depends upon a quantity called pressure which is the

ratio between the force and the area, *i.e.*, Pressure=Force/Area.

We commonly use this word pressure in different contexts like "Pressure of work, pressure of circumstances, Political pressure," *etc.* In scientific terminology, this word pressure is uniquely defined as the force acting normally on a unit area.

If, on a given surface, the force is acting uniformly, then, the pressure P is given by

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}.$$

It is evident that pressure has the dimensions of force per unit area and is measured in newtons per square metre (newtons/m^2). It can also be measured in kilogram force per square metre (kgf/m^2). For example, if a force of 1,000 newtons is acting uniformly and at right angles to a surface of area two square metres, then the pressure is

$$P=1000/2=500 \text{ newtons/m}^2.$$

If the force is not uniform over the given surface, then we can talk of the pressure at a given point on the surface. If we choose a small element of area '$\triangle A$' enclosing this point such that the force $\triangle F$ is acting uniformly on this area, the pressure P at this point is given by

$$P=\triangle F/\triangle A.$$

It is evident that for a given force the pressure increases as the area decreases. In the light of this we can easily understand the advantage in having the edges of the cutting tools to be sharp.

Example : A boy weighing 40 kgf is standing on the floor. The area of his foot is 1/100 square metre. Calculate the pressure he exerts when he stands on one foot.

$$\text{Pressure} = \frac{40 \text{ kg} \times 9 \cdot 8 \text{ m/sec}^2}{1/100 \text{ m}^2}$$
$$=39{,}200 \text{ newtons/m}^2.$$

If he stands on both feet, the pressure becomes

$$P = \frac{40 \text{ kg} \times 9 \cdot 8 \text{ m/sec}^2}{2/100 \text{ m}^2}$$
$$= 40 \times 50 \times 9 \cdot 8 = 19{,}600 \text{ newtons/m}^2.$$

14.2 Pressure due to Liquids

After getting an idea of pressure, let us examine what the pressure is in a liquid and how it varies. Take for example a rectangular glass tub containing water. The water experiences a downward force due to gravitational attraction and exerts some force on the bottom of the container. The solid bottom exerts an equal and opposite force on the liquid and consequently the liquid is at rest. From experience we know that if we drill a small hole in the bottom, the water will be forced out This is because we have removed the solid material from the bottom and consequently removed the force exerted by the solid on the water. It is also our common experience that if we drill a hole on the side wall of the container, the water will still flow out. In general, we can say that wherever a hole is drilled in the solid surface in contact with the liquid, the static equilibrium of the liquid is disturbed and liquid flows out. This shows that the sides of the container exert a force on the liquid to prevent it from flowing out. From Newton's third law, we know that the liquid must be exerting an equal and opposite force on the wall. Now let us consider one brick placed in a rectangular container which can just accommodate it. If the bottom is removed, the brick falls down. But if one of the sides is removed then the brick does not move sideways, that is, the side-walls of the container do not exert pressure nor the brick exerts any pressure on the side-walls of the container. This is one of the differences in the behaviour of solids and liquids.

14.3 Pressure inside a Liquid

We will now construct an apparatus for measuring pressure inside a liquid.

Take a thistle funnel. The narrow end of the funnel is connected by a rubber tube to a manometer made of glass. The manometer consists of a U-tube which is partly filled with coloured water. Stretch a flexible rubber membrane over the funnel.

You will observe that the level of water in both the limbs of the U-tube is the same. If you now press the stretched membrane with your finger, you will observe that the level of water in the outer limb is higher than that in the inner limb. A change of pressure on the membrane is thus indicated by the difference in the water level.

(a) *Pressure at a point is the same in all directions*

Take a rectangular tube containing water. Immerse the funnel of the above apparatus up to a certain depth indicated by a certain horizontal line The pressure exerted on the membrane at this depth is transmitted to the manometer. The position of the funnel is marked as (1) in figure 14.2. Now let us tilt the funnel such that the membrane is vertical but the centre of the membrane is at same horizontal level as shown in position (2). You will observe that the manometer indicates the same value of pressure. In position (3), the funnel is held such that the membrane is again horizontal and at the same level. The manometer indicates the same reading. In fact, we can show by this simple experiment that in whatever orientation the funnel is held, pressure is the same provided the centre of the membrane is at same horizontal level This experiment shows that pressure at a point in a liquid acts equally in all directions. From the above we can show that if we consider any area A, the net force on one side of surface is equal and opposite to the force on other side of the surface If the area considered is horizontal then the upward and downward forces on the surface are equal.

We can demonstrate it by the following experiment.

A glass cylinder open at both ends is taken and a thin metal disc of area of cross-section slightly greater than the cross-section of the cylinder is attached to the lower end by means of a thread fastened to the disc as shown in Fig. 14 3. The cylinder with the

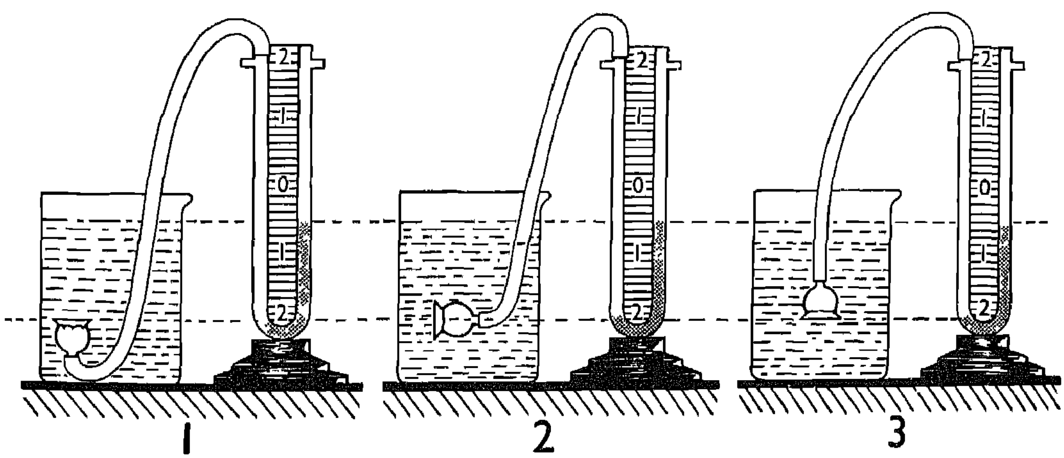

Fig. 14. 2. *The pressure is the same in all directions.*

disc is immersed in a beaker containing a liquid (say water). It is observed that the

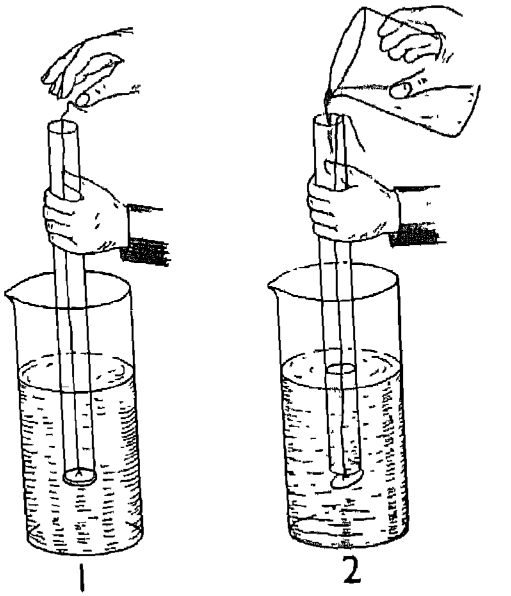

FIG. 14.3 *The pressure at the same depth is different in different liquids.*

disc will remain intact even though the thread is released. This is clearly due to the upward thrust of the liquid on the disc. If now the same lqiuid is introduced into the cylinder from above slowly, the disc will remain undisturbed until the level of liquid in the cylinder is equal to that in the beaker. When the level of liquid in the cylinder is just higher than that in the beaker, the disc sinks to bottom of the beaker. This proves that the upward and the downward thrusts on a given surface in the liquid are equal.

(b) *Pressure varies with depth and density*

Experiment : Take a measuring cylinder and fill it with liquid up to a certain height. Now take the pressure-measuring apparatus of the last experiment. On inserting the thistle funnel inside the jar, it is found that there is a difference in the water levels of the two limbs of the manometer. This is a measure of the pressure at a point due to the liquid in the measuring jar. We have seen in the last experiment that pressure at a given depth is independent of the direction in which the membrane is held Plot a graph between the depth of membrane and the level difference in the two limbs of the manometer. It is found to be a straight line which shows that pressure increased directly with depth.

If we perform the above experiment with different liquids inside the measuring jar (Fig. 14.4), we will find that at the same depth the pressure is different in different liquids and is proportional to their densities.

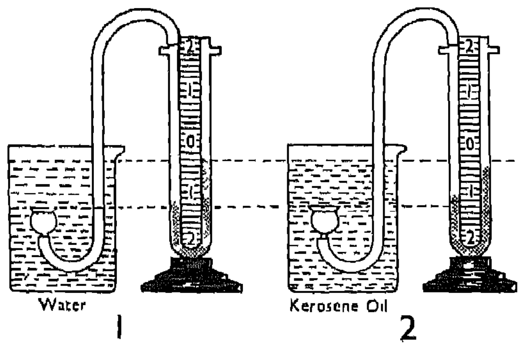

FIG. 14.4. *Experiment showing upward pressure of liquids.*

Put a vessel with some holes in its sides under a water tap (Fig. 14.5). Allow the vessel to be gradually filled with water and watch the flow of water from the side-openings. Why does the water from the lower hole reach a greater distance ?

(c) *Pressure is the same at the same horizontal level*

The experiments so far described show that pressure at a given point in the liquid is the same in all directions and that pressure increases with depth. When vessels of

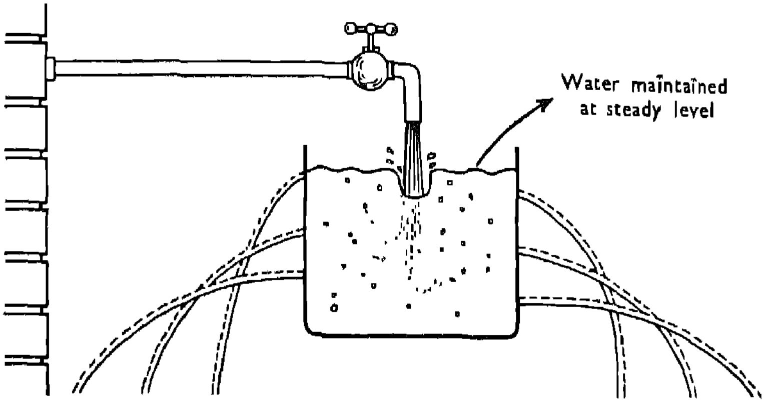

FIG. 14.5. *The pressure at a point below the surface of a liquid varies with the vertical depth.*

irregular shapes are used as containers it becomes less easy to arrive at the above conclusions by a superficial examination. Actual experiment shows that the conclusions are true even for these vessels.

Let us consider for example two vessels A and B (Fig. 14.6) filled with a liquid up to the same height. Now measure pressure at points P, Q, P′ and Q′. It will be observed that pressure at all these points is the same.

It is easy to see why the pressure at P and Q in A is the same; the water columns above P and Q are of the same height. But in B the real height of the column of water standing above point P′ is h_1 and at Q′ is h_2, and yet experiment has shown that they have the same pressure. From this we conclude that at the point Q′ there is some other agency which is exerting pressure. This extra pressure must be numerically equal to the pressure exerted by a column of liquid of height $h_1 - h_2$. This agency must obviously be the wall of container above Q′. We have already seen that liquid exerts pressure on the wall and *vice versa*.

Similar analysis can be done for a vessel of any irregular shape and it can be seen that the pressure at any point depends on the depth from the horizontal level of the top surface of the liquid.

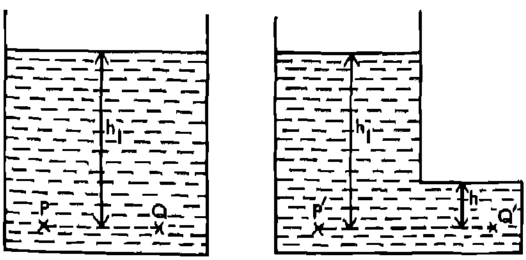

FIG. 14.6. *The pressure at each point in the same level remains the same*

(d) *Liquid finds its own level*

We know that pressure is the same at all points on the same horizontal plane inside a liquid We also know that pressure depends on the height of the liquid column above a given point. Consequently we expect that a liquid in vessels of different shapes and sizes in communication with one another, stands at the same level. This can be illustrated by arranging vessels as shown in Fig. 14.7.

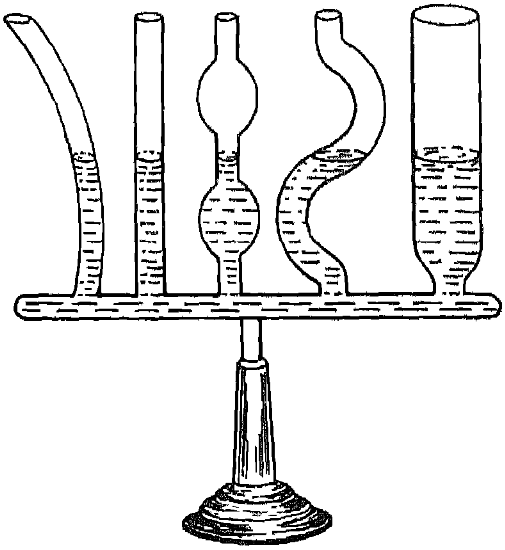

FIG. 14.7 *To equalize the liquid rises to the same level in each container, the shape and size of the container have no effect on the pressure.*

So far, the above experiments have given the following information about the pressure due to fluids :

1. Liquids exert lateral pressures on the surfaces of the container,
2. The pressure at a given point inside a liquid is the same in all directions;
3. The upward thrust and downward thrust on a plane surface inside a liquid are equal;
4. The pressure depends only on the vertical heights of the liquid and the density of the liquid. It does not depend upon the size and shape of the container and the volume of the liquid; and
5. The pressure increases with the depth and the density of the liquid.

14.4 Artesian Wells

In some places, if a hole is bored into the earth, water gushes out through the hole with some force. How does the water rise up against gravity ? The explanation is to be found in Fig. 14.8 and in the law that a liquid finds its own level.

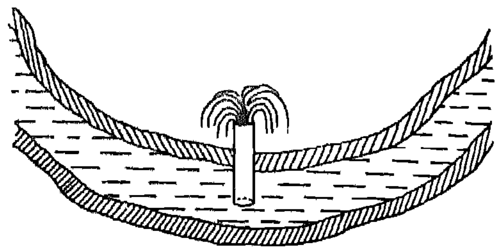

FIG. 14.8. *If the level of water under earth's layer is higher than the level of the ground at which a hole is bored, then the water gushes out through the hole with some force.*

14.5 Pressure at a Point with in a Fluid

We have so far studied some of the properties of liquids at rest, qualitatively. Now we shall discuss it quantitatively.

Consider a liquid in a beaker (Fig. 14.9). To calculate the pressure at a depth h metres below the liquid surface, we consider a small circular area of cross-section a sq m over which the pressure can be assumed to be uniform. The pressure at P is caused by the liquid column in the cylinder of base area a and height h. The volume of the liquid is $a \times h$ m^3. If d is the density of the liquid and g is the acceleration due to gravity—the downward thrust on a due to gravitational pull on the liquid in the cylinder is mass times the acceleration due to gravity. Hence the downward thrust $=(a \times h \times d)$ g newtons.

Then the pressure at $P=\frac{a hdg}{a}=hdg$

newtons/m^2

So, $p \propto h$,
and $\propto d$.

The pressure at a depth of 1/10 metre in mercury is (density of Hg is $13 \cdot 6 \times 10^3$ kg/m^3)

$$P=\left(\frac{1}{10}\right) m \times 13 \cdot 6 \times 10^3 \ \frac{kg}{m^3} \times 9 \cdot 8 \ \frac{m}{sec^2},$$

$$=13,328 \frac{\text{kg m}}{\text{sec}^2\ \text{m}^2} = 13,328 \text{ newtons/m}^2$$

If it is water, the pressure is 980 newtons/m^2.

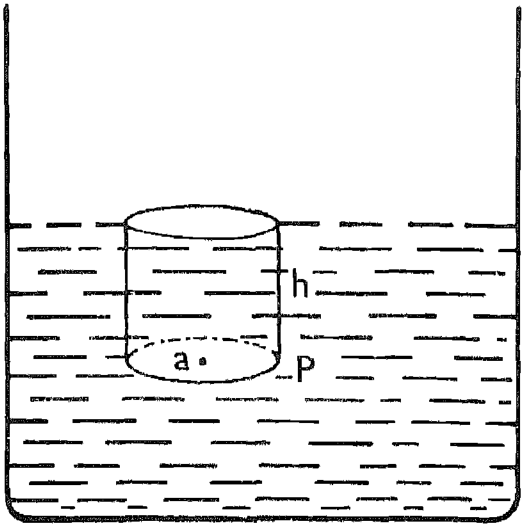

FIG. 14.9. *The magnitude of pressure at any point inside the liquid depends on the depth.*

14.6 Determination of Density of a Liquid—U-tube Method

The principle of equal pressure at two points at the same level in a liquid is used for determining the densities of liquids. The U-tube apparatus shown in Fig. 14·10 utilises this principle for determining density by comparison of two immiscible liquids.

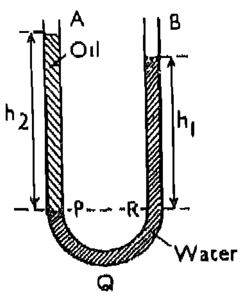

FIG 14.10. *U tube method for determination of density of a liquid.*

BR=height of water=h_1

AP=height of oil=h_2

The pressure at the same horizontal level PR is the same since water is present in the section PQR. Let d_1, d_2 be the densities of water and oil. Then we have

$$h_1 d_1 = h_2 d_2$$

Therefore, $d_2 = h_1 d_1 / h_2$.

If d_1 is taken as unity we have $d_2 = h_1/h_2$. The density of the liquid is numerically the ratio of height of the water column to the height of the liquid column (here we have taken oil).

14.7 Pascal's Law of Transmission of Pressure by Liquids

Consider a vessel of shape shown in the figure 14·11 below. The vessel is completely filled with liquid and has two just fitting rubber stoppers. Now we press the stopper A. We would see that the other stopper, B, is forced out. We did not apply any pressure directly on the stopper B, but we see the force that has been applied on A has been transmitted to B. This could have happened only through the medium of the

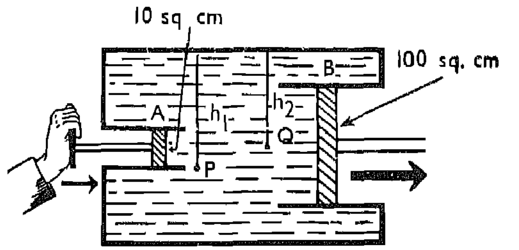

FIG. 14.11. *Ten men were required to hold* B *in place when one man pushed on A*

fluid. The important point is that this pressure is transmitted equally in all directions.

Let us look at the transmission of pressure in some detail Consider the points

P, Q in the liquid. Before any pressure is applied on stopper A, the pressures at P, Q are respectively $\rho g h_1$ and $\rho g h_2$. On pressing the stopper at A, the increase in pressure at both P and Q can be seen to be the same. This increase in pressure is indeed the same for all points in the liquid. We can demonstrate the above statements by the following experiment.

Consider a vessel of shape as shown in figure 14·12 completely filled with water. The heights of water column in the various manometers indicate the pressure at P, Q, R and S.

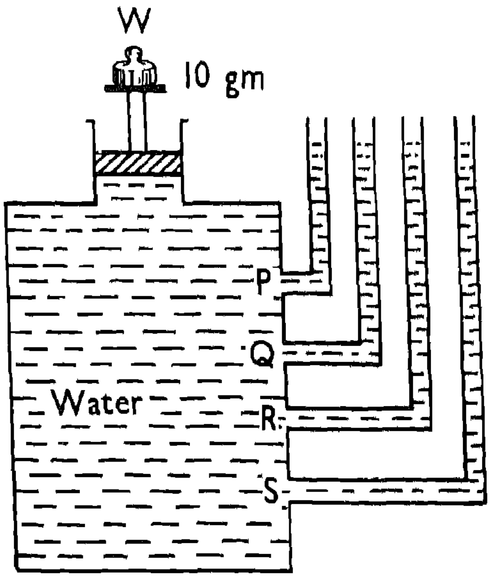

FIG. 14.12. *When a weight of 10 gm is placed on the piston rises up to the same level in all the tubes.*

Now let us put a ten-gram weight on the stopper. The pressures at P, Q, R, and S change and this change can be measured by measuring the change in the height of the water column. The increase in the height of the water column can be seen to be the same in all the manometers. This shows that the increase in pressure is the same, throughout the body of the liquid, though the actual pressures at different points like P, Q, R and S in the liquid are not the same.

The results obtained above are stated in Pascal's Law as follows :

Pressure caused by an external force applied to a liquid is transmitted undiminished throughout the body of the liquid.

The most important application of Pascal's Law can be understood from the following experiment.

Consider the vessel of the shape shown in the figure 14.13.

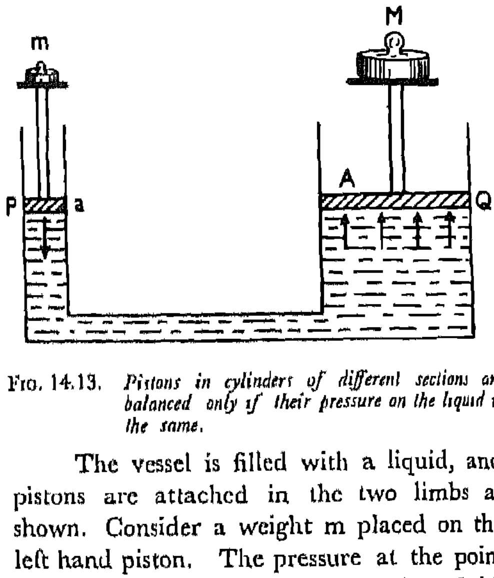

FIG. 14.13. *Pistons in cylinders of different sections are balanced only if their pressure on the liquid is the same.*

The vessel is filled with a liquid, and pistons are attached in the two limbs as shown. Consider a weight m placed on the left hand piston. The pressure at the point just below the piston P is $= m/a$ and this pressure is transmitted to the right hand piston Q. Thus there is an increase in upward thrust equal to $\frac{m}{a}$ A, which can balance a weight M, with $M = \frac{mA}{a}$.

The mechanical advantage of the system is $\frac{M}{m} = \frac{A}{a}$.

The ratio of the cross-sections can be made suitably large in order to get the desired mechanical advantage

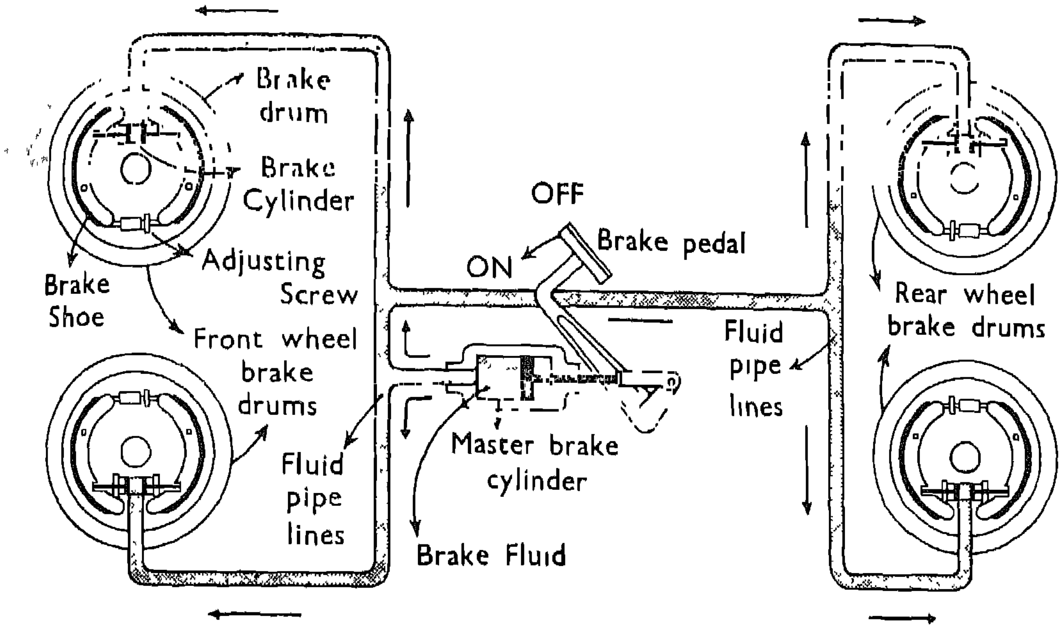

FIG. 14.14. *The hydraulic braking system of a motor-car.*

Brahma's hydraulic press works on the same principle as described above. One can produce with this machine enough force to press bales of cotton, books, metal sheets, *etc.*

Hydraulic brakes system in motor-cars which works on the same principle is shown in figure 14. 14.

14.8 Measurement of Blood Pressure

This refers to blood pressure in the human body. Usually medical people refer to this as B.P. By measuring this doctors are able to judge the patient's physical condition. It helps in the diagnosis of certain diseases. We all know that tubes called arteries lead the blood from the heart to the lung and tissues in all parts of the human body. The return tubes which bring the blood to the heart are called veins. The fine narrow tubes called capillaries in the tissues require considerable pressure to force blood through them. This pressure is supplied by the heart and is called the systolic pressure.

The instrument which the doctors use for measurement of B P. is called Sphygmomanometer. It consists of a flat, thin walled, airtight rubber tube which is attached to a U-tube manometer containing mercury Through the other end of the tube air can be pumped into the tube For taking B.P. the rubber tube is wrapped round the arm and then air is pumped into it until the pressure in it increases and becomes great enough to squeeze the artery which supplies blood to the fore-arm The flow of the blood stops. This can be recognized by the doctor by listening to the pulse beat with a stethoscope placed on the artery in the fore-arm

The doctor then slowly reduces the pressure in the hand by opening a pinch cock. When the pressure is just equal to the systolic pressure, which is the maximum pressure the heart can exert in forcing the

blood through the arteries, the doctor can hear the thumping sound of the blood flow. The corresponding pressure recorded by the manometer is called the blood pressure.

When more air is let off, the continuous flow of blood is established. This is recognized by the doctor as a continuous sound. The corresponding pressure is known as the diastolic pressure. Normally if the systolic pressure exceeds 160 mm or diastolic 120 mm the patient's condition is said to be grave. He suffers then from high blood pressure.

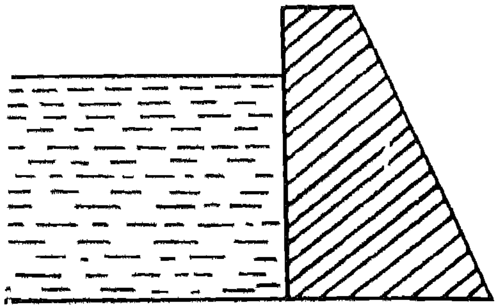

FIG 14.15. *The cut-section of a dam.*

14.9 Water Dams

Most of us have heard of the huge dams like Krishna Raja Sagar, Bhakra and Hirakund constructed in our country. The construction of these dams is based on the principles of hydrostatics. We have seen that the pressure due to a liquid increases with the depth of the liquid. When a dam is constructed for storing water the depth of water depends on the height of the dam. When huge dams are constructed with very high retaining walls, the depth of the water stored is great. Consequently, the pressure at the bottom of the dam is enormous and this high pressure is utilized to turn turbines producing large amounts of electric power.

The huge volume of water, distributed through a proper net work of canals is used for round the year irrigation purposes. The design problems in constructing the dams are, how the retaining wall should be designed and constructed. Since the pressure varies with depth, the wall should have maximum thickness at the base of the dam with deep foundation and the thickness decreases with the height from the bottom. The dam wall is generally buttressed on the valley side of the dam to fortify the wall against huge pressures due to water (Fig. 14.15).

It is important to remember that the thickness of the dam wall does not depend on the catchment area of the dam, that is, on the area of the lake formed by the dam. Some facts about Bhakra Dam are given below :

Height = 225·55 metres (740 feet)
Length of top = 518·16 metres (1,700 feet)
Width at top = 9·14 metres (30 feet)
Width at base = 402·33 metres (1,320 feet)

Examples : 1. A storage dam is filled up to a height of 250 metres. What is the pressure at the bottom ?

The pressure = hdg,

$= 250 \text{ m} \times 1000 \text{ kg/m}^3 \times 9 \cdot 8 \text{ m/sec}^2$,

$= 24 \cdot 5 \times 10^5 \text{ newtons/m}^2$.

2. The height of the water reservoir in a city is 40 metres. With what velocity will water come out of a tap on the ground floor? (Neglect friction due to the pipe).

When the water is at a height h, it has got potential energy equal to gh joules per kg. When it comes out of the tap, the whole of it is converted into kinetic energy, of which the value is $\frac{1}{2}v^2$ joules per kg. Hence,

$$v = \sqrt{2gh},$$
$$= (2 \times 9 \cdot 8 \text{ m/sec}^2 \times 40 \text{ m})^{\frac{1}{2}},$$
$$= 28 \text{ m/sec.}$$

FIG. 14.16 *The Bhakra Dam in Punjab, building in lower right shows the left hand power stations.*

Classroom Activities

1. Choose a rubber cork which will fit into the mouth of a hot water bottle. Bore a hole in it and pass a tube through the cork and then fit it into the hot water bottle. By a rubber tubing connect it with a long tube and pour water into the long tube till the bottle is full of water and the tube is partly filled up. Now put a wooden platform and ask a boy to stand on the platform. Pour more water, if necessary till the boy is supported on the platform Measure the height of the water column (Fig 14·17). Repeat the experiment with different boys and measure their relative weights.
2. With the above apparatus, take platforms of areas 8 cm^2, 12 cm^2 etc and find the weights which will raise the water to the same height in the vertical tube. Plot a graph between the weights and the area of the platforms.

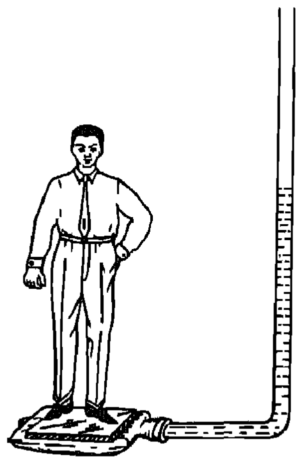

FIG. 14.17.

Questions

1. Consider a metallic cube of side 5 cm held in water. The top face of the cube is at a depth 10 cm below the surface. Calculate:
   (a) The total force on the upper surface and its direction.
   (b) The total force on the bottom surface and its direction.
   (c) The thrust exerted by the liquid on this body and its direction.
   (d) The tension on the string required to hold it if its mass is 1 kg.
   (e) The apparent loss in weight.
2. A tube of cross-section 1 sq cm is filled with 50 g of mercury. Then one end of the tube is filled with 10 g of water and the other end 8 g of oil of density 0·6. Find the positions of the three liquids in the U-tube.
3. In the experiment described in section 14·3, if one fills the glass cylinder with oil of density d instead of water what would be the height of oil level before the disc would start sinking ?
4. A bucket of water is suspended from a spring balance. Immerse a piece of metal in water suspended by means of a string. Does the reading on the spring balance change ? Explain.
5. Calculate the thrust on a diver's bell which is at a depth of 100 metres below the surface of sea. The area of the bell is 10 sq metre. (Density of sea water=1·03 g/c.c.).
6. The height of the Bhakra Dam is 225·55 metres. Calculate the water pressure at its bottom in newton/metre^2.

Further Reading

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CHAPTER XV

Archimedes' Principle and its Applications

15.1 Introduction

Those of us who have drawn water from a well have noticed that as soon as the bucket full of water comes above the water surface, it suddenly appears much heavier. Indeed, it is a common experience to most of us that all solid bodies appear lighter when held under water. We can now understand this apparent loss in weight by considering the following example (Problem 4 Chapter XIV).

Consider a metallic cube held in a fluid, like water as shown in figure 15.1. The cube and the fluid are at equilibrium, even though several forces are acting. Let us discuss these forces in detail.

The cube has six sides and the liquid exerts pressure on all of them. The magnitude of the force acting on any of the vertical sides (*e.g.* BCGF) is $a^2 (h+a/2) \rho g$, where ρ is the density of the fluid.

This force is horizontal, and normal to the face. Thus the force on opposite side ADHE balances against the force on BCGF and similarly the force on ABFE balances that against DCGH. This leaves us to

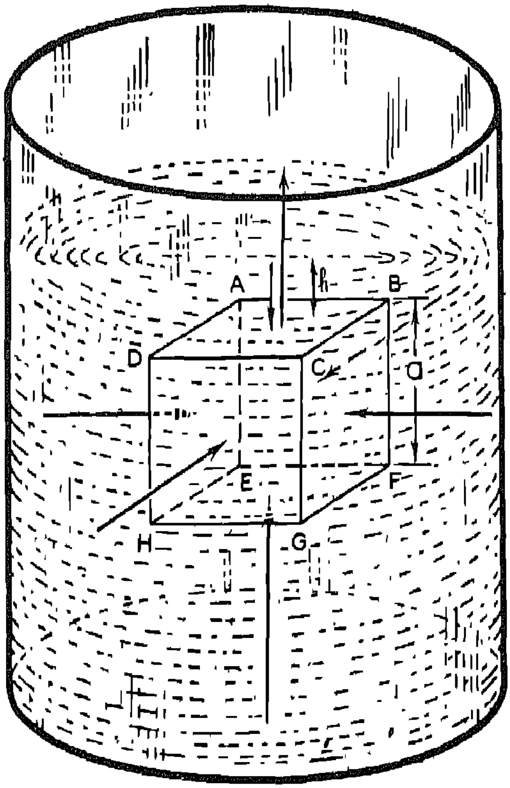


FIG. 15.1. *Forces acting on a cube inside the liquid.*

consider the forces on the horizontal faces ABCD and EFGH. The force on ABCD is downward and acts, on the centre of the face. The magnitude of the force is $a^2\rho gh$. The force on the bottom face EFGH is upwards, and acts at its centre. But its magnitude is $a^2\rho g(h+a)$. These two forces do not balance and there is a resultant *upward* force due to the liquid on the body

$=a^2\rho g(h+a)-a^2\rho gh=a^3\rho g=V\rho g,$

where V is the volume of the cube. Notice that this force depends only on the volume of the immersed body and the density of the liquid. It does not depend upon the depth h. This force is called the force due to buoyancy. It acts along a vertical line passing through the centre of the cube which is called the centre of buoyancy.

If there were no other forces on the body then the body would rise to the surface. However we have the force due to gravity which is acting downward and is of magnitude a^3dg, where d is the density of the body.

If $a^3dg>a^3\rho g$, or $d>\rho$, the body would tend to sink and the upward force due to the tension of the string would have to hold it. If $d=\rho$, the body would stay in position without being held by the string. If $d<\rho$ the body cannot be in equilibrium unless an extra downward force is applied to the body.

We have analysed above the simple case of a cube immersed in a liquid. One can consider the general case of a body of any shape immersed in a liquid and one finds that the force due to buoyancy is upward and is always $V\rho g$, where V is the volume of the body immersed in the liquid, or which is the same as the volume of the liquid displaced by the solid. This force acts along a vertical line through a point called the centre of buoyancy

The above analysis can be summarized thus : The cube, if hung freely in air, would have a weight, as revealed by the tension in the string, equal to dVg (The small buoyancy due to air has been neglected here. We shall learn about this in the next chapter). When this cube is immersed in the liquid the apparent weight, as shown by the tension of the string, is only $V(d-\rho)g$, and therefore the cube appears lighter by an amount which is equal to the upthrust of the liquid. The above analysis holds for a body of arbitrary shape.

This force acts along the vertical line passing through the centre of buoyancy.

Archimedes first discussed this and enunciated a principle which can be stated as follows :

"When a solid body is wholly or partially submerged in a fluid, it experiences an upward force which is equal to the weight of the fluid it displaces and which acts through the centre of buoyancy."

This is known as Archimedes' principle.

15.2 Centre of Buoyancy

In the simple case of a cube the centre of buoyancy coincides with the centre of the cube. If the cube is of uniform density then its centre of gravity is also at the centre of the cube. However in the general case, the centre of gravity of the body and the centre of buoyancy may not coincide.

The centre of buoyancy is defined in the following manner : If we imagine that the space occupied previously by the solid is replaced by the liquid then the centre of gravity of this body of the liquid is called the centre of buoyancy.

From the above diagram the significance of the centre of buoyancy should become clear. Consider the liquid enclosed by an imaginary surface as shown in the figure 15·2. The liquid within the surface is in equilibrium with the surroundings, that is, the forces acting on the surface due to the surroundings

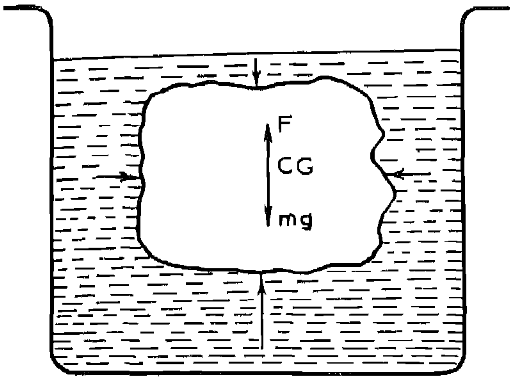


Fig. 15.2. *The net force on an imaginary surface containing a liquid is equal to the weight of liquid displaced and acts at its C G.*

just balance the weight of the liquid. Hence the resultant of all these forces passes through the centre of gravity of the liquid contained in the surface. This point is the centre of buoyancy defined above. In every case when a solid displaces a certain volume of liquid, the centre of buoyancy coincides with the centre of gravity of the liquid displaced.

In the simple case of a cube of uniform density completely immersed in a liquid the centre of gravity of the cube and the centre of buoyancy coincide with each other In fact this is true for solids of any shape, but of uniform density.

15.3 Verification of Archimedes' Principle

The principle of Archimedes can be verified by the following experiment

Take an overflow can (Fig 15·3) and fill it with water. Now weigh any piece of stone in air by tying it with a fine thread and hanging it from a spring balance. Take a catch bucket and weigh it. Now immerse the piece of stone in the overflow can and collect the water which overflows in the catch bucket. Find the weight of the piece of stone while immersed in water and determine the weight of the water collected in the catch bucket. You will find that loss in weight of the stone piece is equal to the weight of the water collected in the catch bucket. Repeat the experiment with different liquids.

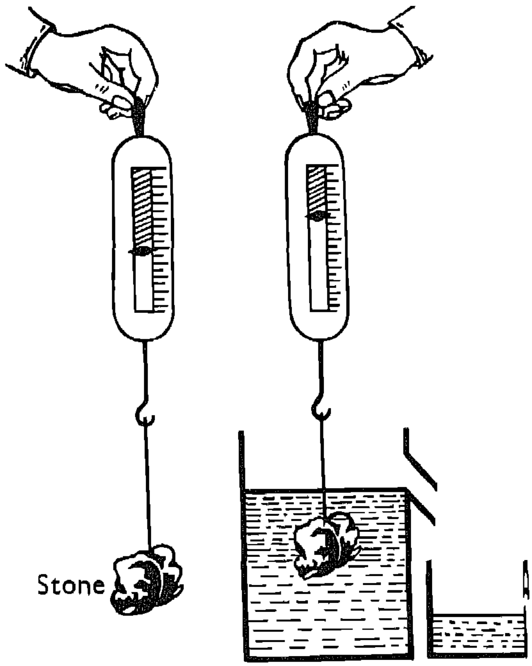


Fig 15 3 *The apparent loss of weight of the stone equals the weight of water displaced.*

15.4 Applications

Relative density of a solid

Let us determine the relative density of a glass stopper. Weigh the stopper in a balance. Let its weight be w_1. Now a beaker of water is put under the stopper on a wooden bridge so that it is completely submerged. The stopper loses weight Let its weight now be w_2.

Loss of weight of stopper $(w_1 - w_2)$. From Archimedes' principle, the weight of

an equal volume of water (w_1-w_2). Relative density of glass=Density of glass/Density of water=mass of glass/mass of an equal volume of water$=w_1/(w_1-w_2)$.

Relative density of cork

The above principle can be used for determining the relative density of substances which float in water, like cork also. The cork is weighed in air. Let the weight be w_1 (Fig 15.4). Then it is tied to a heavy sinker to keep the cork under water. First we put the sinker alone in water and the cork in air. The weight of this system is noted by means of a spring balance. Let it be w_2. Next the cork and sinker are both immersed in water and the weight w_3 is again noted. Then since the sinker is always under water, the loss of weight equal to (w_2-w_3) is that due to the liquid displaced by the cork.

Loss of weight of cork in water$=(w_2-w_3)$. Hence relative density of cork$=w_1/(w_2-w_3)$.

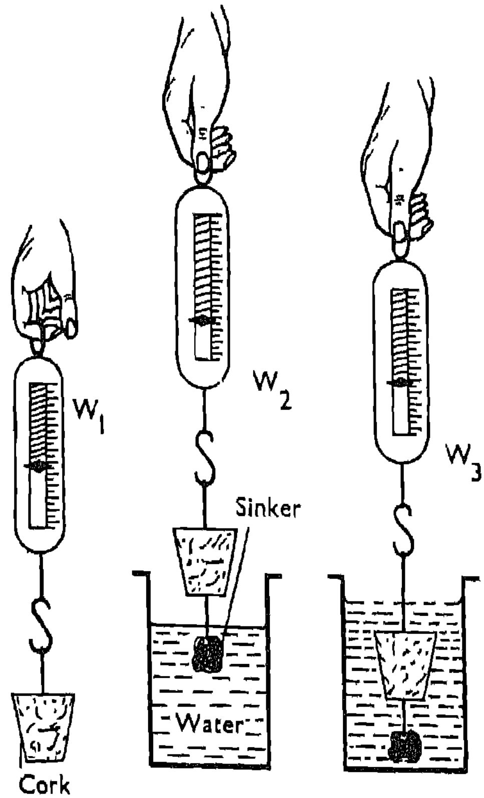


Fig. 15.4 *Finding the relative density of body lighter than water.*

Relative density of liquids

To determine the relative density of a liquid, a solid cylinder is first weighed in air (w_1) then in water (w_2), and then in the given liquid (w_3).

Loss of weight in water$=w_1-w_2$

Loss of weight in liquid$=w_1-w_3$.

By Archimedes' principle

Weight of liquid displaced$=w_1-w_3$.

Weight of water displaced$=w_1-w_2$,

and volume of liquid displaced=volume of water displaced. Hence relative density of liquid=weight of liquid/weight of an equal volume of water,

$$=\frac{w_1-w_3}{w_1-w_2}.$$

Examples : 1. A solid weighs 3·2 newtons in air and 2·4 newtons in water. What is its relative density ?

Loss in weight of the solid in water
$=(3\cdot2-2\cdot4)$ newtons,
$=0\cdot8$ newtons.

Hence the weight of water equal in volume to that of the solid is 0·8 newtons and the relative density of the substance

$$=\frac{3\cdot2 \text{ newtons}}{0\cdot8 \text{ newtons}}=4.$$

2. A solid weighs 3 newtons in air, 2·5 newtons in water and 2·6 newtons in a liquid. What is the relative density of the liquid?

Loss of weight of the solid in the water
$=(3-2\cdot5)$ newtons,
$=0\cdot5$ newtons.

Loss of weight of the solid in the liquid
$=0\cdot4$ newtons.

Hence relative density of the liquid

$$= \frac{0 \cdot 4 \text{ newtons}}{0 \cdot 5 \text{ newtons}},$$
$$= 0 \cdot 8.$$

Archimedes Story 287-212 B.C

Archimedes lived in the ancient city of Syracuse on the island of Sicily.

The story goes that Hiero, the king of Sicily, gave a lump of gold to his goldsmith and asked him to make a crown with it. When the crown was made, its weight was correct but the king suspected that the goldsmith had stolen some of the gold and substituted an equal weight of silver or some less precious metal in its place. Archimedes was asked to find out whether this was so, without destroying the crown. He was unable to solve the problem for some time One day while taking bath in a tub, he noticed that some water overflowed. In a flash of inspiration he got the principle, now known after him and he got so excited by this that he leapt out of the bath and rushed through the streets of Syracuse shouting "Eureka" (I have found it). He obtained a lump of pure gold and a lump of pure silver, each with a weight equal to the weight of the crown. Then he immersed the crown, the gold and the silver in turn in a vessel filled to the brim with water. Each time he collected the water which overflowed. The volumes were all found to be different. This proved that the crown had a density not equal to that of pure gold and hence was impure. He went on to calculate the percentage of gold and silver in the crown which the goldsmith had made. This was perhaps the first application of science in the detection of crime.

15.5 Principle of Floatation

Consider a cube of wood floating in water. You will observe that part of the wood is immersed under water From Archimedes' principle we know that there is an upward force acting on the wooden cube equal to the weight of liquid displaced by that part of the wood under water This upward force must balance the actual weight of the wooden block, which acts in the downward direction. For all bodies which float in a liquid, we have the general condition . The weight of the body is equal to the weight of the liquid displaced, and the centre of buoyancy and centre of gravity are in the same vertical line.

15.6 The Variable Immersion Hydrometer (The Test Tube Float).

A simple form of this instrument is a test tube float. Take a narrow uniform test tube and paste a graduated strip of paper on it parallel to its length.

It is advantageous if the closed end is flat (If it is hemispherical, measure 1/3 of the radius from the closed end and take this point as the zero from which the depth immersed is to be measured). Load the test tube with a suitable amount of lead-shot till it floats vertically in a jar of water. Observe the depth immersed (h_1) (Fig 15.5). Take out and wipe the tube and float it in a jar of the given liquid. Observe the depth immersed now (h_2). In each case see that the test tube floats vertically without touching the jar and remove air bubbles, if any, from its surface.

Let w be the weight of the hydrometer, 'a' its area of cross-section and 'd' the density of the given liquid. Since the weight of the float is equal to the weight of the displaced liquid, in the first case, $w=$ weight of water displaced$=ah_1\, d_w$ where d_w is the density of water. In the second case $w=$weight of liquid displaced$=ah_2\, d$. Since the weight of the float is constant in both the cases.

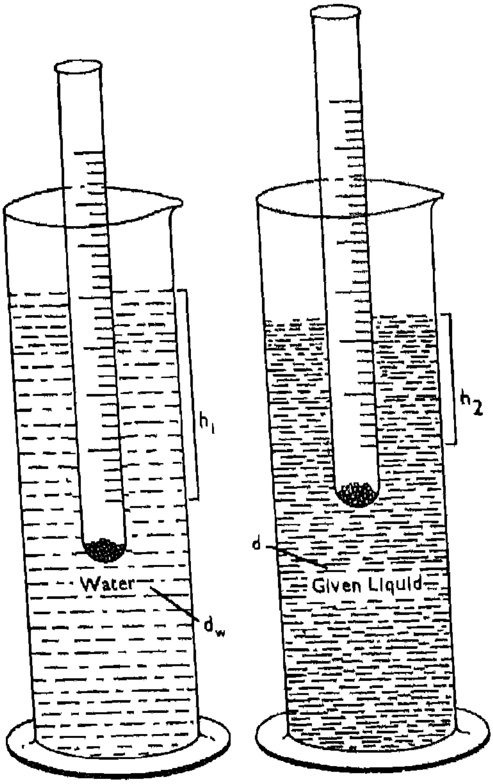


FIG. 15.5. *A loaded test tube, if properly calibrated, can be used to determine the relative density of liquid.*

$$a \times h_1 \times d_w = a \times h_2 \times d$$
$$\text{or } d/d_w = (\text{R. D.}) = h_1/h_2.$$

Since h_1/h_2 is a ratio, h_1 and h_2 may be either in inches or in centimetres.

Repeat the experiment with different amounts of lead-shots.

15.7 The Common Hydrometer

This is a variable immersion hydrometer graduated and ready for rapid use (Fig 15.6). It consists of a uniform, graduated stem with a weighed bulb below (for vertical floatation). When the instrument is floated in a liquid the reading on the stem corresponding to the level of the liquid gives its relative density. The battery hydrometer utilizes this principle. When the battery is fully charged the relative density of the acid in it is 1·280, and during discharge the relative density falls. The hydrometer is inside a glass container with a rubber bulb at the top and a rubber tubing at the bottom. When the tube is inserted in the battery and the top rubber squeezed and released, acid enters the container tube and the hydrometer floats in the acid. Explain why it is advantageous to have the stem of the hydrometer to be of a small area of cross section.

15.8 Icebergs

Ice has a density slightly less than that of water. Sometimes large masses of ice float from the northern seas into the open sea, especially near Newfoundland. Most of the ice is, of course, immersed in water. From a distance they look like floating mountains and are known as icebergs (Fig 15.7). A large percentage of icebergs will be under water.

FIG. 15.6. *A common hydrometer.*

Example:- If the relative density of sea water is 1·03 and that of ice 0·917 and the volume of an iceberg above the sea level is 100 cubic metres, calculate the total volume of the iceberg.

If the total volume of the iceberg is V-cubic metres, its mass is 917 V kg and the volume of sea water displaced (0·917/1·03)V cubic metres.

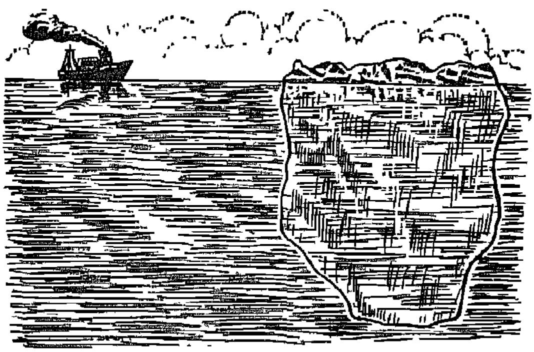


FIG. 15 7. *Ice berg.*

Hence,

$$V-\frac{0\cdot917}{1\cdot03}V=100 \text{ cubic metres,}$$

$$\text{or } V=\frac{1\cdot03}{0\cdot113} \text{ cubic metres,}$$

$$=911\ 5 \text{ cubic metres.}$$

15.9 Ships

The ship, displaces a large amount of water because of its shape (Fig 15.8) Though the ship is made of steel, the upward thrust due to the displaced water is so large that it equals the weight of the ship and so the ship floats.

15.10 Submarines

Submarines are ships that can submerge into the water and travel underneath water (Fig 15.9). In the hull of a submarine there are large tanks known as buoyancy tanks. When a submarine is to be submerged water is allowed to enter these tanks The water runs in and adds to the total weight of the submarine and it begins to

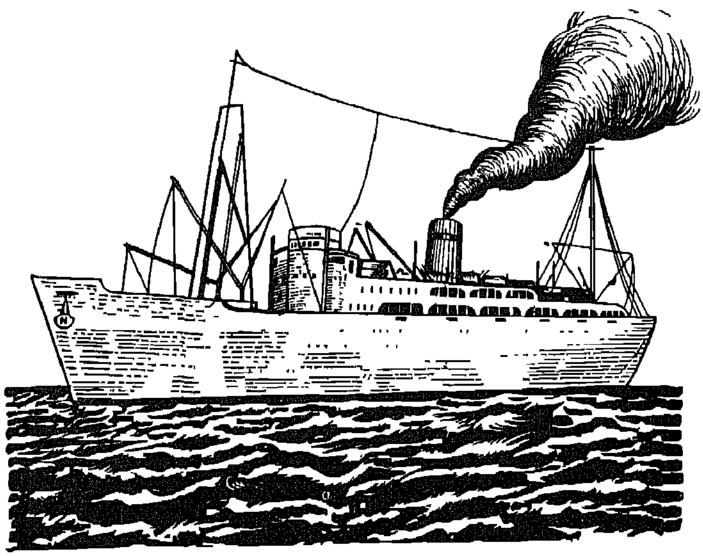
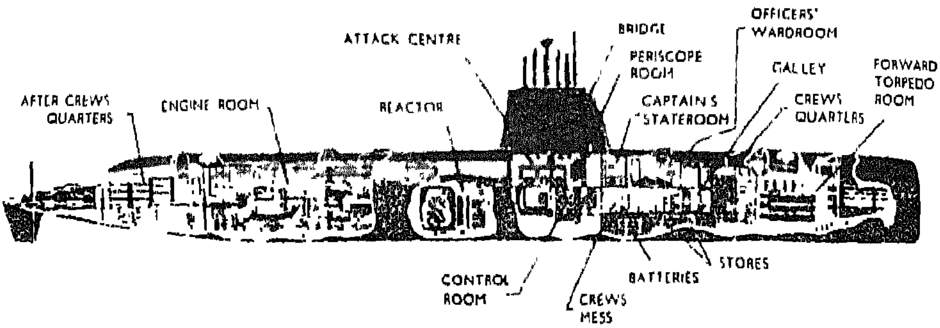


FIG. 15.8. *A floating ship.*



The Atomic Submarine "Nautilus"

FIG. 15.9. *A nuclear submarine.*

sink. At every instant the weight of water displaced is equal to the total weight of the submarine (including the weight of water in its tanks). When the submarine dives, a small reserve of buoyancy is usually left, and the motion of the submarine causes a downward motion on account of the action of the water on horizontal rudders known as hydroplanes. Modern submarines can submerge within 30 seconds of getting the alarm In order to make it rise to the surface, compressed air is pumped into the tanks and it forces the water out. The weight of the submarine decreases while the upthrust on it remains the same and therefore the submarine rises.

It is possible to keep it at a certain level without using trimmers or engines. This is done by pumping water into or out of internal tanks. The weight of water in these tanks is adjusted until the submarine is horizontal and stationary at the desired depth. Since all the valves are now closed no water can enter or leave the tanks and the submarine may remain stationary for a considerable time.

15.11 Stability of Floating Bodies

We have discussed in section 15.5 the principle of floatation of bodies. We will now consider the condition so that a floating object

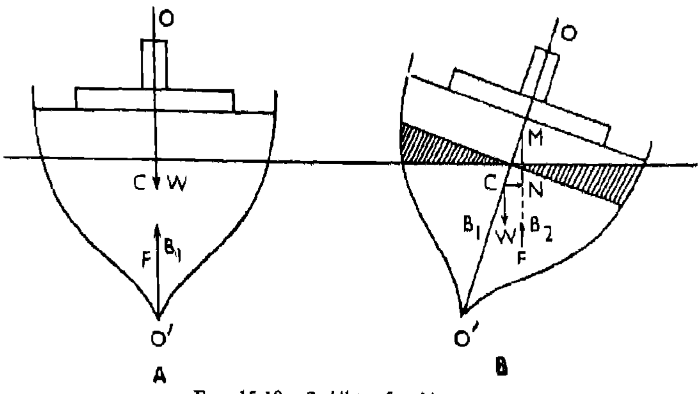


FIG. 15.10 *Stability of a ship.*

like a ship is in stable equilibrium When the ship is floating in the vertical position as shown in Fig. 15 10 A, the centre of buoyancy B_1, *i.e.*, the centre of gravity of the displaced water is just below the centre of gravity of the ship so that the force due to gravity and the force due to the upthrust are in the same vertical line. When the ship rolls, the shape of the water displaced by it changes and the centre of buoyancy B_2 is in a different position The point M, through which the vertical line through B_2 cuts the central line OO′ of the ship is known as the *metacentre* For stability this point must always be above C as shown in Fig 15.10 B. Then the force due to gravity and the force due to the upthrust constitute an anti-clockwise couple which tends to bring the ship back to the untilted position. If the point M is below C, the forces will form a counter-clockwise couple which will tend to tilt the ship further. Hence the ships are designed in such a way that their centre of gravity is as low as possible. When the ships are not carrying any cargo, it is customary to put some ballast to lower their centre of gravity.

Classroom Activities

1. Take water in a graduated jar and put a rectangular block of paraffin wax in it. Mark the position of the lower surface of the piece of wax. Now pour some kerosine oil over the water Does the level of the lower surface of the wax piece change ? Can you explain it ?
2. Take a fresh egg and put it in water. Does it sink or float ? Now put it in salt solution. Explain what you observe.
3. Take some water in a beaker and weigh it by a physical balance. By a spring balance, suspend a piece of light wood so that it is floating in the beaker. The wood piece loses all its weight. What does the physical balance show ?
4. Take a thin sheet of lead. When dropped in water, it sinks. Now bend the sheet in the form of a boat and put it lightly on water. Does it still sink ? How do you explain the difference ?
5. Put some iron filings or pieces of stone in a wide-mouthed bottle and pour some melting wax over it so that when the wax solidifies the bottle will float in an upright position in water (Fig. 15.11). Now bore two holes in a rubber stopper and fit two glass tubes into the bottle (as shown in the figure). Fill the bottle with water and let it sink in water contained in larger vessel. Now blow some air into the bottle to expel the water and see that the bottle rises like a submarine. By withdrawing air you can make it sink.

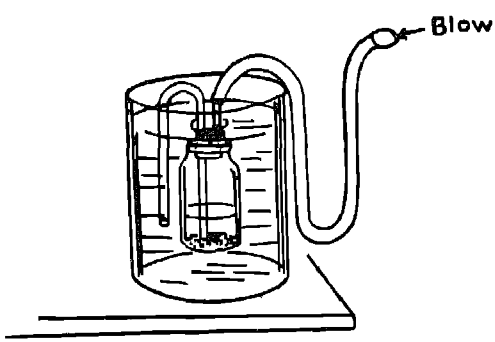


FIG. 15 11

Questions

1. A piece of gold copper alloy weighs 5 kgf, when suspended from a spring balance and submerged in water it weighs 4·5 kgf. Calculate the amount of copper in the alloy if the density of gold is $19 \cdot 3 \times 10^3$ kg/m^3 and of copper $8 \cdot 9 \times 10^3$ kg/m^3.
2. Explain the action of the toy cartesian diver.
3. A man weighing 75 kgf is floating in a river sitting on a circular log of wood. If the length of the log is 5 metres, what must the radius of its section be in order that the man will be just supported ? Density of wood $= 0 \cdot 6 \times 10^3$ kg/m^3.
4. Why does a ship sailing from fresh water into the sea rise ?
5. A portion of the top of an iceberg melts under the heat of the sun. Will the iceberg sink or rise ? Explain.
6. A conical vessel has the area of its bottom equal to 40 cm^2 and of the mouth 10 cm^2. If the vertical height of the vessel is 10 cm, calculate the total force in newtons on the bottom of the vessel when it is full of water. Calculate the total weight of the water in the vessel and explain the difference in the two values.
7. A cube of wood of density $0 \cdot 7 \times 10^3$ kg/m^3 has each side equal to 10 cm. It is floating in a vessel containing water. If oil of density $0 \cdot 6 \times 10^3$ kg/m^3 is poured into the vessel, will the cube rise or sink ? Explain. What will be the depth of oil when the upper surface of the oil and the wood are on a level ?
8. An iron sphere of radius 1 cm is dropped into a vessel containing mercury and water. What volume of the sphere will be inside mercury ? Density of mercury $= 13 \cdot 6 \times 10^3$ kg/m^3 and that of iron $7 \cdot 8 \times 10^3$ kg/m^3.
9. A piece of a solid weighs 50 gf in air and 40 gf in water. How much will it weigh in a liquid of density $1 \cdot 20 \times 10^3$ kg/m^3.
10. A piece of a solid weighs 100 gf in air and 87 gf in water. If it weighs 90 gf in a liquid, calculate the density of the liquid.
11. If the density of ice is 890 kg/m^3 and that of sea water 1030 kg/m^3, calculate the fraction of an iceberg that will be above the sea-surface.
12. A boy is swimming in a river with the help of an inflated car tube. If the weight of the boy is 25 kgf (density $1 \cdot 05 \times 10^3$ kg/m^3) and that of the tube 1 kgf, calculate the volume of the tube so that the boy will be able to swim with 1/5 of his body outside the water.
13. A 15 cm long test tube weighed with lead shots sinks to a depth of 11 cm in water and a depth of 14·2 cm in alcohol. Calculate the density of alcohol.
14. A spherical block of wood of relative density 0·7 and of volume 10,000 cm^3 floats in water. Calculate the vertical force in newtons necessary to keep it completely immersed.
15. A common hydrometer consists of a bulb and a thin stem of uniform cross-section. The stem is graduated in millimetres. When floated in water the reading is 80 mm and when floated in paraffin of relative density 0·8, the reading is 180 mm. When floated in two other liquids the readings are 40 mm and 140 mm. Calculate the relative densities of the two liquids.
16. When a man weighing 80 kgf steps into a boat, the boat sinks by 4 cm. Calculate the area of cross-section of the boat at the water level.

17. 75 grams of wood of relative density 0·6 are submerged in water by attaching to it pieces of iron of relative density 7·9 How much metal will be required so that the wood is just under water?

Further Reading

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CHAPTER | XVI

Atmospheric Pressure and Boyle's Law

16.1 Introduction

In the previous chapters we have discussed the forces acting in liquids. In particular we had seen that the liquids exert pressure because of their weight. In this chapter we shall see that the gases, which are also fluids exert pressure. We know that we are surrounded by a gaseous envelope which we call the atmosphere. This envelope extends up to a height of several hundred kilometres. Thus we are actually living at the bottom of an ocean of air.

As we go up, say in a balloon from the ground, the air gradually becomes less dense, and the decrease in density becomes quite noticeable at about 3 kilometres from the ground. In fact higher up it becomes more difficult to breathe. When Tenzing and party went to the peak of Everest they had to carry oxygen cylinders with them. In fact, the air as we go higher up is so rarified that only 1% of the total air is above 32 kilometres, the rest 99% being contained below this height. In addition to this effect of rarefaction there is a change in temperature as we go up which we shall consider later. This envelope of air round the earth, for most practical purposes, may be regarded to extend to a height of 1,200 kilometres.

16.2 Torricelli's Experiment to Measure Atmospheric Pressure

An experiment to determine air pressure was first performed by Torricelli. Take a long narrow glass tube with one end open and about one metre long and 1 cm in diameter. Fill this tube completely with mercury. See that there are no air bubbles sticking to the sides of the tube. Closing the open end of the tube with your thumb, invert it into a tank of mercury (as shown in Fig. 16.1).

Remove your thumb under the mercury. Now you will notice that mercury in the tube flows into the tank until the height of mercury column stands at about 76 cm from the free surface of the mercury in the tank. We notice that there is an empty space above the mercury column. This is

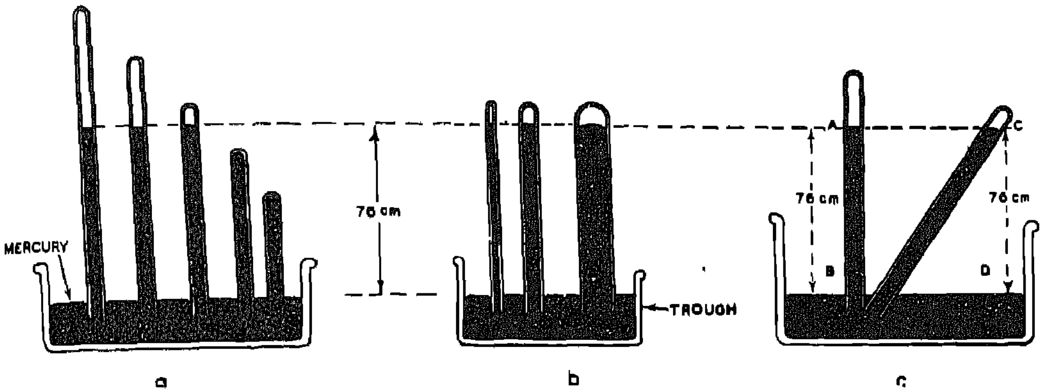


FIG. 16.1 *The Torricellian or mercurial barometer*
The height of mercury column does not depend on :
(a) *The length of the tube*
(b) *Cross-sectional area of the tube*
(c) *Inclination of the tube.*

called Torricellian vacuum after Torricelli who first performed this experiment **

Even if you tilt the long tube you will notice that the vertical height of the mercury column in the tube is always 76 cm from the free surface of the mercury This is called atmospheric pressure. This means that the column is being supported by some pressure, which must be acting on the surface of the mercury This pressure must be due to the air above the mercury surface We can now measure the atmospheric pressure from this experiment. Consider two points P_1, and P_2 in the same horizontal level, the point P_2 being just on the surface The pressure at these two points must be the same The pressure at $P_1 = \rho gh$. where ρ=density of mercury and h the height of the column of mercury. At P_2 the pressure is due to the atmosphere and therefore atmospheric pressure=ρgh.

Experimentally h is about 76 cm at sea level This is a simple barometer, which measures the atmospheric pressure. To measure the pressure the height of the mercury columns above the free surface should be measured. But when the column of mercury rises or falls due to changes in pressure the level of the free surface will also change. Thus our reference level from where we have to measure continuously changes. This is not convenient for measuring the height of the column, because if we have markings on the tube, the zero point adjusted for one day would be above or below the level of free surface for when the pressure changes mercury will fall from the tube into the tank, thus changing the level of mercury in the tank. Fortin designed a barometer which is commonly used in laboratories and which is described below.

**Aristotle held the view that vacuum cannot exist and for nearly two thousand years philosophers had accepted this view without performing any experiment to test it. Even after Torricelli's experiment some people held that there might be some air above the mercury column in the tube. Torricelli had succeeded Galileo as professor of mathematics at the Academia in Florence. When Pascal in France came to know about this experiment, he took such an apparatus to the top of a church in Paris and observed that the height of the mercury column was a little less than on the ground which was against the views of Aristotle Decisive support to the view that above the mercury column in the tube there was vacuum was provided when such an apparatus was taken to the top of the mountain Puy de Dome in Southern France where the column was observed to fall by 3 inches (7·6 cm).

16.3 Fortin's Barometer

The Fortin's barometer is shown schematically in Fig. 16.2. This is based on the principle of the Torricellian experiment. It essentially consists of a long narrow glass tube, the open end dipping into a reservoir of mercury. The reservoir is made of flexible leather bottom, so that the level of free surface of mercury can be raised or lowered. A vertical scale is fixed behind the glass tube and there is a movable vernier attached at the top end of the mercury column. With the help of this, the level of mercury can be accurately read. Before taking a reading the lower level is adjusted by means of a screw so that the level stands at the zero of the vertical scale.

This is achieved in practice like this. An ivory tip projecting downward is provided in the reservoir. The tip exactly coincides with the zero of the vertical scale. When adjusting the zero, the level of mercury in the reservoir is so adjusted that its surface exactly coincides with the tip. This is verified by the coincidence of the tip and its image in mercury. This assures that the mercury level is at the zero of the vertical scale. In this case the single reading of the top level of the column of mercury gives directly the atmospheric pressure. For convenience the scale is graduated both in inches and centimetres.

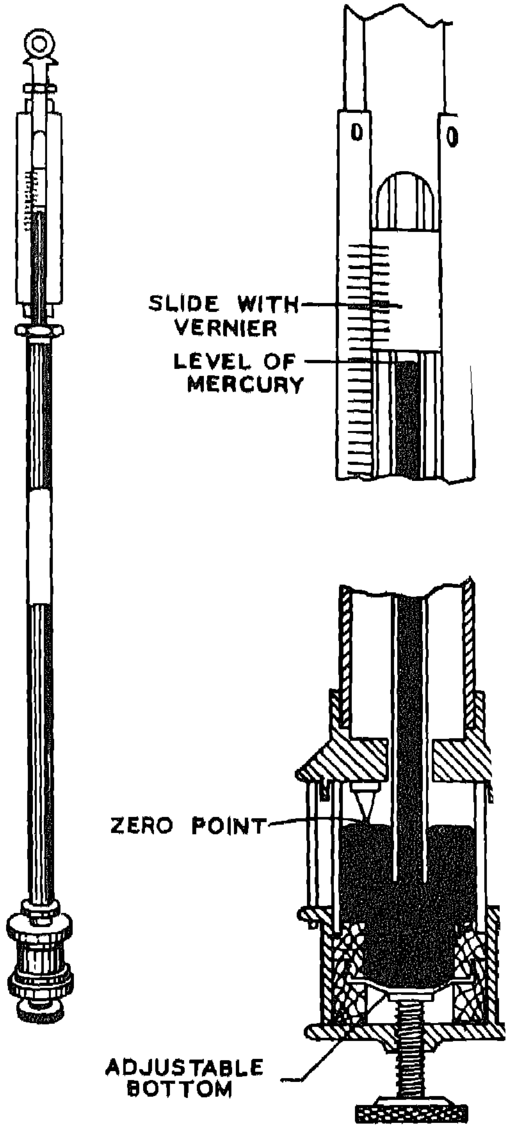
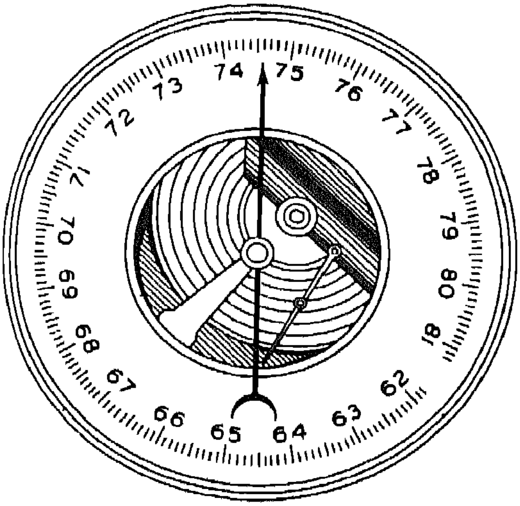


FIG. 16.2. *A mercury barometer for accurate measurements of air pressure. The flexible bottom can be raised or lowered to adjust the level of mercury in the cistern, the vernier scale makes possible accurate reading.*

16.4 Aneroid Barometer

As the very description of the Fortin's barometer shows, the apparatus is bulky and cumbersome and cannot be carried from place to place. It is very accurate for laboratory work. A more convenient type of barometer, called "Aneroid Barometer", is more widely used in aeroplanes etc. As its name indicates it does not contain any liquid It is shown schematically in Fig. 16.3. It consists of a partially evacuated metal box

the centre of whose top moves in or out as the atmospheric pressure changes and this movement is conveyed to a pointer by a series of levers which magnify this movement. By a pervious comparison with a Fortin's barometer the pointer can be made to read the atmospheric pressure directly.



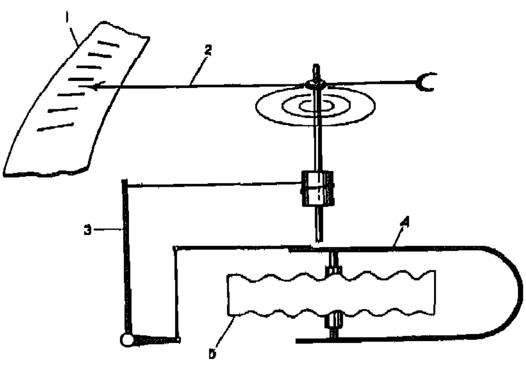


FIG 16.3 (a). *An Aneroid barometer*
(b). *The cut-section of Aneroid barometer.*

1 *Scale; 2. Pointer; 3. Transmission mechanism; 4. Spring, 5. Box.*

16.5 Units of Atmospheric Pressure

We can express atmospheric pressure in terms of the height of the column of mercury and we say that the atmospheric pressure is 76 cm of mercury. Had we used any other liquid, say water, the height of the water column that can be supported by the atmospheric pressure will be equal to ($76 \times 13 \cdot 6$) cm. This is because water is $13 \cdot 6$ times lighter than mercury.

When we say that the normal atmospheric pressure is 76 cm of mercury, we mean that the pressure exerted by the atmosphere is the same as that exerted by a column of mercury 76 cm in height, *i.e.*, the pressure per m^2 is equal to the weight of $0 \cdot 76$ m^3 of mercury. This is W, where

$$W = 0{\cdot}76\ m^3 \times (13{\cdot}6 \times 1000) \times \frac{kg}{m^3} \times \frac{9{\cdot}8 \text{ newtons}}{kg}$$
$$= 1 \cdot 013 \times 10^5 \text{ newtons}$$

Hence the normal atmospheric pressure P_0 is $1 \cdot 013 \times 10^5$ $\frac{\text{newtons}}{m^2}$.

In meteorological data another unit called a millibar is used. A bar is 10^5 newtons/m^2, and therefore millibar is 10^2 newtons/m^2. You will notice that the atmospheric pressure is approximately equal to one bar.

In vacuum technology the unit Torr named after Torricelli is commonly used. A Torr is the pressure due to a column of height one mm of mercury.

16.6 Manometer

We are in a position to understand the working of a manometer used to measure the pressure of a gas in a vessel. The manometer is shown in Fig 16.4 It essentially consists of a U-shaped glass tube bent at one end. The bent end of this tube is

connected to the vessel in which the pressure is to be measured. The other end is open to the atmosphere. Now we can discuss two cases: (i) when the pressure to be measured is smaller than the atmospheric pressure, (ii) when the pressure to be measured is more than atmospheric pressure. In the first case, the manometeric liquid, *i.e.*, liquid used in the manometer will be at a higher level on the receiver side. Let the difference in the levels in the two limbs be 'h' cm as measured on a scale. The pressure at A and B which are on the same horizontal line are equal. The pressure at A is the atmospheric pressure P_0. Hence pressure at B is also P_0. The pressure at C which is also the pressure inside the receiver is 'P' say. Then the pressure at B is given by $P+hdg$. This in turn is equal to P_0, the atmospheric pressure. Therefore,

$$P_0 = P + hdg,$$

and

$$P = P_0 - hdg.$$

(2) When the pressure inside the receiver is greater than P_0, the liquid levels will be as shown in the figure 16.4(b). Now the liquid in the limb open to the atmosphere is at a higher level. Following the same line of argument, we can show that

$$P = P_0 + hdg.$$

In general, we can write

$$P = P_0 \pm hdg,$$

where 'P' is the pressure to be measured, P_0 is the atmospheric pressure, 'h' is the difference in levels of the liquid in the two

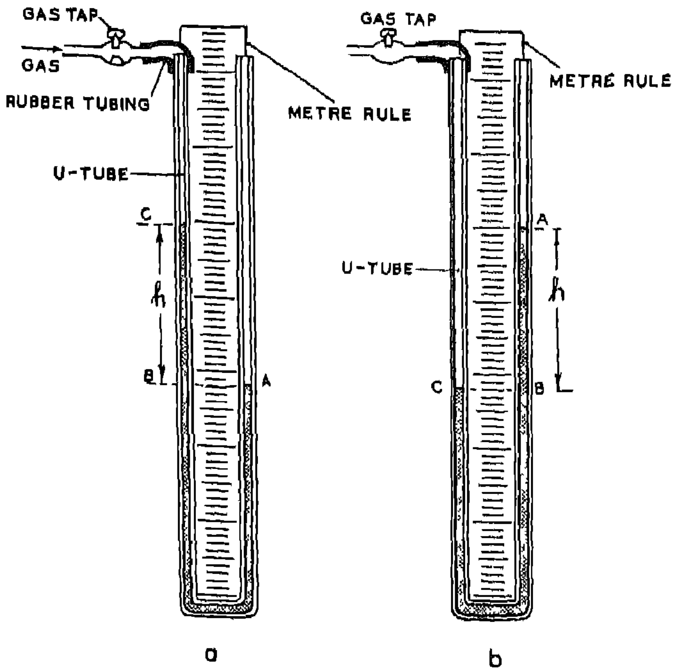


FIG 16.4. *An open tube liquid manometer*
The difference in the level of liquid in the two limbs of the manometer, when
(a) *The pressure to be measured is less than the atmospheric pressure*
(b) *The pressure to be measured is greater than atmospheric pressure.*

limbs, 'd' is the density of the manometric liquid and 'g' is acceleration due to gravity. If 'h' is in metres, d in kg/m^3 and g is m/sec^2, the pressure difference is in newtons/m^2.

It may be useful to remember that for measuring small differences in pressure lighter liquids like water and oil should be preferred as manometric substances. If the pressure differences are large, mercury may be employed as the manometeric liquid because of its high density.

16.7 Bourdon Gauge

The difference between the absolute pressure at a point within the fluid and the atmospheric pressure is called the 'Gauge Pressure'.

Bourdon gauge is a type of pressure gauge used in many technical applications. This gauge contains a sealed spiral of flat metal tubing (1) as shown in Fig. 16.5.

The inside of the tubing is filled with the fluid whose pressure is being measured or with air in communication with the fluid and hence at the same pressure. The outside is exposed to the atmosphere.

The elastic properties of such a tube are such that when the pressure inside is increased, the tube tends to straighten out. This straightening is communicated to a pointer. The position of the pointer depends only on the difference in pressure inside the tube and the atmospheric pressure outside. This gauge can be constructed to cover various pressure ranges either above or below atmospheric pressure.

16.8 Archimedes' Principle and Buoyancy due to Air

We have seen that the atmosphere is a fluid and so any body immersed in it must experience an upward thrust as given by Archimedes' principle. The upward thrust is equal to the weight of the fluid displaced, and it is equal to $V\rho g$, where ρ is the density of air. The density of air, which is about a thousand times smaller than water is $1 \cdot 293$ kg/m^3 and hence the buoyancy due to air is not so noticeable. If one performs

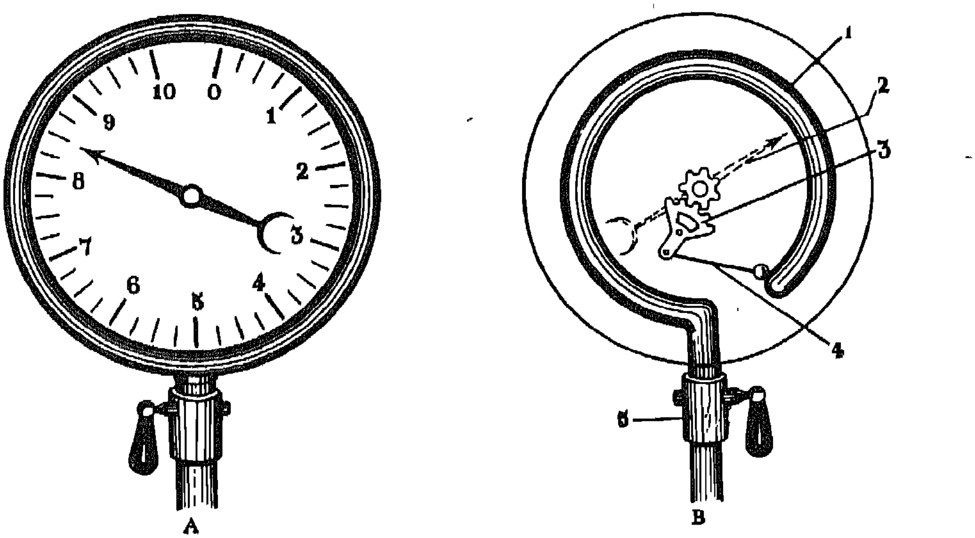


FIG. 16.5. *Bourdon gauge (metal manometer). The internal arrangement is shown in B (back view)* 1. *Metal tube;* 2. *Pointer,* 3. *Ratchet,* 4. *Metal strip;* 5. *Tap.*

accurate experiments, it is possible to show that the weight of a body in vacuum is more than in air.

A balloon filled with a gas lighter than air, like Hydrogen or Helium, would rise because the upward thrust is more than its weight (including the weight of the rubber balloon). As the balloon rises up, the density of the surrounding air is decreasing so that the upward thrust on the balloon decreases. With the gradual fall in the outside pressure, the balloon will have a tendency to expand, and hence would displace a greater volume. The balloon would stop rising at a height where the weight of the displaced air is equal to the total weight of the balloon. Such balloons are used in meteorological and scientific investigations of the upper atmosphere.

16.9 Weather Forecastings

Let us now learn something about the general characteristics of earth's gaseous envelope which determine the weather and climate of different regions. This branch of science is called Meteorology. Our earth is heated by the sun strongly at the equator and feebly at the poles. The moisture in the atmosphere is concentrated over the great ocean basins. The atmosphere, distributes this heat and moisture so that large areas of the land surface become habitable. In general the atmosphere fails miserably in this task in desert regions, on mountain summits and in extreme northern and southern latitudes.

The earth is heated by solar radiation and in turn heats the atmosphere surrounding it. Our atmosphere acts as a trap, admitting the energy of the sun light freely but hindering escape of energy emitted by the earth.

Let us now consider the movements of the atmosphere that distribute heat and moisture over the earth's surface. These movements are governed by two simple facts. (1) Air on heating expands and is lighter than cool air and hence it is displaced by the surrounding cooler, and therefore denser, air and rises above. (2) Air moves horizontally from the high pressure regions to the low pressure ones. Air currents produced in this manner are called convection currents. The earth as a whole is heated strongly in the equatorial belt but less strongly on either side. Rotation of the earth tends to pile up the atmosphere, while high temperature tends to produce a region of low pressure.

In the region along the equator strongly heated air rises and overflows at high altitudes to the North and South, being cooled as it leaves the equator. It is deflected from its poleward course by earth's rotation. Much of this air descends to the earth surface in the latitude about 30° north and south of the equator. The descending air forms belts of high pressure in these latitudes. From the high pressure belts, the air, in part returns along the surface to the equator, in part to the northward and southward polar belts of low pressure. Air currents returning to the equator from 30° latitudes (Horse latitude) form the trade winds. Deflected by the earth's rotation, the trade winds of the northern hemisphere blow from north-east to south-west, those of the southern hemisphere from south-east to north-west. The circulation of air is shown in Fig. 16.6.

The production of rain requires that large masses of humid air be cooled to a temperature below the point at which air is saturated. We shall expect therefore abundant rain wherever air laiden with moisture moves over land where the mountain ranges lie across the path. The equatorial belt with its rapid evaporation and strong rising

air currents provides an ideal situation for super-abundant rain. The land lying in

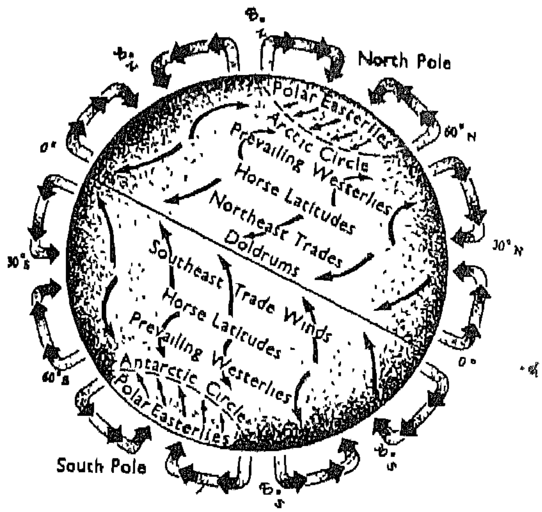


FIG. 16.6. *The arrows in figure show the atmospheric circulation.*

this belt forms steaming jungles as in Africa, South America and East Indies. At 30° south and north latitudes, the region is calm but the climate is not humid. Air in these belts moves downward becoming compressed and heated, hence less and less saturated with water vapour. The climate is very nearly dry. The outer portions of these belts together with the adjacent horse latitudes are the regions of the world's great deserts—the Sahara, the deserts of South Africa and the dry interior of Australia.

In India the Indo-Gangetic plain is strongly heated during May and June and moisture laden winds blow from the Arabian sea and the Bay of Bengal which on striking the western Ghats and the High Himalayan mountains produce abundant rain during the rainy season.

16.10 Applications.

We will describe a few applications in whose working on the atmospheric pressure plays an important role.

Lift-Pump

The utilization of atmospheric pressure for practical purpose is best illustrated in the common water pump (boring well) or lift pump. Its parts are illustrated in Fig. 16.7.

B is a barrel or cylinder in which is arranged an air tight piston P with a handle H. To the bottom of the barrel is attached a long tube AC which dips into the well. There is a valve V_1 at the point where AC communicates with the barrel B and there is another valve V_2 in the piston both open upward. S is a sprout through which water comes out.

1st *Step* (*Upward Stroke*)

V_1 and V_2 will be closing the respective spacings. In the upstroke of the piston, vacuum is created within the barrel and consequently V_2 will be closed. The atmospheric pressure acting on the water surface in the well forces the water into the barrel opening V_1 on its way.

2*nd Step* (*Downward stroke*)

In the downward stroke of the piston, V_1 will be closed, and V_2 opened so that much of the water below the piston goes upward. In the next stroke this water is prevented from coming down by the closure of V_2.

Thus in successive strokes, there is a constant accumulation of water in the barrel above the piston and as soon as its level reaches that of the sprout, water flows out in the upward stroke. The flow of water will not, however, be continuous and will only take place in the upward strokes The height to

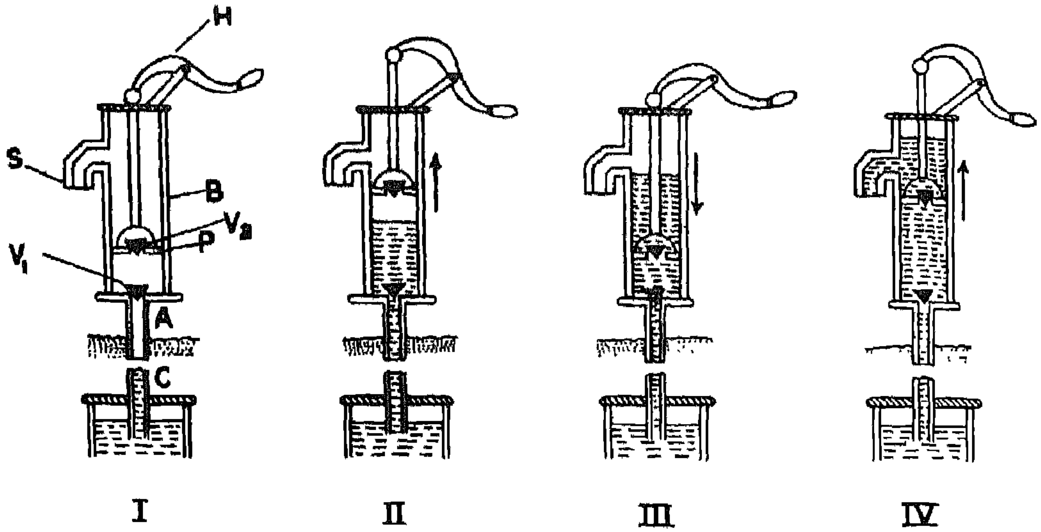


Fig. 16.7. *A schematic diagram of a lift-pump.*

which water can be lifted up with this pump is about 10·3 metres above which height this does not work. Why ?

16.11 Force Pump

It is an improvement over the lift pump in which there is no continuous flow of water. Here the water flow can be made continuous and it can be used for purposes like fire extinguishing of large buildings.

It consists of a barrel B (Fig. 16.8) in which works an air tight piston P having a handle H. It has a long narrow tube CE attached to it at its bottom. V_1 is a valve at the site where CE communicates with the barrel. There is no valve in the piston P and there is a side tube T connecting the barrel B to an air chamber D. At the site of the joint of T and D is another valve opening upwards. D has an air tight stopper through which F, a long spray tube, passes. There is air entrapped in the air chamber D. In the upstrokes, the valve V_1 opens and water is sucked into the barrel B. In the down stroke V_1 is closed and V_2

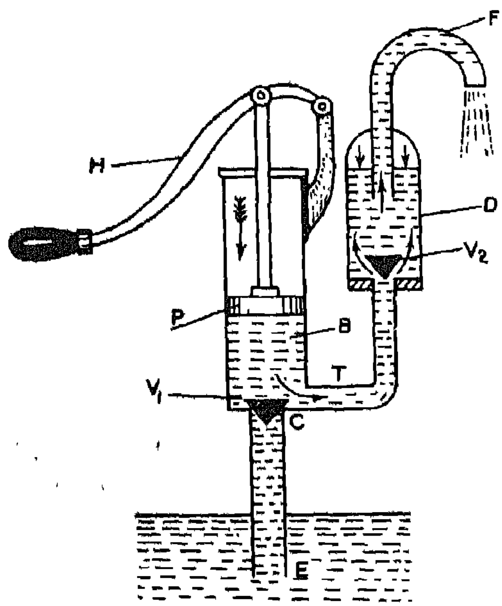


Fig. 16.8. *The force pump.*

In the down stroke V_1 is closed and V_2 opens the water rushing into the air chamber. In the next upstroke V_2 closes, V_1 opens and the process repeats until after a time, regular working of the pump fills the air chamber to a certain optimum level and due to the compressed entrapped air within, the water is forced out of the tube F continuously.

As in the case of the lift pump the length of the limb cannot exceed 10·3 metres. This is because atmospheric pressure is sufficient to raise the water through a height of only 10·3 metres and it cannot support a greater height. But the use of the pressure chamber in the force pump enables us to pump the water upwards to any desired height depending upon the extent of compression of the air inside. Such a device to further increase the height to which water is raised is absent in the lift pump.

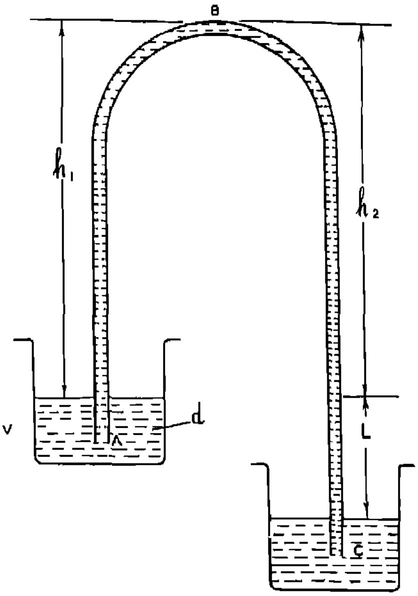


FIG. 16.9. *A siphon transferring liquid from the higher vessel to the lower one.*

16.12 The Siphon

A siphon is a bent tube used for emptying vessels when it is inconvenient to tilt them.

Suppose V is a vessel containing a liquid of density d (Fig. 16·9). Let ABC be a siphon filled with the same liquid. Suppose the height of B above the liquid is h_1 that of B above C is h_2. Imagine the end C to be closed with a cork. We shall find the difference in pressure on the two sides of the cork.

The pressure on the surface of the liquid is atmospheric pressure equal to P.

Pressure at $B = P - h_1dg$.

Pressure at C inside the tube $= P - h_1dg + h_2dg$.

If $h_2 > h_1$ there is a greater pressure outwards from the tube on the cork than inwards. There would then be a tendency for the cork to be thrust outwards and for the liquids to flow.

Hence if a siphon is to work, the lower limb of the tube must be below the level of the liquid in the reservoir

The pressure at B is $P - h_1dg$. This cannot be less than zero. Hence $h_1 = P/dg$, so that the greatest height of a siphon is the height of a barometer filled with that fluid. Thus if water barometer stands at 10·3 metres, for syphoning water, the point B should never be at a height greater than 10·3 metres.

One can imagine that the whole set up is enclosed in a chamber which may be maintained at any pressure P. In that case P in the above equation will represent the pressure in the chamber.

16.13 Lavatory Bell Flush

In this device a tank will be filling up with water upto a certain predeterminate level and stops filling then. When wanted, the tank can be drained off into the septic

latrine by pulling a chain H. Through the inlet the water flows in and the level of water will be rising. There is a floating ball B which is attached to a fulcrum fixed in the wall of the tank. The rod joining the float to the fulcrum has a rubber stopper in the middle. As the level of the water in the tank rises, the float which originally hangs down is raised and at a certain level the stopper attached to the float closes the mouth of the inlet. Thus the level of water is prevented from rising and the tank does not overflow.

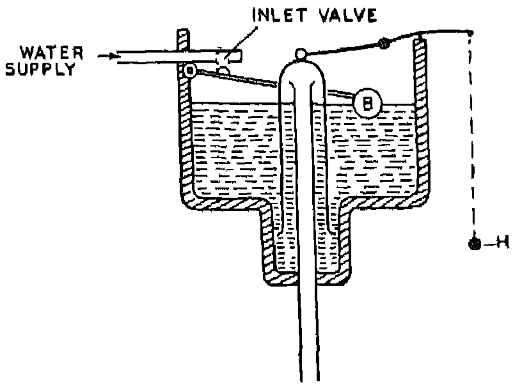


Fig. 16.10 *A bell type lavatory flush tank.*

To drain away the tank when required, there is a bell within which is the outlet whose mouth is just above the constant level of water. When the chain is pulled down, the arrangement raises the bell up. As the bell is dropped down, water inside it cannot escape out quickly through the gap between the rim of the bell and the walls of the tank. Hence the level of water inside rises above the level of the tube in the centre and syphoning through the outlet takes place.

In another type of flushing system shown in Fig. 16.11, there is an inverted U-bend passing through the bottom of the tank and connected to an inverted can. When the chain is pulled, it raises a disc with holes in it. On the disc there is a rubber diaphram so that on raising the disc, the water in the can is raised above the bend and syphoning action takes place. Water rushing through the holes in the disc bends the rubber diaphram and the tank empties quickly.

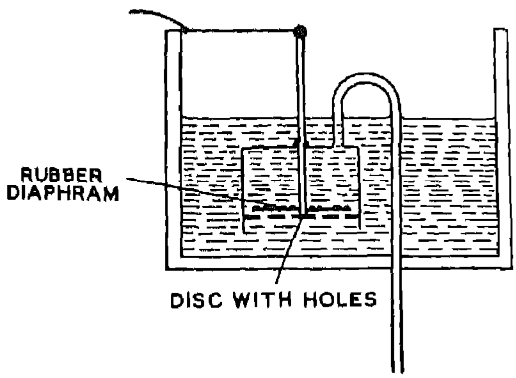


Fig. 16.11. *Another type of flush.*

16.14 Intermittent Flushing Tank

This is a device in which the tank is drained off automatically every half an hour

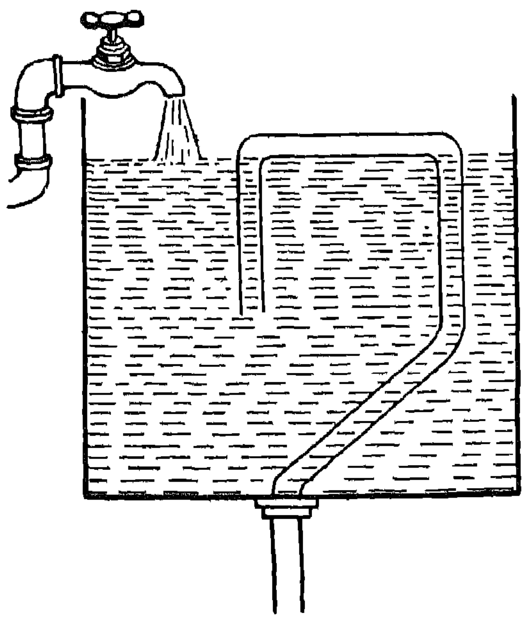


Fig. 16.12. *Intermittent flushing tank.*

or so. This is achieved easily by making use of the syphon principle. The arrangement is shown in Fig 16.12 The inlet fills in the tank and the water level rises until it reaches the level of the bend in the syphon. As soon as the level rises to that bend in the syphon, the water flows out through it and the tank is drained off. Again the water level builds up. Here the inlet is always open and by adjusting the rate of inward flow we can make the drainage as frequent as desired.

16.15 Effect of Pressure on Volume of Gases. Boyle's Law

If one takes an inflated rubber balloon and gradually squeezes it, to reduce the volume of the gas then a stage comes when the rubber balloon bursts. This is because, with the reduction in volume, the pressure gradually increases and reaches a limit when the strength of the rubber is no longer sufficient to hold it. We thus see that if we have a given mass of gas, by reducing its volume the pressure rises correspondingly.

To get a quantitative relationship between pressure and volume of a given mass of gas, let us examine the effects shown in Fig. 16.13.

The product of pressure and volume remains constant for all the three cases,

$$P_1V_1 = P_2V_2 = P_3V_3.$$

This was first studied by Robert Boyle and he stated it in the form of a law known after his name :

A given mass of a gas, occupying a volume V_1, under pressure P_1, would occupy a volume V_2 under pressure P_2 where,

$$V_2 = \frac{P_1V_1}{P_2},$$

provided the temperature is held constant. Thus if the pressure is doubled, that is $P_2/P_1 = 2$, the volume is halved, $\frac{V_2}{V_1} = \frac{1}{2}$, and so on This is mathematically stated as volume is inversly proportional to the pressure of a given mass of gas, provided the temperature is kept constant.

$$V \propto \frac{1}{P}$$

or VP remains constant for a given mass of gas and temperature.

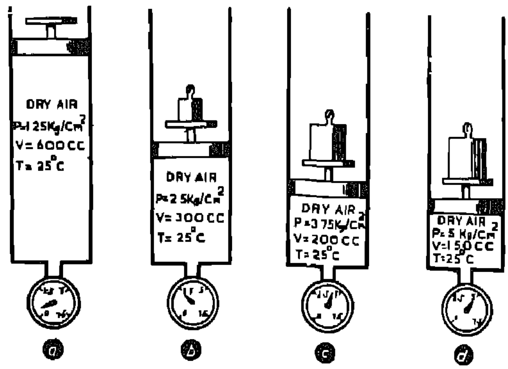


FIG 16.13 *At constant temperature, the effect of pressure on the volume of a fixed quantity of air*

It is important to note that Boyle's law is only true under the mentioned conditions : (1) The mass of the gas must be constant. (2) The temperature of the gas must be constant. For example, consider air being pumped into a rubber balloon The pressure and volume both increase. Conditions for applying Boyle's law do not exist, since the mass of the gas in the balloon is varying Similarly, Boyle's law cannot be applied when the temperature of a gas varies with the change in pressure or volume.

16.16 Pressure and Density

We can express Boyle's law as a relation between the pressure and the density of the gas. Let 'v' be the volume of 'm' kg of gas at a pressure p and at a given temperature. Then its density is m/v at pressure p. Now let the pressure be

increased to 2p. Its volume is then reduced to v/2 and the density becomes m/(v/2), *i.e.*, 2m/v, that is, if the pressure is doubled, the density is also doubled. It is to be noted that the mass remains the same.

In general, if the pressure be made n times p, the volume becomes v/n and consequently its density becomes $m/\frac{v}{n}$ or mn/v, which is n times the original density. That is, temperature being constant, the density of a gas is directly proportional to its pressure.

16.17 Experiment to Verify Boyle's Law

A uniform and narrow glass tube of length about 50 cm and a bore about one mm with one end open and the other closed is taken. A suitable length of mercury thread of about 20 cm is taken in it. A scale is fixed to the tube. The atmospheric pressure p is noted with the help of Fortin's Barometer. The tube with mercury thread is first placed horizontally on a table (Fig. 16·14). The length of the enclosed air column is measured.

The mercury thread is horizontal and so the pressures on either side of it are equal. Hence the pressure of the enclosed air of length l is equal to the atmospheric pressure.

Next the tube is fixed to a retort stand in an inclined position with its open end upwards. Again the length of the enclosed air column along the tube is measured. Also the vertical heights h_1 and h_2 of the two ends of the mercury thread are measured with a scale. The vertical height of the mercury thread is $h=h_1-h_2$. The air enclosed below supports the atmosphere and a mercury column of height h so the pressure inside $P=p+h$ in cm of mercury. The experiment is repeated with various angles of inclination and l and h are tabulated. Also with the open end downwards $P=p-h$, since now the atmosphere supports the air and mercury column. Then in various inclinations, h and l are noted. $P=p\pm h$ and if Boyle's law is correct, $(P\times l)$ will be constant. A graph is drawn between p and l and it will have the shape shown in Fig. 16·15. We have verified the law for small variations of pressure at room temperature. When large pressures are applied and the experiment is carried at low temperatures, it is found that no gas strictly obeys Boyle's law at all temperatures and pressures. But some of the gases like helium, hydrogen, nitrogen, oxygen and air, obey Boyle's law more closely than other gases. A gas which obeys Boyle's law exactly is called a perfect gas.

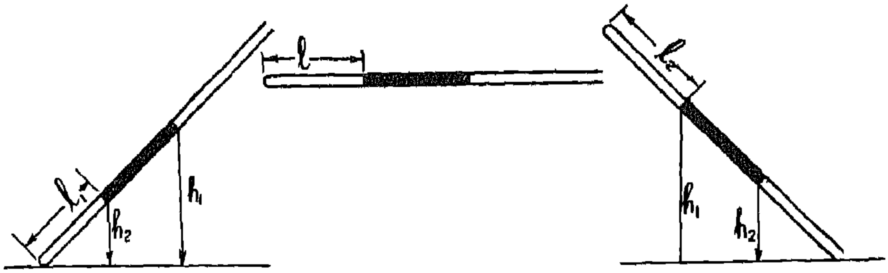


FIG. 16 14 *Experiment to verify Boyle's law*

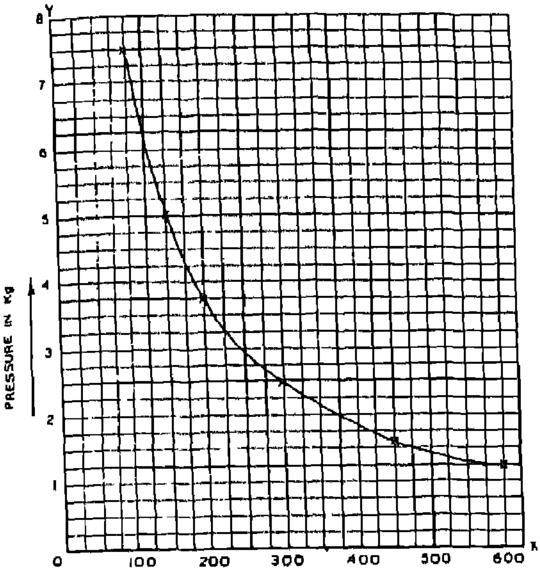


Fig. 16 15. *Graph between pressure and volume.*

16.18 Laboratory Experiment to Verify Boyle's Law

Now we give another more accurate experiment for the verification of this law. The apparatus used for this is shown in Fig. 16.16.

We have shown in the figure three different types of cases for the pressure of the enclosed gas.

In figure (a) the pressure of the enclosed gas is equal to the atmospheric pressure. In figure (b) the pressure is more than the atmospheric pressure by a height h_1 where as in figure (c) the pressure of the enclosed gas is less than the atmospheric pressure by a height h_2. In the following table, the pressure and the volume of the gas are given.

We have only shown three positions. But several readings can be taken for

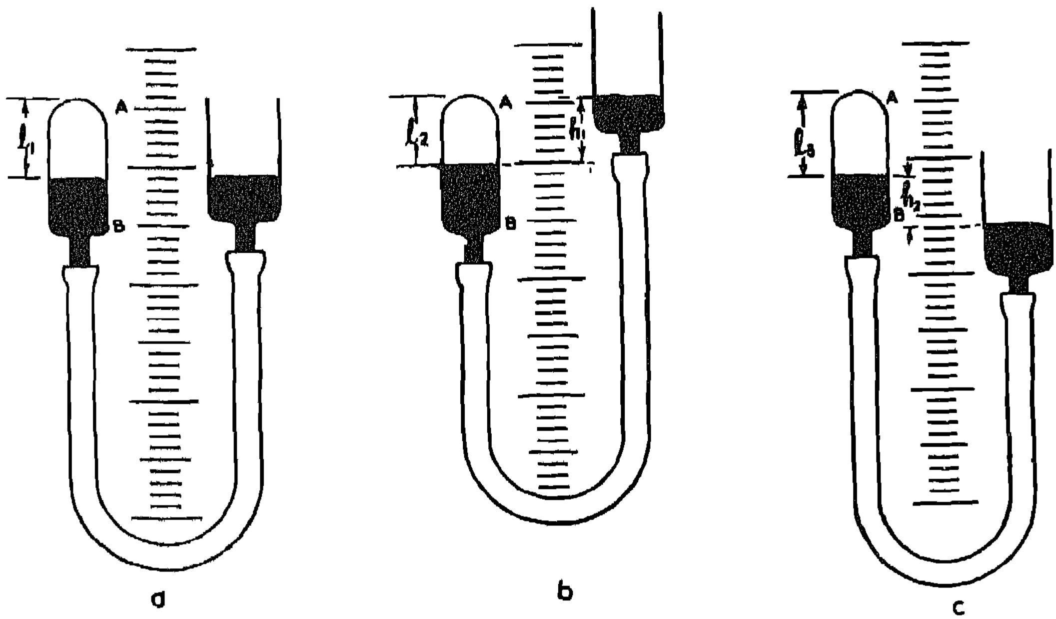


Fig. 16.16. *Boyle's law apparatus in three positions*
(a) *Atmospheric pressure is equal to the pressure of enclosed gas*
(b) *The pressure of the enclosed gas is greater than the atmospheric pressure*
(c) *The pressure of the enclosed gas is less than the atmospheric pressure.*

Pressure of gas in cm of Hg	*Volume*	*Pressure × volume*
p	al_1	apl_1
$p+h_1$	al_2	$a(p+h_1)l_2$
$p-h_2$	al_3	$a(p-h_2)l_3$

P=atmospheric pressure in cm of Hg
a=area of cross-section of the tube AB

different values of h_1 and h_2. The verification of Boyle's law consists in finding that the numbers in the last column come out to be the same.

This can be shown graphically by plotting log (pressure) against log (volume), which will be found to be a straight line with a slope −1, indicating that $p \propto 1/v$

16.19 Compressed Air Manometer

This instrument is very much like the Boyle's law apparatus.

It is an ordinary U-tube closed at one end and open at the other end fixed vertically (Fig. 16·17). The closed limb must be uniform. Mercury is poured into the tube enclosing a column of air in the closed limb. The length of the air column is noted and its pressure is found from the barometer reading and the difference of levels. The open end is then connected to the container of the fluid, whose pressure is to be determined. The lengths of the air column and the difference of levels are again noted By applying Boyle's law the pressure of the enclosed air is calculated from its original pressure and volume. From this the pressure of the fluid is found by adding or substracting the difference of levels as the case may be.

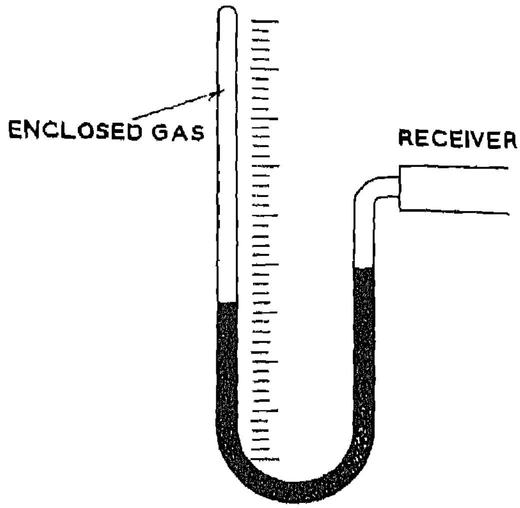


Fig. 16.17. *The compressed air manometer.*

Classroom Activities

1. Tie a rubber membrane to the mouth of a vessel which has a hole at its bottom through which it can be evacuated Seal the vessel at its mouth so that there is no leakage of air. Now evacuate the vessel by a pump and observe the effect of the atmospheric pressure.
2. Insert a rubber tubing through a rubber stopper fixed in a flask which contains some water Heat the flask so that steam is issuing freely through the tube. Now close the rubber tube by a pinch cork and invert the flask so that the open end of the rubber tube is under water in a trough. Open the pinch cork and explain what you observe.
3. Devise an experiment to show that air occupies space.
4. Open the valve of a bicycle and then fix it to the bicycle pump and try to under stand the working of this pump.
5. Explain the working of the syringe used by the doctors.

Questions

1. Calculate the mass of air in a room of dimensions 8 m $\times$ 6m $\times$ 4m. Density of air $= 1 \cdot 293$ kg/m^3.
2. If the radius of the earth is 6400 km, calculate the total mass of the atmospheric air, assuming the barometric pressure to be equal to 76 cm of mercury.
3. If the density of the air in the atmosphere at all altitudes were $1 \cdot 293$ kg/m^3, how much would be the atmospheric height ?
4. What is the buoyancy due to the atmosphere on a 10,000 kg elephant if the mean density of his body is 1000 kg/m^3 and that of air 1 30 kg/m^3?
5. The densities of air, helium and hydrogen are $1 \cdot 293$ kg/m^3, 0·178 kg/m^3, and 0 0899 kg/m^3 respectively What will be the volume of a hydrogen balloon which has a lift of 100 kg? What will be the lift when the same balloon is filled with helium?
6. Assuming the average density of the lower atmosphere to be equal to $1 \cdot 25$ kg/m^3, calculate the barometric height at a height of 4000 metres (Relative density of mercury$=13$ 6).
7. Calculate the total weight of the atmosphere on the roof of a room 5m $\times$ 4m. Why does not the roof collapse ?
8. A cylinder contains 6 kg of oxygen when the pressure of the gas is 10 atmosphres. What will be the mass of the oxygen when the pressure falls to 2 atmospheres ?
9. If the atmospheric pressure is 74 cm of mercury, calculate its value in kg/m^2.
10. A bicycle pump is full of air at a pressure of $1 \cdot 04$ kgf/cm^2. If the length of the stroke of the pump is 40 cm, at what point the air will begin to enter the tyre where the gauge pressure is $2 \cdot 5$ kgf/cm^2 ? Assume the compression to be isothermal.
11. The open end of a narrow tube closed at one end is inserted in a broad vessel containing water upto a height of 50 cm. If the length of the tube is 1 metre, how high will the water in the tube rise if the atmospheric pressure is 75 cm of mercury (Density of mercury$=13{,}600$ kg/m^3)?
12. A barometric tube is 1 metre in length. When the mercury stands at a height of 75 cm some liquid oxygen is introduced into the tube and the height of the mercury column becomes 70 cm. Calculate the amount of oxygen introduced (Density of oxygen$=1 \cdot 429$ kg/m^3).
13. The volume of the tube of a car tyre is 30,000 c c. If the gauge pressure is 2kgf/cm^2 and the atmospheric pressure 1kgf/cm^2, calculate the amount of air in the tube. (Density of air at 76 cm pressure$=1 \cdot 29$ kg/m^3).
14. An air bubble rises from the bottom of a lake. On reaching the surface its volume is five times its original volume. Calculate the depth of the lake (Atmospheric pressure$=76$ cm of mercury and relative density of mercury$=13$ 6).
15. At what depth in the sea will the density of an air bubble be the same as that of the sea-water ? (Density of sea water$=1030$ kg/m^3 and of air$=1$ 293 kg/m^3 at one atmosphere pressure).

Further Reading

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CHAPTER XVII

Fluids in Motion

17.1 Introduction

In the last three chapters we had studied the properties of fluids at rest. We had shown in particular that (a) the pressure at a point in a fluid is the same in all directions; (b) the pressure at two points in the same horizontal level is the same; (c) the liquid finds its own level. All these conclusions were obtained from experiments done with fluids at rest. In this chapter we shall study the laws of fluids in motion.

Consider the following experiment, as shown in the Fig. 17·1. The tube A_1 with a manometer M_1 and the tube A_2 with a manometer M_2 are connected by means of a short rubber tubing. The tube A_1 has been chosen to have a smaller diameter than the tube A_2. The tube A_1 is connected to a reservoir R, containing water. The water flows through the tubes A_1 and A_2, and finally runs out into the bucket B. There is a pinchcock P attached to a rubber tubing connected to A_2 in order to control the flow.

If the pinchcock P is kept pressed so that no water is allowed to flow, then we have the case of a liquid at rest and therefore the water in the manometers will be at the same horizontal level L, as in the reservoir R. On opening the pinchcock it will be found that the level in both the manometers M_1 and M_2 will fall down.

Soon a stage would be reached when the water in the manometers will stand steadily at fixed heights. In this steady state the height L_1 of manometer M_1 will be less than the height L_2 of manometer M_2. This indicates that the pressure of water in A_1 is less than the pressure of water in A_2, when water is flowing, even though the tubes are placed on the same horizontal table.

Let us consider the amount of water flowing through the tubes in one second. The amount of water passing through A_1 must be the same as the amount of water passing through A_2 because there is no accumulation or loss of water in this passage Let a_1 be the area of cross-section of the tube A_1 and v_1, the velocity of water in A_1 then the amount of water flowing per second in this tube is a_1v_1. Similarly the

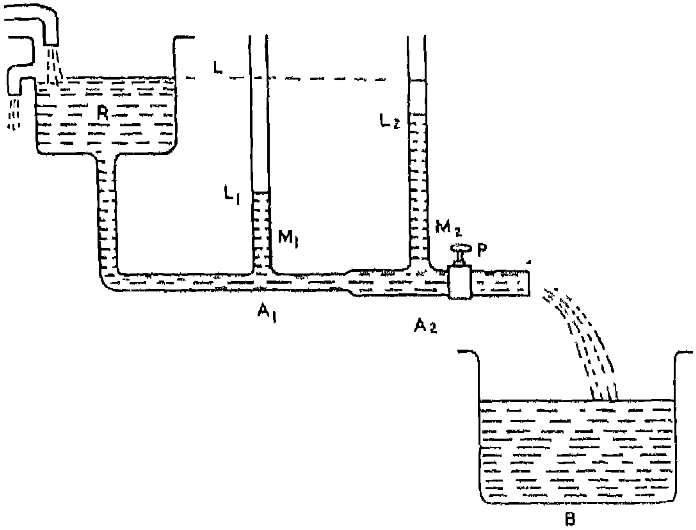

FIG. 17.1. *Illustration of Bernoulli's Principle.*

amount of water flowing per second in tube A_2 is a_2v_2, where a_2 is its area of cross-section and v_2 the velocity of water. Since $a_2 > a_1$, $v_2 < v_1$. But we have seen experimentally that in A_1 the pressure is less than in A_2. Thus we see: In a flowing liquid, the pressure is less at those points where the velocity is greater. This principle is called *Bernoulli's Principle.* We can verify this principle from a series of simple experiments.

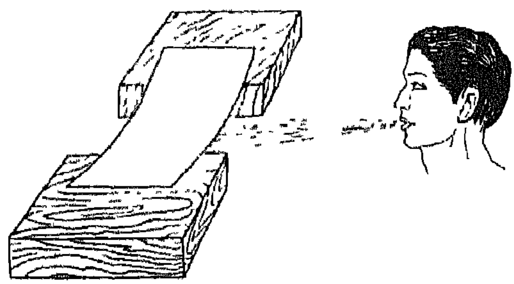

FIG. 17.2. *What happens to the sheet of paper when air is blown under it?*

1. Take two wooden blocks approximately 5 cm thick. Place them on a table with 8 to 10 cm separation between them. Take a piece of paper and place it on the wooden blocks so that it is horizontal as shown in the Fig. 17.2. Now blow air underneath the paper as forcibly as you can. You will notice that the paper bends down with convex surface towards the table. This can happen only if there is an increase in pressure on the upper side or lowering of pressure below, or both. Since the air on the top surface is not disturbed, the pressure at the lower surface must have decreased due to blowing. Hence the paper is forced down. By the blowing action the velocity of air has increased resulting in lowering of the pressure.

2. Take a long glass tube ABC having a constriction at B. (Fig. 17·3). To this tube are attached three small glass tubes of narrower diameter at A, B and C. These dip into three beakers containing water. Air is blown from one end say from left, through the tube by means of a blower.

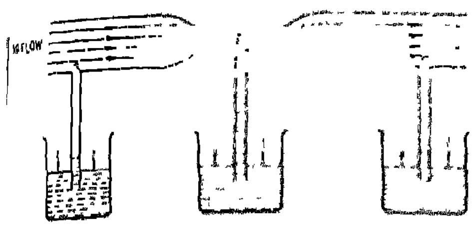

FIG 17.3. *The pressure is lower in the narrow tube.*

We notice that the water rises into the three tubes. The levels in tubes A and C are the same, whereas the level in the tube attached to B is higher than in the other two. This happens because the pressure at B is lower than at A or C. Due to the constriction the air blows faster at B and consequently pressure at B is lower.

17.2 Applications

Bunsen Burner: A bunsen burner is shown in figure 17.4. Gas under pressure from a reservoir issues out in a stream as shown by the arrows. The pressure inside the fast moving gas is lower than the pressure of air outside. This causes air to flow through the opening as shown. The mixture of air and gas burns at the top.

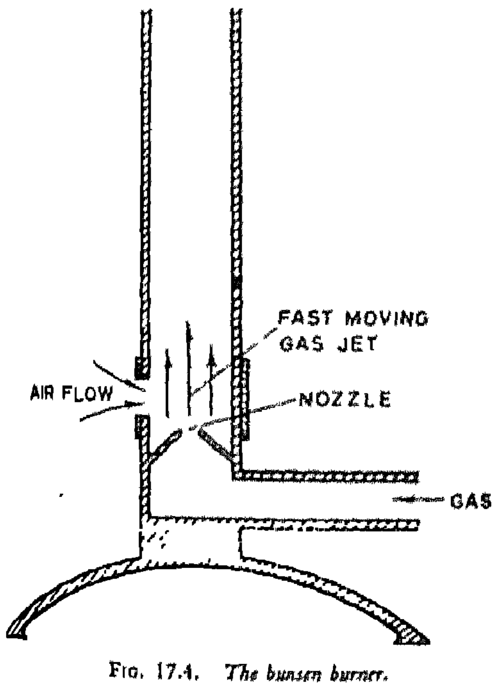

FIG. 17.4. *The bunsen burner.*

17.3 Atomizer

This is a common device used to produce a spray. A diagram of an atomizer is shown in Fig. 17·5. A is a bulb made of rubber. On squeezing it air moves in a stream over the nozzle of the tube B and the pressure in the stream is less than the atmospheric pressure. This causes the liquid to rise in the tube B and the liquid particles come out of the tube and are blown in the form of a spray by the same stream of air.

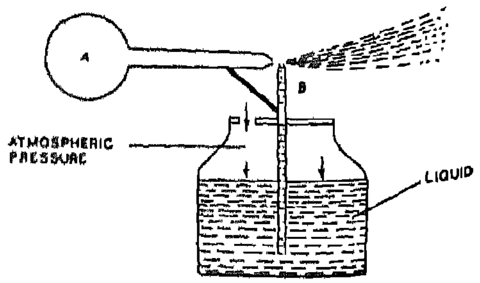

FIG. 17.5. *The atomizer.*

17.4 Filter Pump

This pump is commonly used in chemistry laboratories to hasten filtering. The pump is shown in Fig. 17.6. It should be easy to see that its action depends upon Bernoulli's principle.

17.5 The Carburettor

This is a device used in motor cars to supply a mixture of air and petrol to the cylinder of the engine. The explosion of this mixture inside the cylinders of the engine supplies energy. A diagram of the carburettor

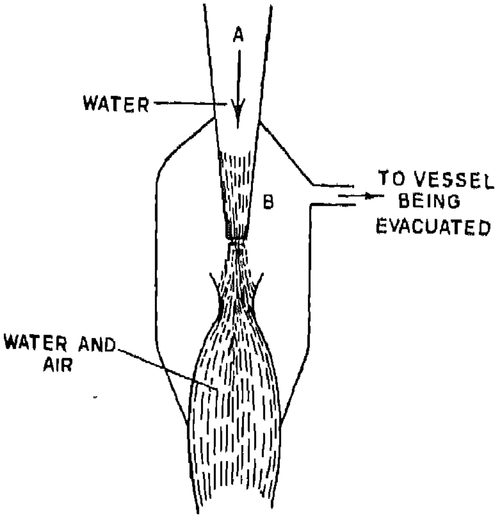

FIG. 17 6. *The filter pump*

is shown below (Fig 17.7). The float chamber contains petrol. Due to the motion of the pistons of the engine there is a decrease in pressure on the side A. This causes the air from outside to be sucked in as a stream. Now the pressure near the nozzle B is

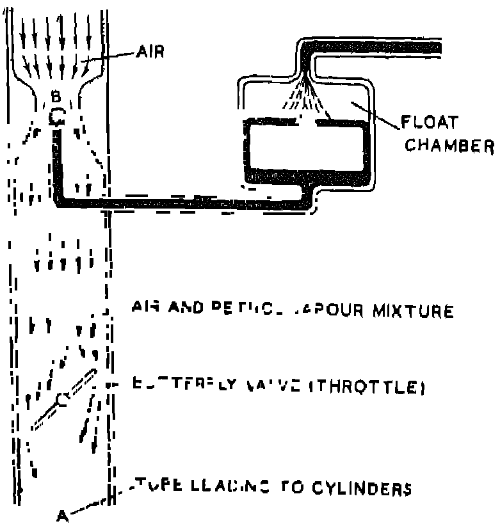

FIG. 17.7. *The principle of operation of the carburettor The fast moving air causes a reduction of pressure at B and petrol is drawn into the air-stream*

reduced, forcing the petrol out. Thus the air and petrol mixture enters the piston. The supply of air and petrol mixture is controlled by the butter fly-valve

17.6 Lift of an Aeroplane

With the understanding of the Bernoulli's principle, we are in a position to understand how an aeroplane stays in air. The propellor or the jet moves the plane forward and this causes the air to stream past the wings. The cross-section of the wings, or the aerofoil, has a special shape and is shown in Fig. 17.8.

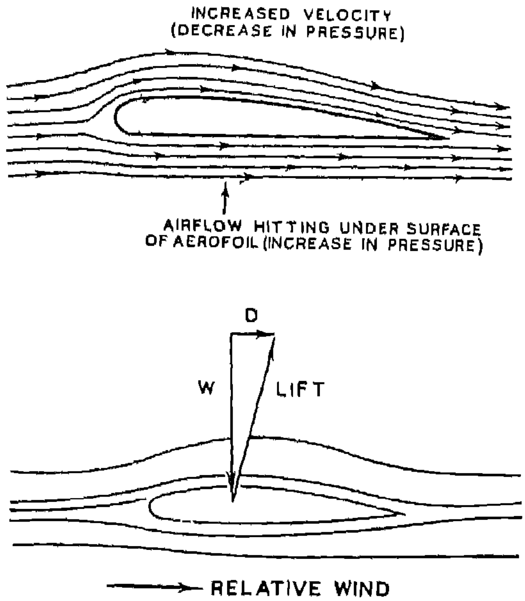

FIG. 17.8. *The lifting effect on an aeroplane wing is due to the greater pressure on the bottom of the wing than on the top.*

In this the air at the upper part has to travel further and therefore, faster than the air travelling at the bottom part This causes a pressure difference resulting in an upward force. About two-thirds of the lift for an aeroplane comes from this pressure difference. The other one-third comes from the air

hitting the bottom part. This action of the wind is similar to that on kites flown by children.

17.7 Helicopter

In an aeroplane, the forward motion of the aircraft causes the air to move over the wings and we have seen that due to Bernoulli's principle there is a force in the vertical direction which gives the lift to the plane. In the helicopter also the same principle is involved in causing its lift Only the mechanism is different The engine rotates the blades of the helicopter and just as the motion of air across the wings causes a lift in the aeroplane, the motion of air across the blades which have a section similar to the aerofoil causes the lift in the helicopter. Thus the helicopter can be lifted and maintained in the air without any forward motion. To give it a forward motion the blades have to be slightly tilted.

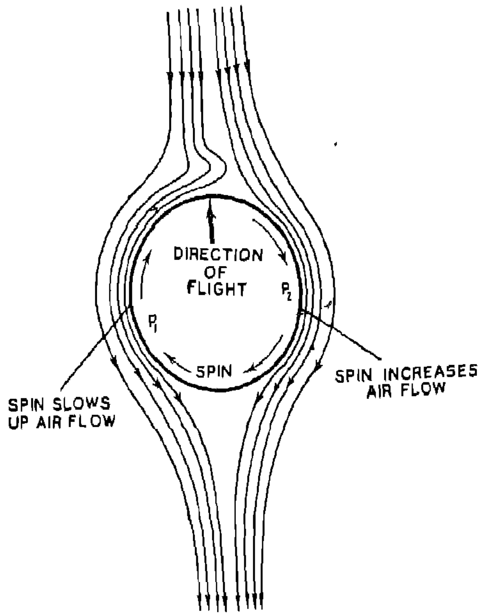

Fig 17.9. *The pressure decreases on the right and increases on the left. The ball swings to the right*

You may have noticed that there is another propeller at the tail end of the helicopter. The function of this is to counteract the rotational tendency of the helicopter When the rotor blades rotate, by Newton's third law, they tend to rotate the helicopter in the opposite direction Due to the motion of the tail propeller there is a force which has a tendency to rotate the helicopter in a direction opposite to that of the main rotor blades thus stablizing the movement of the helicopter By controlling the speed of the tail propellor, the direction of the helicopter can be changed.

17.8 Curved Motion of A Ball in Flight

It is often observed that a tennis ball when served or a cricket ball when bowled does not move in a vertical plane but moves sideways Due to the forward motion of the ball and due to its spin, the air does not move with the same speed on the two sides of the ball (Fig. 17.9). At the side P_2, the velocity of the air with respect to the ball is approximately equal to the forward motion of the ball plus the velocity imparted to the air on account of the drag due to the spinning motion of the ball On the other side these two velocities are in opposition. Hence the velocity of air on the side P_2 is greater than that on the side P_1 Thus the pressure of air on the side P_1, is greater than that on side P_2 and the ball drifts towards the side P_2, *i e.*, towards the side, towards which its nose is moving due to its spin.

Classroom Activities

1. Take a wide tube with a constriction in one part and with two narrow tubes connected to the tube at the wider portion and the constricted portion. Allow water to flow through the tube at different speeds and measure the difference in water level in the two narrow tubes.
2. Blow a continuous blast of air through a vertical nozzle. Place a ping pong ball in the stream of moving air.
3. Make a filter pump and connect it to a manometer and measure the pressure of the evacuated chamber for different rates of flow.

Questions

1. Explain why a spinning tennis ball curves during its flight.
2. Discuss the stability of a ping pong ball placed on a fountain of water.
3. Explain why it is dangerous to stand near the edge of a platform when a fast train is passing.
4. Why do flags flutter in a breeze ?
5. Two boats are moving in the same direction side by side in a calm sea. Will they be drawn together or pulled apart or there will be no effect ?
6. Explain why in a strong wind the roof can be completely lifted although the walls are not appreciably damaged.
7. Two ping pong balls are suspended from the ceiling by thin wires and air is blown between them. Explain how the balls will move.
8. A cylinder is rotated in a clockwise direction as observed from above and air is blown from left to right. Will there be a force acting on the cylinder ? Explain your answer.

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CHAPTER | XVIII

Elasticity, Viscosity and Surface Tension

18.1 Introduction

When you stretch a rubber band slightly, you notice that the band exerts a force on your hand On releasing the rubber band it goes back to its original shape. Similarly if you take a piece of wire and bend it with your hands slightly, you again feel a force exerted by the wire on your hand, which tends to bring it back to the original shape.

When we stretch, compress, bend, or twist a body, we change its size or shape and thus produce a deformation. If the deformation is small enough then the body comes back to its original condition on removing the force which caused the deformation. This body is called elastically deformed. However, if the deformation is large, for example, if we bend a piece of wire by a large amount, it will not return to its original shape when the force is removed. The body is said to be deformed plastically

The restoring forces mentioned above are called elastic forces. There are various kinds of elastic forces.

18.2 Stress

Consider a body being stretched by forces acting in opposite directions. For example, this can be done by fixing one end of a piece of wire to the ceiling of a room and hanging weights at the other end. The reaction at the fixed end is equal and opposite to the weight suspended and thus we have two equal and opposite forces trying to stretch the wire. Such forces are called tensile forces

The wire is experiencing a force T on both sides which produces a change in its length After the stretching has taken place, the wire is in equilibrium, *i.e.*, each small length of the wire must be experiencing equal and opposite forces at its two ends. Thus, due to stretching, internal forces are developed in the wire which oppose further stretching. The internal force is equal to the applied external force. This is true of every section as is shown for an arbitrary section in the Fig. 18.1. Here the part A is pulled by the part B and part B is pulled by the part A In this particular example the internal force is along the

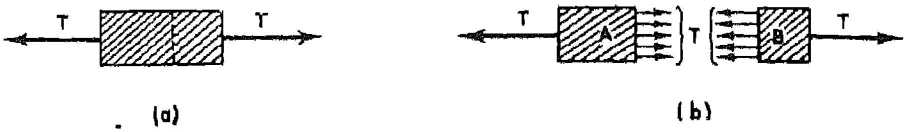

FIG. 18.1. (a) *A bar under tension*
(b) *When a bar is under tension, internal forces develop in the bar.*

longitudinal direction. The force per unit area of cross-section is called *stress*, and in this case it is a *longitudinal stress*. Since the internal force is equal to the applied external force, the stress can be measured as the external force per unit area of cross-section. In MKS system it is measured in newtons/m².

Longitudinal Stress = Force/Area

18.3 Strain

When a body, say a piece of wire, experiences tensile force, it causes a change in the length of the body. In some cases, like the rubber band, the change in length is large. In others like a steel wire, the change in length is small, and needs accurate methods to measure the change.

Experiments show that if a body is elastically deformed under a tensile force, then the change in length for a given stress is directly proportional to the original length.

Therefore it is convenient to define a quantity called *STRAIN* as the ratio of the change in length to the original length. In our example the strain is longitudinal strain. Suppose the initial length of the wire is l m and change in length is Δl m.

$$\text{Longitudinal Strain} = \frac{\Delta l}{l}.$$

Strain has no units because it is a ratio of two similar quantities.

18.4 Elastic Limit

Suppose we fix one end of a long copper wire to the ceiling and load the other end with increasing weights. Each time let us measure the elongation and plot a curve between strain and stress. The curve will be something like that shown in Fig. 18.2. Upto stresses corresponding to a point A on the curve, the strain will be proportional to stress. That is, the curve is linear. Above A, the curve is no longer linear. By definition the stress at A is called the 'proportionality limit'. Between A and B the increase in strain is more for the same increase in stress. Nevertheless, upto the point B, the material is elastic, *i.e.*, the wire regains its original length if the load is removed. The point B is the elastic limit. Above B it acquires a permanent deformation, that is, when the stress is removed the wire does not come back to its original condition and its length is a little more than the original length. At a stress corresponding to C, the wire breaks down. The

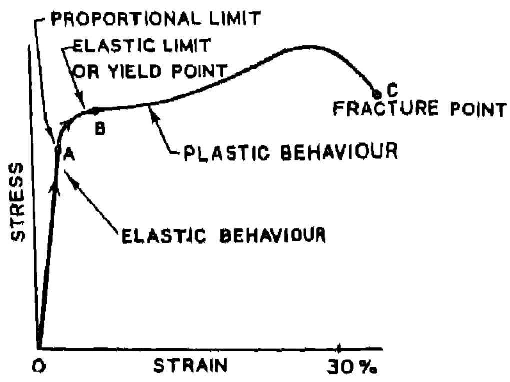

FIG. 18.2. *Typical Stress-Strain diagram for a ductile metal under tension.*

stress at C is called "Breaking stress" Between the points B and C, the wire is said to behave plastically The strains at A and C will be nearly 0·0006 and 0·3 respectively

18.5 Hooke's Law

We have seen that if the stress is small, the body returns to its original shape and size on removing the stress This is the case of elastic deformation. We shall confine our attention to deformations within the proportionality limit. The fact that for small stresses the strain is proportional to the stress was first observed by Robert Hooke and is known as Hooke's Law.

18.6 Different Types of Stresses and Strains

The three principal types of deformation which solid bodies experience under stress are stretching, uniform compression and twisting. Figs 18·3 and 18·4 show a twisted wire and cube, compressed cube and a stretched rubber band, as examples of these three When we remove the forces acting on these objects, the rubber band will begin to shrink, the bent bar to straighten out, and the twisted wire to untwist and the deformed cube regain its

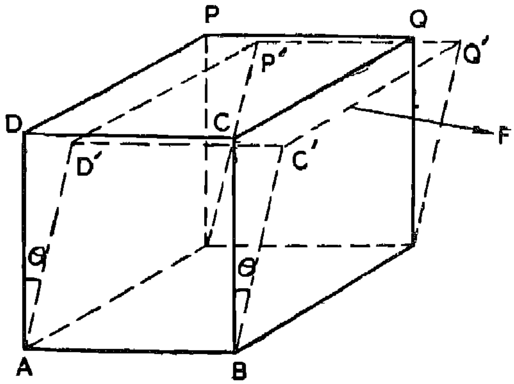

FIG. 18 3 *Shearing of a cubical block through an angle θ by a force F.*

original shape. In a somewhat generalized form, Hooke's law states that for all types of small deformations, stress is proportional to the strain The ratio of stress to strain is therefore a constant for any given elastic object Further, if the definitions of the preceeding paragraphs are used, this constant is, within wide limits, independent of the size and shape of the object and depends only on the characteristics of the substance. The ratio stress/strain is called a "modulus". The ratio longitudinal stress/longitudinal strain is called Young's modulus. The two other moduli are the shearing modulus, and the bulk

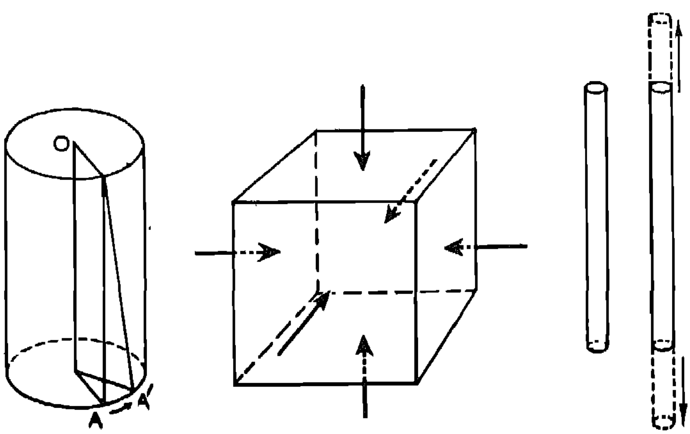

FIG. 18.4. *Different types of stresses and strains*

(a) *twisting a metal wire*
(b) *compressing a cube*
(c) *stretching a rubber band.*

modulus which arise respectively when a body is twisted and subjected to a uniform pressure. We shall not discuss them here.

18.7 Poisson's Ratio

When a tension is applied to a wire, in addition to the longitudinal strain there is a lateral strain which leads to a decrease in the cross-section of the wire. The ratio of lateral to longitudinal strains is called "Poisson's ratio" in honour of the physicist and mathematician S. D. Poisson who introduced it. It may be shown that the value of the Poisson's ratio lies between 0 and $\frac{1}{2}$. Table 18.1 gives the values of Young's modulus of some common substances.

TABLE 18.1

YOUNG'S MODULUS

Material	*Newtons/m²*	*Material*	*Newtons/m²*
Aluminium ..	$6 \cdot 9 \times 10^{10}$	Lead ..	$1 \cdot 6 \times 10^{10}$
Brass ..	$9 \cdot 2 \times 10^{10}$	Nickel ..	$2 \cdot 1 \times 10^{11}$
Copper .	$1 \cdot 0 \times 10^{11}$	Phosphor Bronze	$1 \cdot 2 \times 10^{10}$
Glass ..	$5 \cdot 8 \times 10^{10}$	Quartz ..	$5 \cdot 6 \times 10^{10}$
Iron ..	$2 \cdot 0 \times 10^{11}$	Silver ..	$7 \cdot 5 \times 10^{11}$
India rubber	$5 \cdot 0 \times 10^{8}$	Steel ..	$2 \cdot 2 \times 10^{11}$
Ivory ..	9×10^{10}	Tungsten ..	$3 \cdot 6 \times 10^{11}$

18.8 Viscosity

It is a common experience that it takes less time to empty a glass of water than a glass of honey. In fact, different liquids would take different amounts of time to pour out from a glass.

To study this effect systematically in a laboratory, it is necessary to create identical conditions under which liquids will flow out A possible arrangement is shown in Fig 18.5. R is a reservoir containing

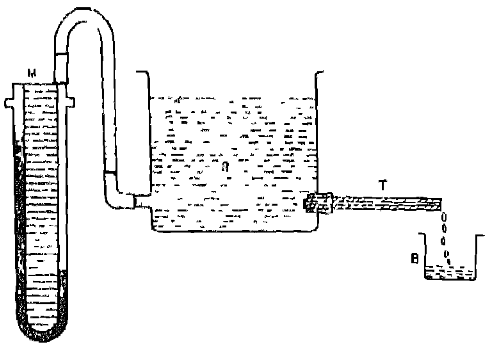

FIG. 18.5. *The flow of a viscous liquid is less for the same pressure.*

the liquid, M is a manometer showing the initial pressure and T is a tube of diameter say a few millimetres and of length about 10 cm. They are all connected by a rubber tubing B is a beaker in which the liquid is collected as it flows out of the tube T. It can be experimentally seen that when different liquids are used, for the same pressure lead as indicated by the manometer M, different amounts of different liquids flow out per unit time.

A liquid of which the amount flowing per unit time is less is said to be more *viscous* and this property of the liquids is called viscosity.

Another experiment which shows the effect of viscosity is shown in the self-explanatory Fig. 18·6.

Along the tube we find the pressure is decreasing, even though the cross-section of the tube is constant (You can strictly apply Bernoulli's principle only to liquids which are not viscous). If you use different liquids, the fall in pressure would be also different, showing that some liquids are more viscous than others.

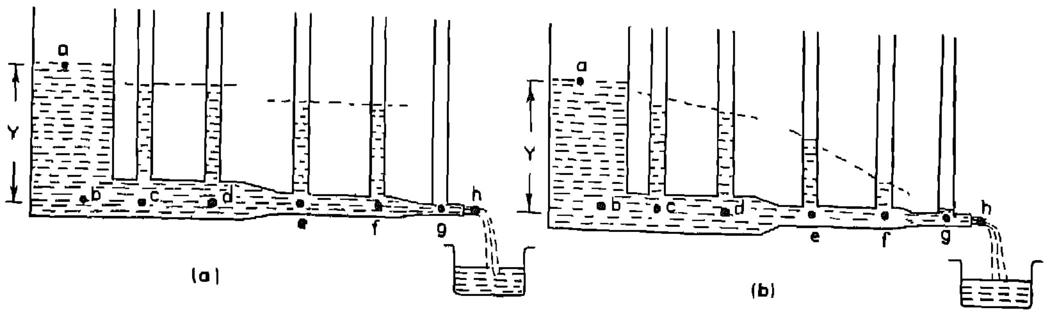

FIG 18.6 *Pressure along a horizontal tube in which is flowing*
(a) *an ideal fluid*
(b) *a viscous fluid.*

Viscosity arises due to the friction between the fluid and walls of the tube. Just as a solid body moving on a surface experiences frictional force, there is a retarding force trying to check the flow of the fluid. However, we not only have the frictional force between the wall and the liquid but also between the various layers of the liquid, when one layer of liquid moves over another layer of liquid. If the frictional force is large, then the liquid is more viscous.

18.9 Surface Tension

We have learned in the chapter on Archimedes' principle that if one placed a piece of steel, say a steel needle, in a beaker of water, it would sink to the bottom for the density of steel is greater than that of water. However, if one places a steel needle carefully on the surface of water it can be made to float. From this experiment we realize that there must be an upward force at the surface of the liquid which balances the weight of the needle. That this upward force comes only from the surface becomes clearer by immersing the needle inside the water completely when it will always sink no matter how carefully you release the needle.

One more experiment to demonstrate that the surface layer behaves in a special manner is shown by the following experiment. Take a capillary tube of about 1 mm diameter and dip the lower end vertically in water. You will find that water rises in the tube above the surface and stands at a height of 1 cm or so This means that there is a certain upward force which can balance the weight of this water column

From these and several other experiments it was concluded that they could be understood only by assuming that the surface of a liquid is under tension and behaves like a stretched membrane. A property called surface tension has been introduced. We shall explain below more clearly what it means.

Consider a steel wire XABY bent in the form as shown in Fig 18 7. Attached to it is another wire DC which can slide over XABY. On dipping it in a liquid, say water, a film of water ABCD can be formed. This film has a tendency to contract, which will be felt by the force on DC.

Consider the part of the film MNQP. We assume that there is a force acting in both directions on the surface shown by arrows This keeps MNQP in equilibrium. On the other hand the wire DC experiences

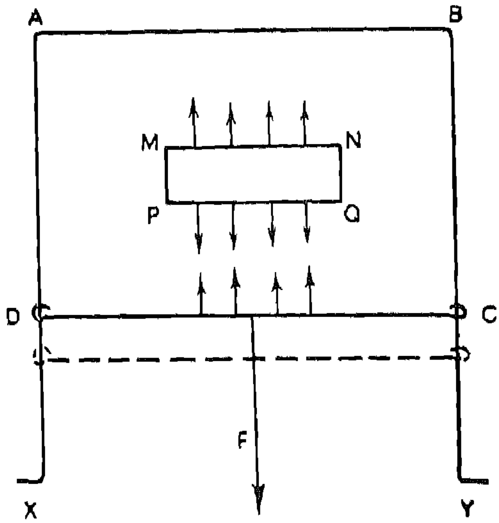

FIG 18.7 *The forces acting on a liquid film.*

an upward force due to the surface and if DC can slide over XABY without any friction, it would move towards the end AB. To hold it in position, would require a force F. The force acting on the surface per unit length is called *SURFACE TENSION*, In our example there are two surfaces. Thus the force required to hold it in position is

F = 2 × Surface tension × length DC

Surface tension is a property of the liquid. It is expressed in terms of newtons/m.

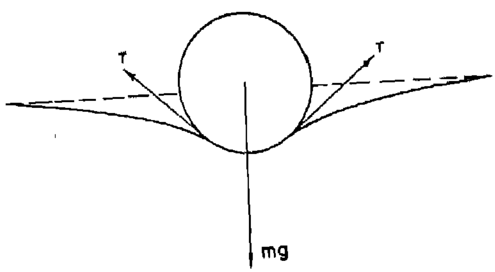

FIG. 18 8. *The force due to surface tension is sufficient to support a needle*

Now consider first the case of the steel needle floating on the surface of water (Fig 18.8). A section has been shown in the figure, where the various forces acting are shown. From the diagram it should be clear how the needle floats.

Next let us consider the case of rise of water in the capillary tube. A magnified diagram is shown in Fig. 18.9 The surface tension pulls the sides of the glass downwards with a force $2\pi r T \cos\theta$ where θ is the angle shown in the figure. The side of the glass in turn pulls the water surface upward with a force $2\pi r T \cos\theta$. The weight of water column balances this upward force and thus,

$$(\pi r^2 h)\,\rho = 2\pi r T \cos\theta$$

$$\text{or, } h = \frac{2T \cos\theta}{r\rho}$$

where ρ is the density of water, θ is the angle made by the tangent to the surface of water with the glass. This angle is called the angle of contact. The angle of contact depends upon the nature of the liquid and the surface it comes in contact with. θ can be greater than 90° as for example is the case between mercury and glass. From the formula we notice that if $\theta < 90°$, $h > 0$ and if $\theta > 90°$, $h < 0$. This means that if the angle of contact is less than 90°, the liquid will rise in the capillary, and if it is greater than 90°, the liquid will be depressed in the capillary compared to the outer level.

Notice that the height to which a liquid rises is inversely proportional to the radius of the capillary and the density of the liquid. That is why the effect is most pronounced in capillaries of small diameter

The rise of oil in the wick of a lamp is due to the capillary action. It is again capillarity that is to a large extent responsible for raising water from the soil up to the green leaves.

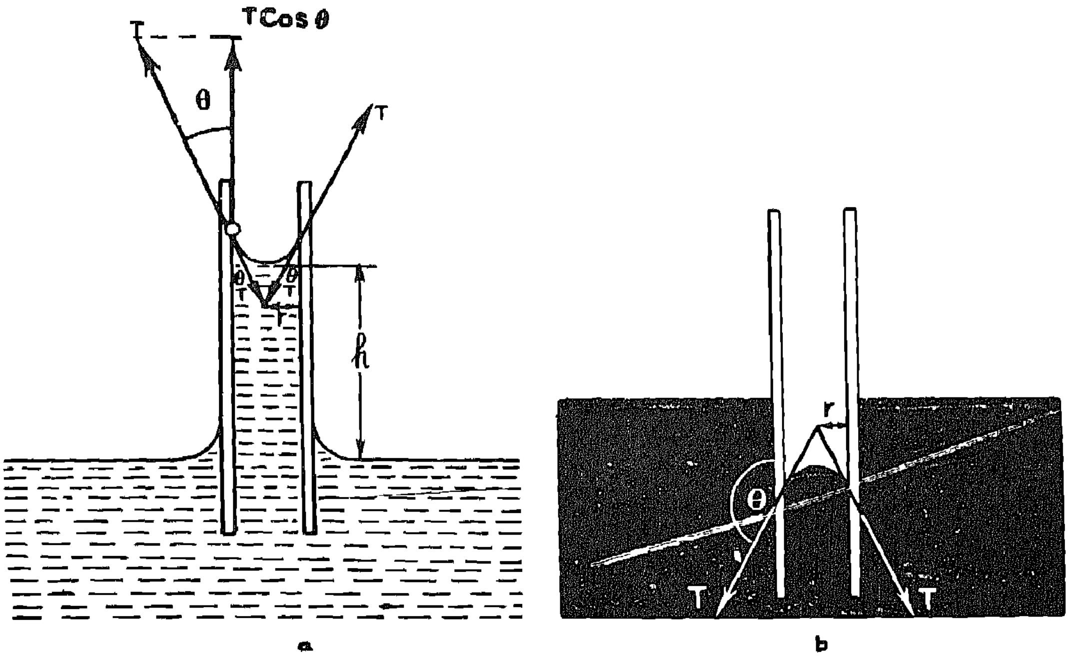

FIG. 18.9. *Surface tension forces on a liquid in a capillary tube. The liquid rises if $\theta < 90°$ and is depressed if $\theta > 90°$.*

Classroom Activities

1. Take two pieces of fine aluminium wire and hang them from a common support. Attach pans to them and put small weights to keep the wires taut Attach to them a vernier scale. Add on one pan weights in steps of 0 5 kg till the elastic limit of the wire is reached. Now remove the weights and determine the permanent change in the length of the wire for the load of your experiment.
2. Hang five spring balances by hooks attached to a beam. Hang another beam from the lower ends of these springs. Now hang from the lower beam a heavy weight, observe the increase in the readings of the spring balances The sum of these increases should be equal to the heavy weight. This shows how a number of small forces balance a large force and how the stress in a thicker wire is less than that in a thinner wire for equal applied forces.

Questions

1. In an experiment to measure Young's modulus, a load of 50 kg hangs from a wire 3 m long and of cross section 0·2 sq cm. It was found to stretch the wire 0·3 cm above its no load length Calculate (a) the stress, (b) the strain, (c) the young's modulus of the material of the wire.

2 If in the above experiment, the diameter of the wire is doubled, the length made 5 m, calculate the extension for (a) double the load (b) half the load.

3 A load of 3 kg is hung from a steel wire of length 5 metres and cross section 0 1 sq cm. Calculate the (a) stress (b) the strain and (c) the increase in length of the wire. Young's modulus for steel is $2 \cdot 0 \times 10^{11}$ newtons/m^2.

4 If the proportionality limit of the material of a wire is reached for a value of the strain equal to 0·3 per cent, calculate (a) the limiting stress, (b) the total change in length (c) the load for a wire of length 1 m and area of cross section 0·189 cm. Young's modulus of the material of the wire is 7×10^{10} newtons/m^2.

5 In a capillary tube the water rises 4 cm above the surface of the surrounding liquid. Calculate the diameter of the capillary tube. Surface tension of water is 75×10^{-3} newton/m.

6. If the same tube is now dipped in mercury, calculate the depression of the mercury column below the surface of the surrounding mercury. Surface tension of mercury is $46 \cdot 5 \times 10^{-2}$ newton/m.

7. Explain why small water droplets are spherical in shape.

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CHAPTER XIX

Molecular Theory

19.1 Introduction

We have in the previous few chapters learnt some of the properties of solids, liquids and gases. We learned these properties by performing experiments, collecting and analysing data. These properties were stated in the form of laws or principles; for example, the Hooke's law, the law of friction, the Bernoulli's principle, etc Quite often scientists are able to make a set of assumptions (usually a few in number) which enable them to understand several of these laws, which are apparently unrelated. The set of assumptions is usually called a theory If a theory is a good one, it leads to an understanding of several phenomena in nature We shall describe in this chapter one such theory, the molecular theory. The basic assumptions of this theory are described below.

All substances are composed of extremely small particles, called molecules. All molecules of the same pure substance are identical The molecules of different substances are different. In fact, the differences in properties of substances are due to the differences in their molecules. For each substance there is a molecule such as molecules of water, iron, sugar, oxygen, etc. Since there are a large number of different substances there are an equal number of different types of molecules. The molecules are composed of still smaller particles called atoms

Suppose we take a piece of sugar and break it into smaller and smaller bits, a stage will come when there is only one molecule of sugar On breaking it further we would no longer have sugar Thus, according to this assumption, a piece of sugar would consist of several molecules held together in much the same way as the bricks in a wall are held together In a wall the bricks are held together in place by cement. We assume that in a solid the molecules attract each other strongly and this attractive force holds them together. These molecules are fixed in relation to each other in space and do not change places. But the molecules are not quite static We assume that a molecule can vibrate about its mean position but cannot leave its neighbours and wander

around. This accounts for the fact that solids maintain their shape.

One finds in nature that the external appearance of solids usually has a definite geometrical shape, *e.g.*, sodium chloride or sugar crystals. In the interior of the crystals, the molecules (or the atoms or ions which constitute the elementary bricks) are arranged in a regular manner in three dimensions. The external regular geometrical appearance of a crystal is the result of the internal regular arrangement. By breaking a crystal into irregular bits, the external appearance is changed but the regular internal arrangement is not destroyed. We can study the internal regular and periodic arrangement of a solid with the help of X-rays and other methods However, it may be mentioned that all solid looking substances are not crystalline, for example, glass lacks the regular internal arrangement of atoms and its structure is similar in many ways to that of a liquid. One may therefore regard glass as a supercooled liquid.

Further, we assume that the molecules in a solid are packed closely. This would explain why solids are almost incompressible. Now we know that the density of a substance is mass per unit volume. If the mass of a molecule is m kg and there are n molecules per unit volume (in metre^3), its density would be $m \times n$ kg/m^3.

In a liquid, the molecules are not regularly arranged and are also not fixed in position. They are free to move about and because of this the liquid takes the shape of the vessel which contains it. Since on the melting of a solid, the change in volume is very small, we conclude that the molecules in a liquid are as closely packed as in a solid. This explains why liquids are also nearly incompressible.

In gases, the molecules are not very close to each other, in contrast to the case of liquids and solids. Because of the comparatively large space available to them, the molecules of a gas are free to wander around. They are continuously moving, travelling almost along a straight line till they collide with another molecule or the walls of the container. That is why they occupy the entire container. Further, the gases are easily compressible. This is because the molecules are far apart and when compressed, can be brought closer together as long as the molecular forces remain weak.

Consider ordinary air. Its density is about $1 \cdot 3$ kg/m^3. Liquid air has a density of $9 \cdot 2 \times 10^3$ kg/m^3. Thus air in the gaseous form is about 10^3 times lighter than air in the liquid form. The molecules are the same in both the cases but the density is different. Therefore, in a given volume there are 10^3 times as many molecules in the liquid form as are in the gaseous form. This means that the distance between two molecules of air in the gaseous form is about $(10^3)^{\frac{1}{3}} = 10$ times the distance between two molecules of air in the liquid form. If we assume that in the liquid state the molecules were just touching each other, in the gaseous state the distance between them is ten times their diametres. Because of such a large distance between two molecules in the gaseous form, the molecules hardly exert any force on each other.

We have till now given a description of the molecular theory and the picture of solids, liquids and gases Now the question arises how we know that these assumptions are correct. What the size of one molecule is, what forces there are between molecules which hold them together and what the nature of these forces is. The answers to these questions are difficult. It took over a hundred years to answer these questions satisfactorily and show that the molecular theory indeed provides a good understanding. We will not discuss it in detail here but assume

that the molecular theory is true. We will use molecular theory to explain several points which we have studied up to now.

19.2 The Size of a Molecule

There is one simple experiment which gives an idea of the size of a molecule. Take a dilute solution of oil, *e.g.*, dilute solution of stearic acid in benzene. Put a drop of known volume on the surface of water. You will find that the oil spreads out till it no longer can. But it can only spread till the thickness of the oil is that of one molecule or so. Actual experiments show that 1 cubic millimetre (or $10^{-9}m^3$) oil spreads to an area of 1 square metre Thus the thickness is 10^{-9} metre or 10^{-7} cm Of course the sizes of different molecules can be different, but the order of magnitude of the size of the molecules is 10^{-7} cm.

An idea of the average distance between the molecules of a gas at N T.P. can be obtained as follows. We learn in chemistry that the gram-molecular weight of a gas (the weight of a gas in grams equal to its molecular weight) at N.T.P. occupies a voulme of 22·4 litres. Further, that the number of molecules contained in this volume is $6\cdot025\times10^{23}$, which is called Avagadro's number. Therefore the average space occupied by a molecule is $\frac{22\cdot4\times10^3}{6\cdot025\times10^{23}}$ cc. Assuming this space to be a cube, its edge or the average linear separation between the molecules is $\left(\frac{22\cdot4\times10^3}{6\cdot025\times10^{23}}\right)^{1/3}$ which is of the order of 10^{-7} cm

A similar calculation can be done for solids to get an idea of the linear separation between the molecules (or atoms). Suppose V is the volume in cc occupied by each molecule (or atom). The space occupied by $N=6\cdot025\times10^{23}$ molecules (atoms) will be NV cc. The mass of these N molecules (atoms) will be equal to the molecular (atomic) weight M in grams. Since the density D of this solid will be $D=\frac{M}{NV}$, substituting it for the case of diamond which is carbon (M=12 g and experimentally known value of density D=3·5 g/cc), we get $V=5\cdot5\times10^{-24}$ cc. Again assuming that the space occupied is a cube, we get the linear separation between the molecules is nearly 1 or 2×10^{-8} cm. Thus the molecules in a solid are more closely packed.

In the last few decades, certain big molecules have actually been observed and photographed and their sizes measured by an instrument called electron microscope.

19.3 Properties of Solids

We know now that the molecules in a solid are close together and there is a force of attraction between the molecules which holds them in place.

Since the molecules are very close together it is difficult to stretch or compress a solid, but using a large enough force it can be done. Suppose a piece of steel wire is stretched, it increases in length and the actual increase can be calculated if we know the length of the wire, the stress and the Young's modulus of steel. The molecules change their positions on applying an external force When the force is removed the molecules go back to the original place, provided the external force has not displaced the molecules too much.

It is also difficult to compress a solid substance, say a beam of steel. This means that when two molecules are brought too close together, they repel each other Only an external force can keep the molecules at a distance less than the normal distance. Once the external force is removed the force of repulsion would push the molecules back to the original place and the solid regains its shape.

We may now summarize the nature of the forces in solids. At the normal distance, the force of attraction between the molecules just equals the force of repulsion. If the molecules are brought closer than the equilibrium distance, the force of repulsion increases and tries to repel them back to the equilibrium distance. Lastly if the molecules are too far away, there is almost no force between them.

It is the molecular forces and the molecular arrangement which determine most of the properties of solids and liquids. For example, consider the two forms of carbon: (1) diamond and (2) graphite. Both diamond and graphite have a structure, in which each carbon atom is connected to four other carbon atoms. In diamond if we imagine a carbon atom at the centre of a regular tetrahedron, the other atoms are placed at the corners of the tetrahedron, as shown in Fig 19·1(a). It is clear from the figure that each molecule is held strongly in all directions making diamond the hardest of all substances. In Fig 19.1(a), representing the crystal of diamond, each of the hatched carbon atom is tetrahedrally connected to the surrounding four carbon atoms.

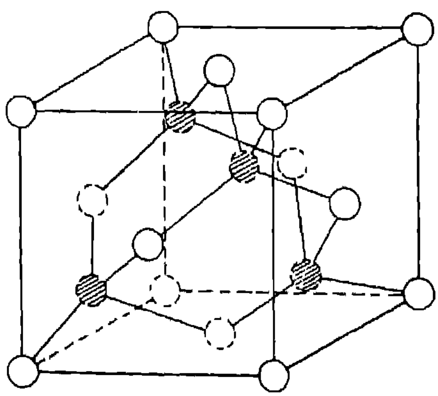

FIG. 19.1(a). *In diamond each carbon atom is tetrahedrally joined to four carbon atoms making diamond the hardest substance known.*

On the other hand, graphite has a structure in which the carbon atoms are placed in layers as shown in the figure. Atoms in each layer are held together strongly, but different layers are held rather loosely because the layers are far apart. The layers can thus slide over one another easily making graphite a very soft substance and a dry lubricant.

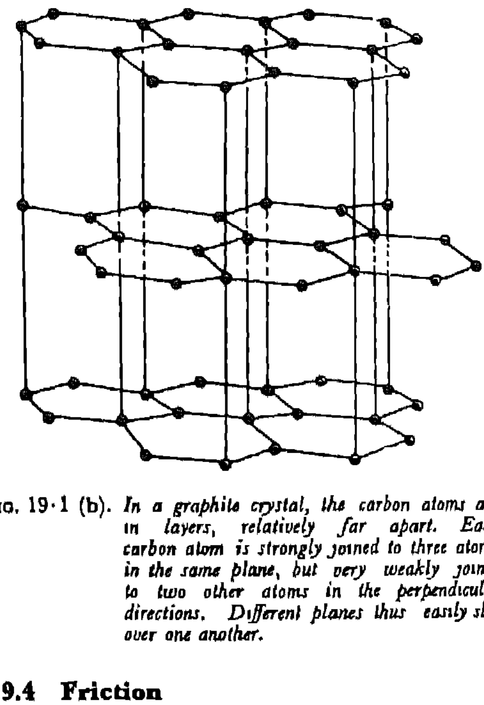

FIG. 19·1 (b). *In a graphite crystal, the carbon atoms are in layers, relatively far apart. Each carbon atom is strongly joined to three atoms in the same plane, but very weakly joined to two other atoms in the perpendicular directions. Different planes thus easily slip over one another.*

19.4 Friction

Friction between two surfaces is partly due to irregularities of the surfaces. A microscopic hill in the surface of one body fits into a valley of the other surface. Thus when the body has to be moved it has to be lifted up a bit. But this is only a part of the cause of friction. The other part arises due to molecular forces. Some of the molecules of one surface come so close to the molecules of the other surface that there is a strong attractive force between them. This force

has to be overcome if the surfaces have to slide over one another.

19.5 Properties of Liquids

In the last chapter we had studied two interesting properties of a liquid : (1) Surface Tension and (2) Viscosity. We shall now explain surface tension from the molecular theory.

19.6 Surface Tension

Consider a liquid say water in a beaker. As already explained all the molecules of the liquid attract each other. Consider a molecule in the interior of the liquid, say at P. It is attracted by molecules from all directions. (Only the forces by the molecules near it are important). All the forces cancel each other However, consider the molecule Q on the surface. All the forces acting on this molecule do not cancel each other, there is a net downward force acting on it. This inward force on every molecule on the

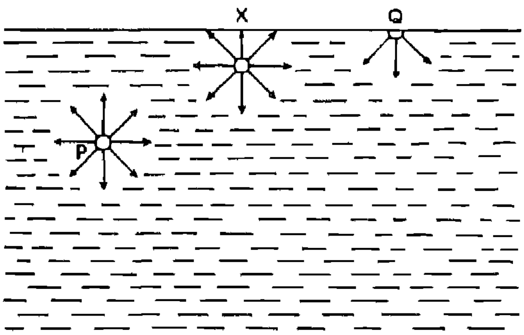

Fig. 19.2. *Molecular forces explain surface tension*

surface makes the surface behave like a stretched membrane giving rise to surface tension.

19.7 Capillary Action

We have seen that if a capillary glass tube is placed in water, water rises in the tube. This is explained as follows .

The molecules of water are attracted by the molecules in glass (such an attractive force between different molecules is called force of adhesion) The attractive force between similar molecules is called the force of cohesion). In the case of a capillary, such an attractive force is sufficient to lift a small column of water. A detailed study of this would require knowledge of the various forces between glass molecules and water molecules, and between water molecules themselves. This would take us far beyond the scope of this book.

19.8 Viscosity

Viscosity, as we saw in the previous chapter, is the tendency to oppose motion of liquid in a tube or to oppose the relative motion between different layers of the liquid. This can again be understood from the molecular point of view. The molecules of different layers of liquid attract each other. The molecules of the walls attract the molecules of the liquid touching the tube. This attraction opposes relative motion giving rise to viscosity.

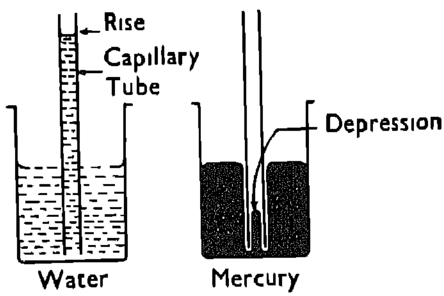

Fig. 19.3. (a) *Strong adhesion pulls the water up inside the tube with a capillary bore.*
(b) *Strong cohesion pulls the mercury down away from the glass.*

19.9 Properties of Gases

We had stated that the inter-molecular forces in gases are very small. This makes

the study of gases easier because several properties can be understood without the knowledge of the molecular forces. The molecular theory of gas is also known as the kinetic theory of gases. We shall briefly discuss this theory and provide an explanation for Boyle's law. We have seen that the average distance between the molecules in a gas is 10^{-7} cm and the diameter of the molecule is of the order of 10^{-8} cm. According to the kinetic theory, the molecules of a gas are moving in random directions. This means that on account of their finite, though small, size they will collide with one another. The molecules are continually colliding and changing the direction of their paths. The distance travelled by a molecule between two collisions is known as its free path. However, the distance travelled by any two molecules between two collisions is different for any two collisions and it is also different for different molecules. The average distance which the molecules travel between two collisions is called the mean free path. At ordinary temperature and pressure this distance is of the order of 10^{-5} cm We thus get the following picture of a gas. An enormous number of molecules ($N = 3 \times 10^{22}$ molecules per litre) are continuously moving. In their movement they collide with one another as well as with the walls of the vessel. Now suppose we decrease the number of molecules by evacuating the container with the help of a vacuum pump. This means that the molecules have more room and so the mean free path increases. At a pressure of 10^{-4} mm of mercury, it can be calculated that the molecules will have to travel, on an average, a distance of 10 cm before colliding with another molecule. Suppose now that the vessel in which the gas is contained is a cube of side 1 cm. The molecules would then collide more with the walls than among themselves.

19.10 Boyle's Law

We shall first understand Boyle's law in the case of such a low pressure that the mean free path of molecules is much larger than the dimensions of the vessel. This helps us in developing a simple theory where the collisions between molecules can be totally ignored.

Consider a gas enclosed in a cubic vessel of side 'a'. We will take the three edges of the cube meeting at a point as the three co-ordinate axes of X, Y and Z. The molecules of the gas keep hitting the walls of the vessel and therefore the walls experience a force This force per unit area, acting on the wall, is what we call pressure. We assume an ideal situation.

(1) There is no force of attraction between molecules of the gas

(2) There is no loss of energy in the collision between walls and molecules.

Let the vessel contain N molecules each of mass m. To derive an expression for the pressure exerted on the walls, we will make the simplifying assumption that, at any moment, exactly one third of the molecules are moving in the direction of the X-axis, one third along the Y-axis and one third along the Z-axis The wall is much heavier than the molecule and if the molecule hitting the wall perpendicular to the X-axis has the velocity v, it will rebound with the same velocity since the wall hardly moves and so does not take any energy from the molecule. From conservation of energy it means that the molecule bounces back exactly with the same velocity, (k.E. $= \frac{1}{2}$ mv^2). This means that the molecule goes to and fro with the same velocity. Thus it hits one of the opposite walls every 2a/v seconds or the number of collisions per second with that wall is $\frac{v}{2a}$.

At each collision, a molecule transfers a momentum of 2mv to the wall, since

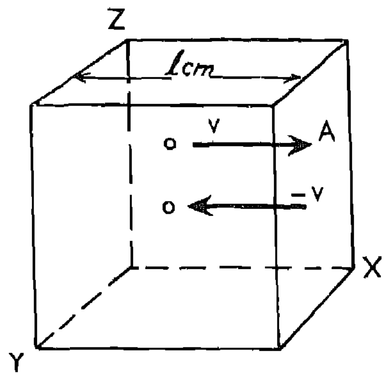

FIG. 19.4 *Pressure in a gas is related to molecular motion.*

the momentum before the collision is mv and after collision it is $-mv$. Thus the total momentum transferred to the wall per unit time by a molecule is $2mv \times \frac{v}{2a} = \frac{mv^2}{a}$. From Newton's second law, this is the force acting on the wall, due to one molecule The total force is obtained by considering all the molecules. Hence pressure $= \frac{\text{Total force}}{\text{area}}$

$= \frac{m}{3a^3}(v_1^2+v_2^2+ \ldots\ldots +v_N^2)$, where v_1, v_2...... refer to the velocities of the N molecules. The factor 3 in the denominator arises from the fact that we have assumed that one third of the molecules are moving in the direction of X-axis

or, pressure $= \frac{mN\bar{v}^2}{3a^3}$, where

$\bar{v}^2 = \frac{1}{N}(v_1^2+v_2^2+\ldots\ldots v^2_n)$ and is called the mean square velocity of the molecules.

This is the pressure on the wall perpendicular to the X-axis. But the pressure along the other two directions is just the same as we have assumed that exactly one third of the molecules are moving along each axis. A more rigorous discussion without making this simplifying assumption gives the same result.

Thus,

$$P = \frac{mN\bar{v}^2}{3V},$$

where $V=a^3$=volume occupied by the gas.

Or, $PV = \frac{1}{3}\, m N\bar{v}^2 = \frac{1}{3} Mv^2$

$= \frac{2}{3}$ K.E of the gas.

If the gas does not gain energy from outside, *i.e.*, if the temperature remains constant and the kinetic energy of the gas does not change, we have PV=constant, which is Boyle's law The expression for pressure can be written, $P=\frac{1}{3}\frac{M}{V}\bar{v}^2 = \frac{1}{3}\, dv^2$, where d is the density of the gas. The last formula gives a way of estimating $(\bar{v}^2)^{\frac{1}{2}}$, the root mean-square velocity of the molecules of a gas. We have $\bar{v} = (v^2)^{\frac{1}{2}} = \left(\frac{3P}{d}\right)^{\frac{1}{2}}$.

For 0°C, using tables one can find that for air, $\bar{v}$= 484·8 m/sec which is roughly the velocity of a bullet from a high powered rifle

It can be shown that the formula for the pressure of a gas remains the same even if we take account of collisions between molecules

We have discussed here the kinetic theory of gases from an elementary point of view. In the last century, Maxwell and Boltzmann gave a detailed mathematical theory in order to understand several phenomena such as diffusion, viscosity, Charles' law etc. The great amount of research, both experimental and theoretical, has put the molecular kinetic theory of gases on a quantitative basis. This theory forms one of the foundation stones of modern physics.

We have assumed above that the molecules have different velocities, *i.e.*, some of the molecules have a velocity much higher than the average while others have a velocity much lower than the average. Fig. 19 5 gives a graph of the distribution of molecular velocities in oxygen at two different

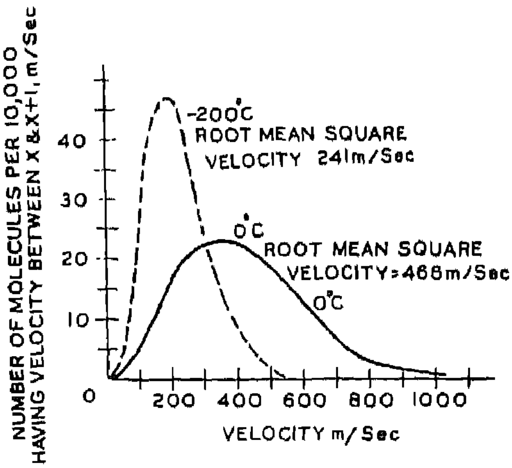

FIG. 19.5. *The Maxwell-Boltzmann distribution curve for oxygen at two different temperatures.*

temperatures The theoretical expression for this distribution was first derived by the British physicist Maxwell and is known as *Maxwell's distribution law.* It has been experimentally verified by the German physicist Stren.

Questions

1. The mass of an atom of hydrogen is $1 \cdot 66 \times 10^{-24}$ g. Calculate the number of atoms in one gram of hydrogen. Is this the same as Avogadro's number ?
2. The density of sodium chloride is $2 \cdot 165$ g/cc. Its molecular weight is $58 \cdot 5$. Assuming that the sodium and chlorine ions occupy the corners of a cube in alternate positions, calculate the distance between them.
3. The density of bromine is $3 \cdot 12$ g/cc and its molecular weight is 160. Calculate the average distance between the molecules.
4. The density of hydrogen is $0 \cdot 0893$ kg/cu m. If 1 kg of hydrogen occupies 1000 cubic metres, calculate the average distance between the centres of hydrogen molecules.
5. At a height of 200 km the density of air is 10^{-10} kg/cu m. Calculate the number of molecules per cc.
6. Calculate the K.E. of 10 grams of hydrogen at N.T.P.
7. Calculate the average kinetic energy of the molecule of a gas at a temperature of 300° C.
8. On a graph paper draw the graph of the velocity distribution as given in Fig. 19.5. Find the total area under the graph. This is proportional to the total number of molecules. Now determine the area lying between the abscissa, $\frac{1}{2}$ V_m and 2 V_m where V_m is the velocity corresponding to the mean value on the graph. What fraction is this area of the total area ? What do you conclude from this ?

9 Draw a graph between the number of students in your class and the marks obtained by them in physics at the last examination. Is this graph smooth? Would the graph become smoother as the number of students increases?

10. Assuming that the diameter of the molecule of oxygen is 3×10^{-11}m and its average speed 5×10^{2} m/sec, calculate the number of collisions suffered by an oxygen molecule per second if the number of oxygen molecules is $2{\cdot}69 \times 10^{25}$ per cubic metre.

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CHAPTER XX

Gravitation

20.1 Introduction

If you look on any evening towards the north-eastern horizon in the month of March, you will observe a group of seven stars shown in Fig. 20.1. The ancients imagined them to form the animal bear and called it the Great Bear. If you look at these groups of stars day after day, you will find that they do not change their positions with respect to each other or with respect to other stars although they rise on any day a little earlier than on the previous day and during summer evenings will be visible in the northern sky. Two of their group of stars always point towards the north pole. If you similarly observe other groups of stars, you will notice that they also do not change their positions amongst themselves although every night they seem to rotate from east to west.

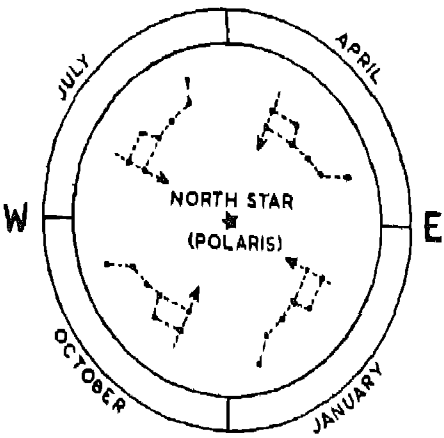

Fig. 20 1 *The position of the Great Bear in the evening sky in different seasons*

On the other hand if you observe the moon, you will notice that the new moon rises in the western sky and on succeeding days is observed to have moved a little to the east, *i.e.*, its position amongst the stars changes. Similarly there are other heavenly bodies called planets which also move amongst the stars. Two of these, the Jupiter and the Venus, are bright objects and can be easily recognized. Three others, the Mars, the Saturn and the Mercury can be observed with some effort. The three other planets, the Uranus, the Neptune and the Pluto, can only be observed with a telescope.

20.2 Kepler's Laws of Planetary Motion

The five planets visible to the naked eye, as well as the sun and the moon, were considered by the ancients to be planets and they tried to explain their motion amongst the stars as arising from their motion round the earth in circular orbits. However, it was found that the motion of the planets, except that of the sun and the moon, was quite irregular. In addition to their regular motion from west to east amongst the stars, it was found that they have a retrograde motion, *i e*, instead of moving from west to east, they stop in their path, turn round and begin to move from east to west After sometime they would again stop and begin their regular motion, thus forming a loop

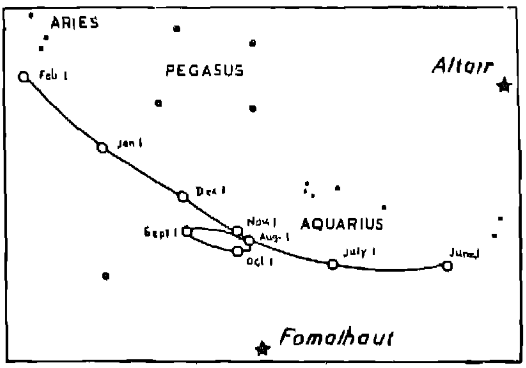

Fig. 20.2. *The retrograde motion of Mars, 1939*

Many attempts were made by the ancient astronomers to explain this irregular motion. Ptolemy constructed a whole system of epicycles in which the planet was assumed to revolve round a small circle the centre of which was moving on another circle round the earth. Although an elaborate system of epicycles was assumed and the motion of the planets became quite complicated, it was found that the predicted positions of the planets did not agree with their observed positions.

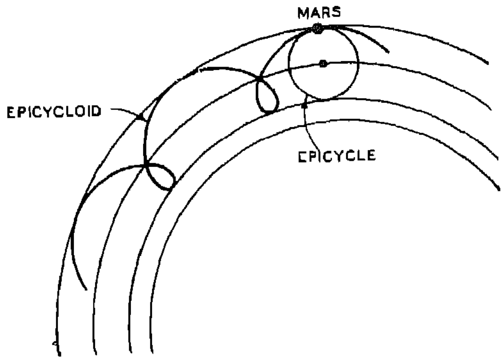

Fig. 20 3 *Ptolemy's explanation of retrograde motion of Mars and other planets*

The number of epicycles was slightly reduced when the Polish astronomer Copernicus revived the ancient theory of Aristarchus that the earth and other planets went round the sun in circular orbits The problem was finally solved by Kepler on the basis of the very accurate observations of Tycho Brahe, the Danish astronomer. Brahe had developed great sextants and compasses which he used for very accurate determination of the position of the planets amongst the stars. His observations were accurate to 1′ of arc which was far in advance of any thing known previously and has only been surpassed after the use of the telescope. Kepler who was an assistant of Tycho Brahe was assigned by him the problem of explaining the motion of the planet Mars. After making various unsuccessful attempts to explain the motion of Mars on the basis of circular orbits, he finally hit upon the idea that the planets move round the sun in elliptical orbits.

In 1609 Kepler formulated his first two laws of planetary motion which are as follows :

1 Each planet moves around the sun in an ellipse with the sun at one focus (The law of orbits).

2. The radius vector from the sun to the planet sweeps out equal areas in equal intervals of time (The law of areas).

Finally Kepler set himself the problem of discovering a connection between the sizes of the planetary orbits and their periods, *i.e.* the time that they take to go around the sun once. After ten years of strenuous work, he discovered what is now known as Kepler's third law which can be stated as follows :

3. The squares of the periods of any two planets are proportional to the cubes of the semi-major axis of their respective orbits.

$\frac{T}{a^{3/2}}$=constant (The law of periods).

The semi-major axis is half the greatest radius of the ellipse. Since the orbits are very nearly circular, we can make the simple statement that the time for a planet to go round the sun will be proportional to the 3/2 power of the diameter of the orbit. This is illustrated in the following table.

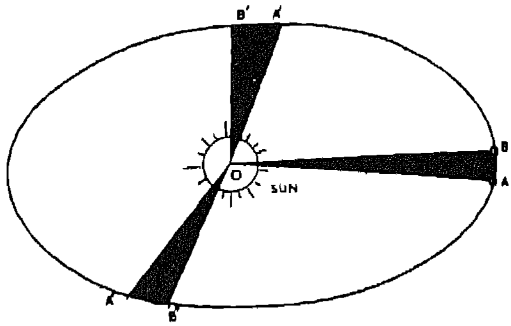

Fig. 20.4 *Kepler's equal areas law*

to the motion of the planets around the sun but also to the motion of the moon around the earth.

20.3 Law of Universal Gravitation

According to Newton, he was led to the correct law of force between the sun and the planets by working backwards from Kepler's third law of motion. We have seen in chapter X, that a body moving in a

KEPLER'S THIRD LAW

Planet	*Radius R of Orbit of planet in A.U.*	*Period T in days*	a^3/T^2 $(A.U.)^3/(Day)^2$	*Modern values* a^3/T^2 ; m^3/sec^2
Mercury	0·389	87 77	$7\cdot64\times10^{-6}$	$3\cdot354\times10^{18}$
Venus	0·724	224·70	7·52	3·352
Earth	1·000	365·25	7 50	3·354
Mars	1·524	686·90	7·50	3·354
Jupiter	5·200	4,332·62	7·490	3·355
Saturn	9·510	10,759·20	7·430	3·353

Jeremiah Horrocks, who was born in the year 1619 and died at the young age of 21 before the birth of Newton, proved in 1639 that Kepler's first law applied not only circular orbit with constant speed has an acceleration equal to V^2/R directed towards the centre. Since $T=\frac{2\pi R}{V}$,

the acceleration 'a' can be expressed as $a=\frac{4\pi^2R}{T^2}$ and the centripetal force acting on the body is given by $F=ma=\frac{m4\pi^2R}{T^2}$ ....(i)

According to Kepler's third law R^3/T^2 = constant = K, *i.e*, $1/T^2=K/R^3$. Substituting this value of $1/T^2$ in the above expression for F, we have

$$F=4\pi^2K\frac{m}{R^2}\cdots\cdots(ii)$$

The force is proportional to the mass of the planet and inversely proportional to the square of the distance of the planet from the sun.

Finally, Newton showed that any object moving under a central law of force varying inversely as the square of the distance from a point will describe an elliptical orbit and the radius vector will sweep equal areas in equal intervals of time which are Kepler's first and second laws respectively.

The constant $4\pi^2K$ in the expression for force is the same for all planets and must be characteristic of the sun alone. Newton applied the same ideas to the motion of the moon around the earth. The force between the earth and a body of mass m can be written as

$$F=\frac{4\pi^2K_em}{R^2}\cdots\cdots\cdots\cdots(iii)$$

where K_e is a characteristic constant for the earth and R is the distance of the body from the centre of the earth. On the surface of the earth, R = Radius of the earth = R_e and for the moon $R=R_m$ = distance of the moon from the earth. Hence

$$g=\frac{4\pi^2K_e}{R_e^2}\text{ and }a_m=\frac{4\pi^2K_e}{R^2_m}\cdots(iv)$$

where a_m is the acceleration of the moon. Therefore,

$$\frac{a_m}{g}=\frac{R^2_e}{R^2_m}\cdots\cdots\cdots\cdots(v)$$

Since the distance of the moon from the earth is approximately equal to 60 earth radii, we have

$$a_m=\frac{g}{60^2}=\frac{9\cdot8}{60^2}=2\cdot72\times10^{-3}\text{m/sec}^2\cdots(vi)$$

Also knowing the distance of the moon (384, 000 km) and the time which the moon takes to go around the earth ($27\frac{1}{3}$ days), we get

$$a_m=\frac{4\pi^2R_m}{T^2_m}=\frac{4\pi^2\times384{,}000\times10^3\times3\times3}{(82\times24\times60\times60)^2}$$
$$=2\cdot72\times10^{-3}\text{m/sec}^2\;\ldots\qquad\ldots\;..\;(vii)$$

These two values of a_m which Newton deduced in two different ways were found to be the same and he concluded that the motion of the moon is governed by the earth according to the same law which governs the motion of a falling stone. Finally, he thought about the property of the body which determines the constant $4\pi^2k_s$ for the sun and the constant $4\pi^2k_e$ for the earth. The simplest assumption is that the force is proportional to the mass of the sun and the earth respectively as it is for planets and the moon Thus $4\pi^2K_s=GM_s$ and $4\pi^2K_e=GM_e$, where G is a universal constant

Newton made this assumption so that the gravitational force of attraction between two bodies of masses m_1 and m_2 separated by a distance R can be stated as

$$F=G\frac{m_1\,m_2}{R^2}\quad\ldots\ldots\ldots\ldots\;(viii)$$

This is Newton's law of Universal Gravitation which is stated as follows:

Every particle of matter in this universe attracts every other particle of matter with a force which varies directly as the product of their masses and inversely as the square of the distance between them

The value of the universal constant of gravitation has been determined experimentally in the laboratory and is found to be

$$G=6{\cdot}67\times10^{-11}\;\frac{\text{newton}\times\text{metre}^2}{\text{Kg}^2}$$

We should note that equation (viii) expresses the force between mass particles having an infinitesimal size. The force between two bodies of finite size must be computed by regarding them as composed of particles and adding vectorially the attractive forces between these individual particles. It is not correct to apply equation (viii) and take for 'r' the distance between the centres of the two bodies and the masses of bodies for m_1 and m_2. Newton was well aware of this point and there is some evidence to indicate that he had invented calculus for attacking this type of problems

For spherical bodies of uniform density it can be proved that the gravitational force can be obtained by taking its entire mass to be concentrated at the centre. So the gravitational force between two separate spheres having a spherically symmetrical distribution of mass will be equal to the product of the masses divided by the square of the distance between the centres of the two spheres. That is why the simple inverse square law of gravitation can be applied to astronomical bodies

20.4 Determination of the Gravitational Constant

The gravitational force between bodies of ordinary sizes is extremely small. Hence an extraordinarily sensitive apparatus should be used to measure the amount of force and calculate the value of G from equation (viii). Such an experiment was performed by Henry Cavendish in 1798 using a type of balance known after him. Fig. 20·5 shows the arrangement. m and m are two small masses. They are placed at the two ends of a T-shaped light rod. At M the light rod is suspended from a fine quartz thread carrying a mirror. After a time the system would be in equilibrium. Now two larger lead spheres m′ and m′ were introduced.

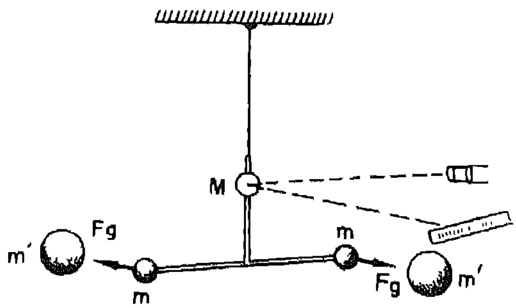

FIG. 20.5. *Principle of Cavendish's balance.*

The attraction between m and m′ causes the rod to rotate slightly towards the larger masses. The twist in the wire is measured by a telescope and scale arrangement. This gives an idea of the gravitational force between the masses. G could now be calculated.

Cavendish's balance experiment gave a value for G. He is also said to have 'weighed the earth'. The term is unscientific because the earth cannot have a weight in the same sense as the weight of a body determined by the spring balance. What is meant by this expression is that he determined the mass of the earth Suppose the earth has a mass M_e and radius R. Its force on a small mass m on its surface is then given by

$$\frac{GM_e m}{R^2}.$$

Now this should be equal to the weight of the body mg.

$$\text{Therefore, } \frac{GM_e m}{R^2} = mg.$$

This shows that whatever m, the mass of the attracted body be, the acceleration is the same. If we know G, R and g, the mass of the earth, M_e can be evaluated from the equation above.

$$M_e = (g/G)R^2$$

and it comes out to be $5 \cdot 97 \times 10^{24}$kg.

20.5 Gravitational Potential

Let us take the case of two bodies of masses M and m and try to see what would be the work needed to separate them from each other against gravitational attraction. We will compute the work needed to move 'm' from a point 'a' to a point 'b' (Fig. 20·6)

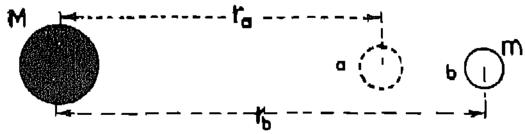

Fig 20 6

The gravitational force when m is at a distance of r_a will be $\frac{GMm}{r_a^2}$. At b it will be $\frac{GMm}{r_b^2}$ If we take the average of forces at 'a' and at 'b' and multiply it by the distance $(r_b - r_a)$, then we get the work done in moving m from a to b against the gravitational attraction There are two ways of taking averages. One is the arithmetic mean and the other is the geometric mean. Calculus shows that it is the geometric mean in which we are interested here

The geometric mean of the forces between points a and b, which we call F_{ab} is

$$\sqrt{\frac{GMm}{r_a^2} \quad \frac{GMm}{r_b^2}} = \frac{GMm}{r_a r_b}.$$

The work done is this force multiplied by the distance moved.

$$W = \mathbf{F}_{ab}\ (r_b - r_a)$$
$$= \frac{GMm}{r_a\ r_b}\ (r_b - r_a) = GM\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

Now what is the work done in moving m to an infinite distance from 'a'? This is obtained when we put $r_b = \infty$.

Thus,

$$W_{a\infty} = \frac{GMm}{r_a}.$$

This is the amount of work which has to be done to move the body of mass m from the point 'a' to infinity If the body now comes from infinity to the point 'a', it is capable of doing this amount of work, *i.e.*, the body possesses more energy at infinity by virtue of its position. Or the body possesses more potential energy at infinity. If we take the potential energy at infinity to be zero the potential energy of the body at the point 'a' will be $-\frac{GMm}{r_a}$. The potential energy per unit mass at any point is the potential due to the mass at the point. Hence the potential at the point 'a' is $-\frac{GM}{r_a}$. This is called the gravitational potential as it arises on account of the gravitational attraction of the mass M

20.6 Escape Velocity

Escape velocity is the minimum velocity required for a projectile in order that it may break through the earth's gravitational attraction to escape into outer space. It is about 11 km per second. Any projectile should be given this much of initial velocity if it has to leave the earth Or else it will stop and retrace its path towards the earth. Since the earth's gravitational field extends to infinite distances, however small it may be, the escape velocity is the velocity required to move the projectile to infinity.

From the previous section we know that the work required to move a body from the earth's surface (*i.e.*, a distance of r_e from the centre of the earth) to infinity is

$$W = \frac{GMm}{r_e}.$$

If a projectile pointing away from the earth is given the above amount of kinetic energy, it will have reached an infinite distance before its K.E has been converted into potential energy; in other words, it will have

the escape velocity. Hence, if v is the escape velocity,

$$\frac{GMm}{r_e} = \frac{1}{2} mv^2,$$

$$\text{or, } v^2 = \frac{2GM}{r_e} = 2r_e g,$$

$$\text{or, } v = \sqrt{2r_e g} = 11 \cdot 3 \text{ Km/sec}$$

The fact that m cancels out in the above expression shows that the escape velocity is the same, whatever may be the mass of the projectile The speed of a satellite in its orbit, whether it be moon or sputnik, must obviously be less than the escape velocity or it would fly into infinite space As we go away from the earth the gravitational pull of the earth decreases. That is the weight of the bodies decreases. This pull decreases as $1/r^2$, where r is the distance of the body from the centre of the earth.

20.7 Einstein's Work on Gravitation

Einstein in 1916 formulated his general theory of relativity which gave a new explanation of gravitation. He has attempted to explain gravitational attraction as a natural consequence of the property of space which is curved.

Example :—Assuming that a satellite moves in a circular orbit round the earth at a distance of 600 Km from the surface of the earth, calculate its (a) velocity and (b) the time of one revolution when the radius of the earth is 6400 Km and $g = 9 \cdot 8$ m/sec^2.

If V is the velocity at a distance R from the centre of the earth,

$$\frac{V^2}{R} = \frac{GM}{R^2}.$$

$$\text{Also } \frac{GM}{R_e^2} = g,$$

$$GM = g R_e^2.$$

$$\text{Hence } V^2 = \frac{GM}{R} = \frac{gR_e^2}{R},$$

$$\text{or, } V = R_e \left(\frac{g}{R}\right)^{1/2},$$

$$= 6 \cdot 4 \times 10^6 \text{m} \left(\frac{9 \cdot 8 \text{m/sec}^2}{7 \times 10^6 \text{m}}\right)^{1/2},$$

$$= 6 \cdot 4 \times 10^3 \left(\frac{9.8}{7}\right)^{1/2} \text{m/sec},$$

$$= 7.6 \times 10^3 \text{m/sec}.$$

$$\text{Also } T = \frac{2\pi R}{V},$$

$$= \frac{2 \times 3.1416 \times 7 \times 10^6 \text{m}}{7 \cdot 6 \times 10^3 \text{m/sec}},$$

$$= 5787 \text{ sec},$$

$$= 96.45 \text{ minutes}.$$

Questions

1. A small mass is held on a frictionless table at a distance of one metre from the centre of a sphere of mass 100 kg. Calculate its acceleration when it is released. What will be the acceleration if the distance is reduced to 50 cm ?
2. Calculate the distance from the earth of a satellite whose period of revolution is 24 hours. What will be its velocity ?
3. Taking the distance of the earth from the sun equal to 150 million km and the time of revolution equal to $3 \cdot 15 \times 10^7$ seconds, calculate the mass of the sun.
4. Taking the mass of the sun equal to 2×10^{30} kg and the period of revolution of Jupiter to be $3 \cdot 74 \times 10^8$ seconds, calculate the distance of Jupiter from the sun
5. Assuming the distance of Venus from the sun to be equal to $1 \cdot 08 \times 10^8$ km, calculate the time of revolution of Venus around the sun.
6. The distance of one of the satellites of Jupiter is $20 \cdot 8 \times 10^6$ km and its time of revolution around the planet 617 days. Calculate the mass of Jupiter.

Further Reading

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